MINIMUM-OUTPUT-ENERGY METHOD FOR BLIND EQUALIZATION OF OFDM AND SYSTEMS WITH SUFFICIENT OR INSUFFICIENT CYCLIC PREFIX

Taewoo Han and Xiaohua(Edward) Li

Department of Electrical and Computer Engineering
State University of New York at Binghamton
Binghamton, NY 13902
{bg26845, xli}@binghamton.edu

ABSTRACT

In this paper, a new blind equalization method is proposed for systems with cyclic prefix such as OFDM. With constrained minimum-output-energy (MOE) optimization, a bank of equalizers can be derived without the help of FFT/IFFT. Algorithms are developed for blindly estimating the constraints and equalizers. When the cyclic prefix is sufficient, i.e., longer than the channel length, the complexity of the method is comparable to traditional FFT-based receivers. A special property is that the new method works similarly even if the cyclic prefix is insufficient, i.e., shorter than the channel length. Therefore, it provides an effective way to improve bandwidth efficiency with shorter cyclic prefix length. Simulations demonstrate the superior performance of the proposed method.

1. INTRODUCTION

Intersymbol interference (ISI) is a severe degradation factor for the performance of communications and must be mitigated by equalizations before a higher throughput can be obtained. Training based equalization methods may be the most widely applied approaches in practice. However, since training sequences reduce system throughput, blind equalization is more promising [4].

To simplify the problem of equalizations, transmitters with cyclic prefix or some other precoding techniques are widely investigated [6]. The most well-known one is OFDM. With the help of FFT, frequency selective channels are converted into a parallel of flat fading channels. Then equalization can be performed by one-tap equalizers or blindly with differential encoding [1]. OFDM has already been used in some broadband systems such as wireless LAN, ADSL, DVB/DAB, etc. It has been proposed as a candidate for the future 4th generation mobile communications. Considering that OFDM introduces large peak-to-average power ratio, another candidate is to use cyclic prefix but without IFFT in the transmitter. Signal detection is performed in the frequency domain at the receiver with both IFFT and FFT, similarly as OFDM.

However, traditional systems with cyclic prefix require that the length of the cyclic prefix be no less than the channel length. Then either the block length should be large, which increases system delay and computational complexity, or, with a short block length, the efficiency becomes low due to the overhead of cyclic prefix. Especially in the future broadband systems with long channels, the system efficiency may greatly suffer from the overhead of cyclic prefix. Channel shortening techniques were studied to improve the efficiency [5]. However, similarly as time-domain equalization of single-input-single-output (SISO) channels, SISO channels can usually be shortened approximately only [2].

On the other hand, because for frequency selective channels some subcarriers have lower SNR than others, the performances of signal detection on these subchannels are not uniform. In some systems, it is impractical to use the optimal water-filling algorithms to adjust the distribution of data among subcarriers because of unknown or time-varying channels. Therefore, there is a reduction in performance.

In this paper, we propose a blind equalization method for the receivers in OFDM and other systems with cyclic prefix. Taking the advantage of the special correlation property generated by the cyclic prefix, we will show that blind equalization can be performed without FFT/IFFT. Instead, the well-known minimum-output-energy (MOE) receivers [3], [7] can be used with some proper constrained optimization. With the conveniently available adaptive implementations, the computational complexity can be generally comparable to traditional approaches. However, the performance is higher since every symbol (or subcarrier) is treated fairly. More important, the new method works in systems where the length of cyclic prefix is shorter than that of channels. Therefore, shorter cyclic prefix length can be used to enhance system efficiency.

This paper is organized as follows. In Section 2, the system model with cyclic prefix is described. In Section 3, we develop the new method for systems with sufficient cyclic prefix. Then this method is extended to systems with insufficient cyclic prefix in Section 4. In Section 5, simulations are performed to demonstrate the performance. Finally, conclusions are given in Section 6.

2. SYSTEMS WITH CYCLIC PREFIX

Consider the baseband digital communication system with cyclic prefix, as shown in Fig. 1. Note that although we consider the OFDM system, our method does not depend on the IFFT/FFT blocks. The input symbol sequence s_n is grouped to form blocks, each with N symbols. Then each block may be processed by the optional IFFT or other multi-rate filterbank precoders [6]. The result is a data sequence u_n. Then L cyclic prefix (CP) samples are added to each block.

