NON-DATA AIDED ADAPTIVE CHANNEL SHORTENING FOR EFFICIENT MULTI-CARRIER SYSTEMS

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ABSTRACT

Multi-carrier (MC) systems are known for being very effective to cancel intersymbol and multi-user interference provided that a cyclic prefix (CP) of length at least equal to the channel order is inserted at the beginning of each transmitted block. However, this insertion causes a reduction of the transmission speed which is not negligible when the channel delay spread is not small with respect to the block-length. In this paper we propose an algorithm for designing time-domain pre-equalizers which shorten the channel impulse response. The method does not require neither the a-priori knowledge of the channel impulse response nor the transmission of training sequences. The only basic assumption underlying our method is that the transmitted MC stream contains null (virtual) sub-carriers, a condition which is verified in many current MC transmission systems.

1. INTRODUCTION

Multi-carrier systems are commonly adopted in broadband communications as very effective tools to compensate for the time dispersion encountered in wireless channels affected by multipath or in subscriber lines [8]. MC systems simplify the channel equalization task considerably, as they convert a time dispersive channel onto a set of flat fading sub-channels whose equalization is obtained with a simple multiplication. This property is achieved by using block transmissions and inserting a cyclic prefix (CP) of length at least equal to the channel order, at the beginning of each block so as to convert the linear convolution operated by the channel into a circular convolution. The only price paid for inserting the CP is the reduction of the transmission data rate. Specifically, if N is the number of symbols multiplexed in each block and L is the cyclic prefix length, the insertion of the prefix implies a reduction of the transmission rate by a factor $\epsilon = N/(N+L)$. This efficiency loss is not negligible in applications where L is big and N cannot be chosen too large because of constraints on decoding delay or on receiver complexity.

To limit this loss without increasing the receiver complexity excessively, it is common practice to insert, at the receiver side, a pre-equalizer, or channel shortener, whose task is to concentrate the energy of the impulse response of the combined channel/pre-equalizer response within a window smaller than the initial channel support. Ideally, if the channel shortener yields a final combined response of order L_S (with $L_S < L$), it is sufficient to use cyclic prefixes of order L_S, instead of L, and still achieve perfect ISI/MUI elimination with a simple FFT processor. Several works have already addressed this problem finding alternative solution for the design of the pre-equalizer [2], [4], [7], [3], [1]. In all these works, the channel was supposed known or estimated by transmitting known preamble sequences. Furthermore, the overall channel was single-input/single output (SISO). In such a case, using an FIR pre-equalizer we can at most concentrate the energy of the combined channel/pre-equalizer response within the desired window, but we cannot ISI completely. Perfect channel shortening using FIR filters is only possible if the channel is single input/multiple output (SIMO), as already suggested in [5]. In such a case, it is possible to find a *bank* of FIR filters able to squeeze *all* the energy of the final impulse response within the desired window. In a wireless scenario, SIMO structures are obtained by using multiple receive antennas, whereas in a wired link one can resort to fractional sampling. The design of perfect pre-equalizers for OFDM/DMT systems in SIMO structures was considered in [5], under the hypothesis that the channel was a-priori known or estimated by transmitting known sequences. In this work, we propose an adaptive strategy for designing a SIMO pre-equalizer, capable of perfect ISI/IBI cancellation, which does not require neither the knowledge nor the estimate of the channel impulse response and it is able to derive the impulse response of the pre-equalizers *directly* from the data, without requiring the transmission of training sequences. The proposed approach exploits the presence of null (virtual) sub-carriers. Clearly, transmitting virtual sub-carriers is equivalent to transmit null, and thus known, symbols. However, differently from training symbols, transmitting virtual sub-carriers does not imply any waste of power and is, in any case, implicit in many transmission systems such as, for example HIPERLAN/2, where virtual sub-carriers are introduced for synchronization purposes, or in multi-user OFDM systems where some sub-carriers are not used when the system is not fully loaded, or, finally, in ADSL modems, where, because of bit loading made according to the water-filling principle, the most faded sub-carriers are not used.

2. TRANSCEIVER STRUCTURE

In this paper we refer to a single user scenario, but the same formulation can be extended to the broadcast (downlink) channel of a multi-user context.

We denote by $s(p;m)$, with $m = 0, \ldots, M-1$, the m-th information symbol transmitted with the p-th block. We assume that our system may incorporate frequency hopping (FH). We will show in Section 5 that FH facilitates the convergence of our adaptive shortening method. Each transmitted block is then composed

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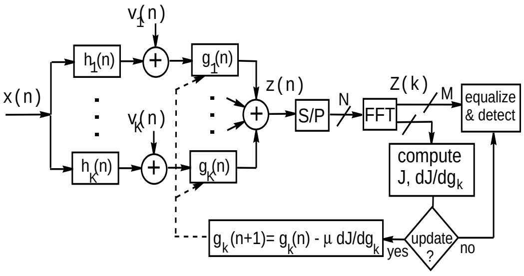

Fig. 1. Receiver block diagram.

of the following $N+L$ samples

$$x(p;i)=\sum_{m=0}^{M-1} s(p;m)e^{j\frac{2\pi}{N}(i_m+i_p)i} \quad ,i\in[-L,N-1], \tag{1}$$

where i_m is the frequency index associated to the m-th symbol and i_p is the (possible) frequency hopping index associated to the p-th block. Because of the periodicity of order N of the exponentials in (1), the first L samples in (1) are equal to the last L sample and constitute the cyclic prefix. We assume that the indices i_m, $m=0,\dots,M-1$, are all different from each other (i.e. different symbols are associated to different sub-carriers) and that $M<N$, i.e. there exists a set of $N-M$ null (virtual) sub-carriers which are not used for transmitting information symbols. We denote by $\mathcal{S}_M$ the set of indices not used in (1), i.e. $\mathcal{S}_M$ is such that $\mathcal{S}_M \cup \{i_0, i_1, \dots, i_{M-1}\} \equiv \{0,1,\dots,N-1\}$.

In Fig. 1 we report the receiver structure, composed of K parallel channels and pre-equalizers, having impulse responses $h_k(n)$ and $g_k(n)$, with $k=1,\dots,K$, respectively. We denote with L_h and L_g the maximum order of the channel and pre-equalizer filters, respectively and we assume that

A1) $L_h+L_g-1<N$.

This assumption guarantees that interblock interference involves no more than two consecutive blocks. Under **A1**), the q-th received block is composed of the following entries

$$\begin{aligned} y_k(q;n) &= \sum_{i=n+L+1}^{L_h} h_k(i)x(q-1;n+N+L-i) \\ &+ \sum_{i=0}^{n+L} h_k(i)x(q;n-i) \quad ,n\in[-L,L_h-L-1], \\ y_k(q;n) &= \sum_{i=0}^{L_h} h_k(i)x(q;n-i) \quad ,n\in[L_h-L,N-1]. \end{aligned} \tag{2}$$

The first term in the first of these equations represents the interference on the q-th block from the previous one. Clearly, removing the first L samples of each block, we eliminate IBI only if $L_h \geq L$; otherwise, there will be IBI. The stream of data received from the k-channel is

$$y_k(n)=\sum_{p=-\infty}^{\infty}\sum_{i=-L}^{N-1} y(p;i)\delta(n-p(N+L)-i)$$

and thus the sequence $z(n)$ at the output of the pre-equalizer filterbank is

$$z(n)=\sum_{k=1}^{K}\sum_{l=0}^{L_g} g_k(l)[y_k(n-l)+v_k(n-l)], \tag{3}$$

where $v_k(n)$ is the additive noise in the k-th channel.

3. CONDITIONS FOR PERFECT EQUALIZATION

Denoting with $H_k(z)$ and $G_k(z)$ the $\mathcal{Z}$-domain transfer function of the k-th channel and filter, respectively, perfect ISI/IBI cancellation is achieved if the combined channel-equalizer transfer function, corresponding to the system of Fig. 1, satisfies the relationship

$$C(z)=\sum_{k=1}^{K} H_k(z)G_k(z)=z^{-d}P(z), \tag{4}$$

where d is a possible delay and $P(z)$ is a polynomial of degree L.

In the SISO case, corresponding to $K=1$, the equation $H(z)G(z)=z^{-d}P(z)$ can be satisfied by using an FIR equalizer, only if the channel is ARMA. However, ARMA modeling may be too critical to adopt in a practical context becuse of possible instabilities and we will not consider this case.