Let $P = N + L$. Define $\mathbf{s}_N(nN) = [s_{nN}, \cdots, s_{nN-N+1}]^T$ and $\mathbf{u}_P(nP) = [u_{nP}, \cdots, u_{nP-P+1}]^T$. If there is IFFT (or other precoders), we have

$$[u_{nP}, \cdots, u_{nP-N+1}]^T = \mathbf{F}^{-1}\mathbf{s}_N(nN). \qquad (1)$$

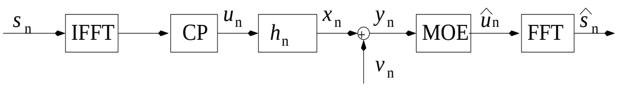

Fig. 1. Baseband OFDM transmitter and MOE receiver. Note that the IFFT/FFT blocks are optional.

where $\mathbf{F}^{-1}$ is the inverse discrete Fourier transformation matrix (or other precoder matrices). We add cyclic prefix to construct the other L elements in the block $\mathbf{u}_P(nP)$ as

$$u_{nP-N-i} = u_{nP-i}, \quad i = 0, \cdots, L-1. \tag{2}$$

Then the cyclic-prefixed data sequence $\{u_n\}$ is transmitted.

At the receiving end, the received baseband signal is

$$y_n = x_n + v_n \triangleq \sum_{k=0}^{L_h} h_k u_{n-k} + v_n \tag{3}$$

where v_n and x_n denote noise and noiseless signal, respectively. The channel $\mathbf{h} = [h_0, \cdots, h_{L_h}]^T$ is assumed to be FIR with length $L_h + 1$. In traditional OFDM or other multicarrier systems, the condition of sufficient cyclic prefix should be satisfied, i.e., $L \geq L_h$. Because the transmission efficiency is $N/(N+L)$, large L value decreases transmission efficiency.

Traditionally, equalization is performed after FFT. In this paper, however, as shown in Fig. 1, we perform MOE-based equalization before FFT. In fact, as can be seen obviously, the IFFT/FFT blocks are optional only.

Since we allow that the length of the cyclic prefix L may be shorter than the channel length L_h, for the flexibility of manipulating correlations of cyclic prefix, we construct the received data vectors with dimension M

$$\mathbf{y}_M(k) = \mathbf{x}_M(k) + \mathbf{v}_M(k) \tag{4}$$

where $\mathbf{y}_M(k) = [y_k, \cdots, y_{k-M+1}]^T$, $\mathbf{x}_M(k) = [x_k, \cdots, x_{k-M+1}]^T$ and the AWGN $\mathbf{v}_M(k) = [v_k, \cdots, v_{k-M+1}]^T$. Considering the FIR channel, the noiseless received signal vector is

$$\begin{aligned} &\mathbf{x}_M(k) \\ &= \mathcal{H}\mathbf{u}(k) \\ &\triangleq \begin{bmatrix} h_0 & \cdots & h_{L_h} & & \\ & \ddots & & \ddots & \\ & & h_0 & \cdots & h_{L_h} \end{bmatrix} \begin{bmatrix} u_k \\ \vdots \\ u_{k-M-L_h+1} \end{bmatrix} \end{aligned} \tag{5}$$

where the channel matrix $\mathcal{H}$ is with dimension $M \times (M + L_h)$.

As a special case, if $L_h \leq L$, we can choose $M = N$ to construct

$$\begin{aligned} &\mathbf{x}_N(nP) = \mathbf{H}\mathbf{u}_N(nP) \\ &\triangleq \begin{bmatrix} h_0 & \cdots & h_{L_h} & & & \\ & \ddots & & \ddots & & \\ & & \ddots & \cdots & h_{L_h} \\ \vdots & & & \ddots & \vdots \\ h_1 & \cdots & h_{L_h} & & h_0 \end{bmatrix} \begin{bmatrix} u_{nP} \\ \vdots \\ u_{nP-N+1} \end{bmatrix} \end{aligned} \tag{6}$$

where $\mathbf{H}$ is an $N \times N$ circulant matrix similarly as traditional OFDM systems.

The general idea of this paper is to find linear filter equalizers $\{\mathbf{f}_d\}$ to equalize the channel matrix $\mathcal{H}$ or $\mathbf{H}$ before performing FFT, i.e.,

$$\mathbf{f}_d^H \mathcal{H} = \mathbf{e}_{D+d}, \quad \text{or} \quad \mathbf{f}_d^H \mathbf{H} = \mathbf{e}_d, \quad d = 0, \cdots, N-1, \tag{7}$$

where $\mathbf{e}_d$ is a unit vector with a value 1 in the $(d+1)$th entry, and D denotes some proper equalization delay. Note that in (7) the vectors $\mathbf{f}_d$ and $\mathbf{e}_d$ should have proper dimensions. We will show that this task can be achieved with constrained MOE optimization.