Let us then consider a proper SIMO structure, as in Fig. 1, with $K>1$. In this case, we will show that *perfect* channel shortening is possible if the K channels satisfy a generalized disparity condition, which is an extension of the one required in [6]. To derive the conditions for perfect shortening, it is better to express (4) in matrix form. We start with $K=2$, for simplicity, and then we will extend the formulation to the most general case. Our notation is the following: $\boldsymbol{h}_1=[h_1(0),h_1(1),\dots,h_1(L_h)]^T$ and $\boldsymbol{h}_2=[h_2(0),h_2(1),\dots,h_2(L_h)]^T$ are the channel vectors; $\boldsymbol{g}_1=[g_1(0),g_1(1),\dots,g_1(L_g)]^T$ and $\boldsymbol{g}_2=[g_2(0),g_2(1),\dots,g_2(L_g)]^T$ represent the pre-equalizers vectors; the vector $\boldsymbol{p}=(p(0),p(1),\dots,p(L))$ is formed with the coefficients of $P(z)$. We also introduce the $(L_h+L_g+1)\times(L+1)$ matrix $\boldsymbol{T}^T := (\boldsymbol{0}^T_{d,L+1}, \boldsymbol{I}_{L+1,L+1}, \boldsymbol{0}^T_{L_h+L_g-L-d,L+1})^T$ Then we build the $(L_h+L_g+1)\times(L_g+1)$ Toeplitz matrices $\mathcal{T}(\boldsymbol{h}_1)$ and $\mathcal{T}(\boldsymbol{h}_2)$, where $\mathcal{T}(\boldsymbol{h}_k)$ has first column $[\boldsymbol{h}_k^T, \boldsymbol{0}^T]^T$ and first row $[h_k(0), \boldsymbol{0}^T]^T$, with $k=1,2$. We are now able to formulate (4) in matrix form as follows

$$\begin{bmatrix} \mathcal{T}(\boldsymbol{h}_1), & \mathcal{T}(\boldsymbol{h}_2), & -\boldsymbol{T} \end{bmatrix} \begin{pmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \\ \boldsymbol{p} \end{pmatrix} = \boldsymbol{0}. \tag{5}$$

We wish to find now the conditions under which (5) admits a non null solution for $\boldsymbol{g}_1$, $\boldsymbol{g}_2$, and $\boldsymbol{p}$. To enforce $\boldsymbol{p}\neq\boldsymbol{0}$, we set one of its elements, let us say its δ-th one, to be equal to one so that $\boldsymbol{p}$ has the form $\boldsymbol{p}:=[\boldsymbol{p}_1^T,1,\boldsymbol{p}_2^T]^T$. Thus, (5) becomes

$$\begin{bmatrix} \mathcal{T}(\boldsymbol{h}_1), & \mathcal{T}(\boldsymbol{h}_2), & -\bar{\boldsymbol{T}} \end{bmatrix} \begin{pmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \\ \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{0} \\ 1 \\ \boldsymbol{0} \end{pmatrix}, \tag{6}$$

where $\bar{\boldsymbol{T}}$ is equal to $\boldsymbol{T}$ deprived of its δ-th column. We are now able to prove the following

Theorem: A set of sufficient conditions for the existence of a non-trivial solution of (6) is that:

A2) $L_g > L_h - L - 1$;

A3) the two polynomials $H_1(z)$ and $H_2(z)$ do not share more than

L common zeros.

Proof: The system (6) admits a non trivial solution if the matrix $\boldsymbol{B} := \quad \mathcal{T}(\boldsymbol{h}_1), \quad \mathcal{T}(\boldsymbol{h}_2), \quad -\bar{\boldsymbol{T}}$ is fat and full row rank. Condition **A2**) insures that $\boldsymbol{B}$ is fat. Let us examine now when $\boldsymbol{B}$ is full row rank. We wish to prove that, under **A2**), **A3**), there is no $(L_h + L_g + 1)$-size vector $\boldsymbol{r} \neq \boldsymbol{0}$ such that $\boldsymbol{r}^H\boldsymbol{B} = \boldsymbol{0}^T$. Every $(L_h + L_g + 1)$-size vector $\boldsymbol{r}$ can always be expressed as the linear combination of a set of $L_h + L_g + 1$ Vandermonde vectors $\boldsymbol{v}(\rho) = (1, \rho_l^{-1}, \ldots, \rho_l^{-(L_h+L_g)})^T$ having all different ρ_l, since such vectors are linearly independent and thus they form a basis for a $(L_h + L_g + 1)$-dimensional space. If we multiply $\boldsymbol{B}$ from the left side by $\boldsymbol{r}^H := \sum_{l=1}^{\bar{L}} \alpha_l \boldsymbol{v}^H(\rho_l)$, where $\bar{L}$ is still to be determined, we obtain

$$\begin{aligned}\boldsymbol{r}^H\boldsymbol{B} &= \sum_{l=1}^{\bar{L}} \alpha_l [H_1(\rho_l), \ldots, \rho_l^{-L_g} H_1(\rho_l), H_2(\rho_l), \ldots, \\ &\ldots \ \rho_l^{-L_g} H_2(\rho_l), -\rho_l^{-d-\delta-1}, \ldots, -\rho_l^{-d-\delta-L}]. \quad (7)\end{aligned}$$