In this paper, we assume that the symbols s_n are i.i.d. with zero mean and unit variance. The noise v_n is AWGN with zero-mean and variance σ_v^2, and is uncorrelated with s_n.

3. BLIND MOE-BASED EQUALIZATION FOR SYSTEMS WITH SUFFICIENT CYCLIC PREFIX

3.1. Constraint parameter estimation

For simplicity, we consider first the special case where the cyclic prefix length L is no less than channel length L_h. We consider the noiseless system first.

In order to apply minimum-output-energy optimization, we need to find a proper constraint. The optimal constraint might be the columns in the channel matrix $\mathcal{H}$ or $\mathbf{H}$ in equations (5) and (6). It is therefore similar to blind channel estimation. However, the channel length is not required to be estimated. In addition, some scalar phase ambiguity in estimation is also permitted.

Let $M = L + 1$ in (5). Then we construct $L + 1$ dimensional data sample vectors and calculate the following three correlation matrices

$$\begin{aligned} \mathbf{R}_x &= E\{\mathbf{x}_{L+1}(nP)\mathbf{x}_{L+1}^H(nP)\}, \\ \mathbf{R}_{L+1}(\ell) &= E\{\mathbf{x}_{L+1}(nP+\ell)\mathbf{x}_{L+1}^H(nP-N+\ell)\}, \\ &\qquad \ell = 0, \quad L+1. \end{aligned} \tag{8}$$

Proposition 1. If $P > 3L$, then channels can be estimated as the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_{L+1} = \mathbf{R}_x - \mathbf{R}_{L+1}(0) - \mathbf{R}_{L+1}(L+1)$.

Proof. Because $P > 3L$, from (5), we have

$$\mathbf{R}_x = \mathcal{H}E\{\mathbf{u}(nP)\mathbf{u}^H(nP)\}\mathcal{H}^H = \mathcal{H}\mathcal{H}^H \tag{9}$$

and similarly

$$\begin{aligned} \mathbf{R}_{L+1}(0) &= \mathcal{H}E\{\mathbf{u}(nP)\mathbf{u}^H(nP-N)\}\mathcal{H}^H \\ &= \mathcal{H}\begin{bmatrix} \mathbf{I}_L & \\ & \mathbf{0}_{L_h+1} \end{bmatrix}\mathcal{H}^H \end{aligned} \tag{10}$$

$$\mathbf{R}_{L+1}(L+1) = \mathcal{H}\begin{bmatrix} \mathbf{0}_{L+1} & \\ & \mathbf{I}_{L_h} \end{bmatrix}\mathcal{H}^H \tag{11}$$

where the matrices $\mathbf{I}_L$ and $\mathbf{0}_L$ are $L \times L$ identity and $L \times L$ zero matrices, respectively. Therefore $\mathbf{R}_{L+1} = \mathcal{H}\text{diag}\{\mathbf{0}_L, 1, \mathbf{0}_{L_h}\}\mathcal{H}^H$. Hence the channel can be identified [4]. □

Note that the condition $P > 3L$ is reasonable because of the requirement of high efficiency.

The estimated channel is in fact $\alpha[h_L, \cdots, h_0]^T$ with a scalar phase ambiguity α. It can be looked as zero-padding when $L_h < L$.

3.2. Blind MOE equalization

Since the length of the cyclic prefix L is no less than the channel length L_h, we construct the sample vector with $M = N$ as in (6). Then we calculate the following correlation matrix

$$\mathbf{R}_N = E\{\mathbf{x}_N(nP)\mathbf{x}_N^H(nP)\}, \tag{12}$$

and construct an $N \times 1$ dimensional vector with $N-L-1$ padding zeros as constraint

$$\hat{\mathbf{h}} = \alpha[h_L, \cdots, h_0, 0, \cdots, 0]^T. \tag{13}$$

Consider the following constrained minimization problem

$$\mathbf{f}_L = \text{argmin } \ \mathbf{f}_L^H \mathbf{R}_N \mathbf{f}_L, \quad s.t., \ \ \mathbf{f}_L^H \hat{\mathbf{h}} = 1. \tag{14}$$

It is well known that the optimal solution $\mathbf{f}_L$ in (14) is a generalized eigenvector of the matrix pencil $(\mathbf{R}_N, \hat{\mathbf{h}}\hat{\mathbf{h}}^H)$ [7].

Proposition 2. If the channel matrix $\mathbf{H}$ is full column rank, then $\mathbf{f}_L^H \mathbf{H} = \alpha^* \mathbf{e}_L$.