Therefore, if $H_1(z)$ and $H_2(z)$ are co-prime, there is no $\boldsymbol{r} \neq \boldsymbol{0}$ such that $\boldsymbol{r}^H\boldsymbol{B}$ is equal to zero. From (7), the only possibility for $\boldsymbol{r}^H\boldsymbol{B}$ to be null requires $H_1(z)$ and $H_2(z)$ to share at least $L + 1$ zeros so that, taking $\bar{L} = L + 1$ and choosing ρ_l equal to the common roots of $H_1(z)$ and $H_2(z)$, for $l = 1, \ldots, L + 1$, the first $2(L_g + 1)$ entries of $\boldsymbol{r}^H\boldsymbol{B}$ in (7) are null and we can always find a set of $L + 1$ coefficients α_l, $l = 1, \ldots, L + 1$ that null also the last L components of (7). This concludes the proof.

Generalizing these arguments to a SIMO structure composed of K channels, we can prove the following

Theorem: Given a SIMO scheme composed of K parallel channels, a set of sufficient conditions for the existence of a nontrivial shortening filterbank is that:
A2) $L_g > (L_h - L - K)/(K - 1)$;
A3) the polynomials $H_k(z)$, with $k = 1, \ldots, K$, do not share more than L common zeros.

From conditions **A2)** and **A3)** we can draw some important remarks:
Remark 1. SIMO equalizers are known to suffer from the so called *disparity* condition [6], requiring that the K channels do not have common zeros. Perfect equalization is in fact a particular case of (4) corresponding to $L = 0$. Interestingly, the combination of the SIMO structure with the OFDM transmission scheme including cyclic prefixes of length L allows the existence of perfect pre-equalization even if the channels share up to L zeros.
Remark 2. In [5], the conditions for perfect pre-equalization in a SIMO structure with an underlying OFDM structure were derived assuming the polynomial $P(z)$ to be known. Here, we have relaxed this constraint to give the adaptive equalizer all the freedom to evolve towards the right solution without assuming any knowledge neither about the channels nor about $P(z)$.
Finally, note that the theorem establshes only a sufficient condition, which means that if **A2** and **A3** are not satisfied, there could still exist a non trivial shortening filterbank.

4. ADAPTIVE PRE-EQUALIZATION

In this section we propose our adaptive algorithm for estimating the coefficients of the pre-equalizer filterbank directly from the received data. With reference to Fig. 1, we introduce a cost function measuring the energy falling in the virtual sub-channels. Specifically, we denote by $Z(q;l)$ the l-th sample of the N-point FFT of the q-th received block extracted from $z(n)$, namely

$$Z(q;l) = \sum_{n=0}^{N-1} z(q;n) e^{j\frac{2\pi}{N}(l+i_q)n}, l \in \mathcal{V}_M, \quad (8)$$

where $z(q;n) := z(q(N + L) + n)$, with $n = 0, \ldots, N - 1$, and we define our cost function as the energy falling in the virtual sub-carriers, averaged over N_b consecutive blocks:

$$J_{N_b}(\boldsymbol{g}_1, \ldots, \boldsymbol{g}_K) := \sum_{l \in \mathcal{V}_M} \frac{1}{N_b} \sum_{q=1}^{N_b} |Z(q;l)|^2. \quad (9)$$

Ideally, if the combined channel impulse response $c(n)$, corresponding to $C(z)$ in (4), had an order at most equal to the cyclic prefix length, such energy would be null. However, if the channel $c(n)$ is longer than $L + 1$, there is some energy falling in the virtual sub-channels. Based on this remark, we propose an adaptive algorithm which estimates the pre-equalizers' impulse responses as the coefficients that minimize $J_{N_b}(\boldsymbol{g}_1, \ldots, \boldsymbol{g}_K)$. More specifically, using a conventional iterative steepest descent approach, the k-th filter response $\boldsymbol{g}_k(n + 1)$ at the $(n + 1)$-th step is obtained as

$$\boldsymbol{g}_k(n+1) = \boldsymbol{g}_k(n) - \mu \nabla_{\boldsymbol{g}_k} J_{N_b}(\boldsymbol{g}_1(n), \ldots, \boldsymbol{g}_K(n)), \quad (10)$$

where μ is the step size and $\nabla_{\boldsymbol{g}_k} J_{N_b}(\boldsymbol{g}_1(n), \ldots, \boldsymbol{g}_K(n))$ is the conjugate gradient of $J_{N_b}(\boldsymbol{g}_1(n), \ldots, \boldsymbol{g}_K(n))$ with respect to $\boldsymbol{g}_k$, computed at the n-th step. In formulas,

$$\frac{\partial J_{N_b}}{\partial g_r^*(i)} = \frac{1}{N_b} \sum_{l \in \mathcal{V}_M} \sum_{q=1}^{N_b} Z(q;l) \frac{\partial Z^*(q;l)}{\partial g_r(i)} = \frac{1}{N_b} \sum_{l \in \mathcal{V}_M} \sum_{q=1}^{N_b}$$
$$\sum_{n=0}^{N-1} Z(q;l) y_r^*(q(N+L)+n-i) e^{-j\frac{2\pi}{N}(i_l+i_q)(q(N+L)+n-i)}. \quad (11)$$

We are not able to prove the absolute convergence of our method, without FH. Conversely, if we incorporate FH, we have proved the absolute convergence property. Specifically, we proved that, under ergodicity assumptions about the transmitted sequence and allowing the FH index i_q to sweep all the range $[0, N - 1]$, as q goes from 0 to $N-1$, the cost function tends asymptotically, i.e. as the number of blocks N_b tends to ∞, to the following expression

$$J(\boldsymbol{g}_1, \ldots, \boldsymbol{g}_K) := \lim_{N_b \to \infty} J_{N_b}(\boldsymbol{g}_1, \ldots, \boldsymbol{g}_K) = \sum_{i=L+1}^{L_c} \alpha_i |c(i)|^2, \quad (12)$$

where the coefficients α_i are all positive and independent of the channel. From (12), we infer that the only possibility for $J(\boldsymbol{g}_1, \ldots, \boldsymbol{g}_K)$ to reach its absolute minimum is to have $c(n) = 0$, for $n > L$. Furthermore, (12) shows that the cost function is a hyperparaboloid and thus the convergence of our steepest descent method towards the absolute minimum is guaranteed *irrespective* of the inizialization (at least asymptotically, i.e. using an infinite number of blocks).

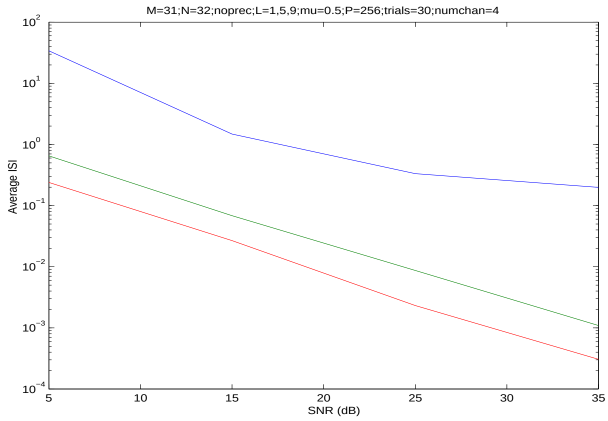

Fig. 2. Average ISI for $L = 1, 5$, and 9 ($L_h = 15$).

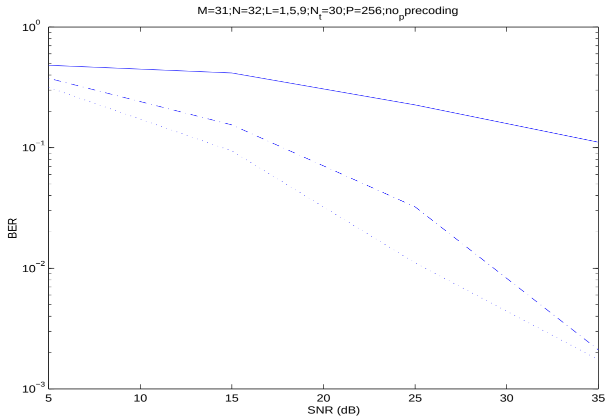

Fig. 3. Average BER for $L = 1$ (solid), 5 (dash and dots), and 9 (dots); $L_h = 15$.

5. NUMERICAL RESULTS AND CONCLUSION

Ideally, at the end of our adaptation we would like to have $c(n) = 0$ for $n > L$. Thus, to quantify the performance of our method, we have introduced the following parameter, which we will term ISI (we use Matlab notation)

$$ISI := \frac{||c(L+1 : L_h + L_g + 1)||^2}{||c(0 : L)||^2}. \tag{13}$$

As a numerical test, we have considered an OFDM system with $N = 32$ samples per block and a cyclic prefix of variable length L. The number of information symbols per block is $M = 31$, so that there is only one null sub-carrier per block. The number of receive channels is $K = 4$ and each channel has order $L_h = 15$. The channel coefficients are generated as complex i.i.d. Gaussian random variables with zero mean and unitary variance. In each iteration, the gradient is averaged over 256 blocks. The order L_g of the pre-equalization filters is computed as $L_g = \lfloor (L_h - L - K)/(K-1) \rfloor + 1$, where $\lfloor a \rfloor$ denotes the integer part of a. At the beginning of each iteration, the coefficients of the pre-equalization filters are all set to 0, except the center one set equal to 1. In Fig. 2 we report the average ISI (averaged over 30 independent channel realizations), for $L = 1, 5$, and 9, as a function of the SNR, obtained after 200 iterations of the steepest descent method (the upper, middle and lower curves refer to $L = 1, 5$ and 9, respectively). As we can see, the performance improves as the CP-length increases. In fact, as L increases the probability of being close to an ill-conditioned situation (the channels share more than L zeros) decreases and thus the performance improves. Furthermore, since the ultimate performance factor in a digital link is BER, in Fig. 3 we report the BER vs. the SNR, obtained with a QPSK constellation (solid, dash and dotted, and dotted lines refer to $L = 1, 5$ and 9, respectively). The number of iterations used to obtain the pre-equalization filters is 200. Again, for $L = 1$ there may be some situations where one out of the 15 channel zeros of one channel is close to one of 15 zeros of another channel. In such cases the algorithm does not converge and the final BER is very poor and thus it affects the average BER considerably. However, as L increases, the probability of having an ill-conditioned problem decreases and the BER decreases accordingly. In conclusion, channel shortening is a useful tool to limit the efficiency losses in wireless OFDM systems and the choice of the CP-length must result from a trade-off between efficiency and performance: The shorter is the prefix, the smaller is the efficiency loss but the bigger is the performance loss, and viceversa. Interesting developments of our approach should incorporate the design of a shortening filterbank which dos not necessarily aims at nulling the ISI exactly, but it minimize the signal to interference plus noise ratio. This should give more flexibility to the design and then achieve better performance in terms of SNR.

6. REFERENCES

[1] G. Arslan, B.L. Evans, S. Kiaei, "Optimum channel shortening for discrete multitone transceivers", *Proc. of ICASSP '00*, Vol. 5 , pp. 2965 –2968, Istanbul, June 2000.

[2] J.S. Chow, J.M. Cioffi, J.A.C. Bingham, "Equalizer training algorithms for multicarrier modulation systems", *Proc. of 1993 IEEE Int. Conf. on Communications, ICC '93* Vol. 2, pp. 761 -765.

[3] B. Farhang-Boroujeny, M. Ding, "Channel classification and time-domain equalizer design for ADSL transceivers", *Proc. of 2000 IEEE International Conference on Communications, ICC 2000*, Vol. 2, pp. 1059 -1063.

[4] P. J. W. Melsa, R. C. Younce, C. E. Rohrs, "Impulse response shortening for discrete multitone transceivers", *IEEE Trans. on Commun.* , pp. 1662–1672, Dec. 1996.

[5] D. Pal, G.N. Iyengar, J.M. Cioffi, "A new method of channel shortening with applications to discrete multi-tone (DMT) systems", *Proc. of 1998 IEEE Int. Conf. on Communications, ICC '98*, Vol. 2 , pp. 763 -768.

[6] L. Tong, G. Xu, T. Kailath, "Blind identification and equalization based on second order statistics: A time domain approach", *IEEE Trans. on IT*, pp.340-349, 1994.

[7] B. Wang, T. Adali, "Time-domain equalizer design for discrete multitone systems", *Proc. of ICC 2000*, Vol. 2, pp. 1080 –1084.

[8] Z. Wang and G. B. Giannakis, "Wireless Multicarrier Communications: where Fourier meets Shannon," *IEEE Signal Processing Magazine*, vol. 17, no. 3, pp. 29–48, May 2000.