Proof. We can subdivide the channel matrix $\mathbf{H}$ into two parts: the Lth column $\mathbf{b}$ and all other columns $\mathbf{B}$. Note that $\hat{\mathbf{h}} = \alpha \mathbf{b}$. Therefore, $\mathbf{f}_L^H \mathbf{R}_N \mathbf{f}_L = \mathbf{f}_L^H \mathbf{b}\mathbf{b}^H \mathbf{f}_L + \mathbf{f}_L^H \mathbf{B}\mathbf{B}^H \mathbf{f}_L \geq 1$, where the equality holds iff $\mathbf{f}_L^H \mathbf{B} = \mathbf{0}$. □

Because of the circulant property, once $\mathbf{f}_L$ is estimated, then we can shift $\mathbf{f}_L$ circularly to obtain $\mathbf{f}_d$ for all $d = 0, \cdots, N-1$ such that $\mathbf{f}_d^H \mathbf{H} = \alpha^* \mathbf{e}_d$. Hence

$$\mathbf{f}_d^H \mathbf{x}_N(nP) = \alpha^* u(nP-d), \quad d = 0, \cdots, N-1. \tag{15}$$

Then symbols can be estimated with a phase ambiguity α^* which can be removed by differential encoding.

If considering noise, then from (4) we use $\mathbf{y}_N(nP)$ instead of $\mathbf{x}_N(nP)$. The optimization (14) converges to the minimum-mean-square-error solution [3], [7].

The constraint $\hat{\mathbf{h}}$ and the equalizer $\mathbf{f}_L$ can be estimated from Proposition 1 and (14) with adaptive implementations such as LMS algorithm. The computational complexity is on the same order as the traditional FFT based receivers because the equalizer bank (15) can be implemented efficiently via FFT.

If the column $\mathbf{b}$ is independent from all other columns, we have the same results. On the other hand, if $\mathbf{b}$ falls in the subspace spanned by other columns, we can utilize the L discarded samples, which we discuss in the more general scenario with possibly insufficient cyclic prefix.

4. BLIND MOE-BASED EQUALIZATION FOR SYSTEMS WITH INSUFFICIENT CYCLIC PREFIX

4.1. Constraint parameter estimation

In this section, we consider the general scenario where the length of the cyclic prefix L may be less than the channel length L_h. If L_h is unknown, we can use an over-estimated one. Obviously, the traditional FFT-based method fails. We will show that the MOE-based method, with some modifications, can still be used.

If L_h is possibly longer than L, the constraint estimation method in Section 3.1 should be modified to apply repeatedly the correlation matrices

$$\begin{aligned} \mathbf{R}_K(\ell) &= E\{\mathbf{x}_K(nP+\ell)\mathbf{x}_K^H(nP-N+\ell)\} \\ &= \mathcal{H} \begin{bmatrix} \mathbf{0}_\ell & & \\ & \mathbf{I}_L & \\ & & \mathbf{0}_{K+L_h-L-\ell} \end{bmatrix} \mathcal{H}^H, \end{aligned} \tag{16}$$

where $\ell = 0, \cdots, K+L_h-L$. On the other hand, if $K+L_h-L \leq \ell \leq K+L_h$, we have

$$\mathbf{R}_K(\ell) = \mathcal{H} \begin{bmatrix} \mathbf{0}_\ell & \\ & \mathbf{I}_{K+L_h-\ell} \end{bmatrix} \mathcal{H}^H. \tag{17}$$

The dimension K, i.e., M in equations (4) and (5), can be chosen as

$$K \geq L_h + 1. \tag{18}$$

Then, we evaluate the summation of a sequence of matrices

$$\mathbf{R}_K = \sum_{\ell=0}^{Q} [\mathbf{R}_K(L_h + \ell L) - \mathbf{R}_K(L_h + 1 + \ell L)], \tag{19}$$

where $Q = \lfloor (K-1)/L \rfloor$. Then the correlation matrix $\mathbf{R}_K$ satisfies

$$\mathbf{R}_K = \mathcal{H}\text{diag}\{\mathbf{0}_{L_h}, 1, \mathbf{0}_{K-1}\}\mathcal{H}^H. \tag{20}$$

Hence the channel coefficients can be estimated as the eigenvectors corresponding to the largest eigenvalue of $\mathbf{R}_K$ in (20), which will be in the same form as (13).

4.2. Blind MOE equalization

In case of insufficient cyclic prefix, circulant channel matrix may not be available. However, the cyclic prefix can still be exploited for equalization. We construct the data sample vector $\mathbf{x}_M(nP)$ according to equation (5) with properly chosen dimension M. The channel matrix $\mathcal{H}$ is with dimension $M \times (M + L_h)$.

Define $m = \lfloor (M + L_h)/P \rfloor$. Considering the L cyclic prefix in every P symbols, there are at most $M + L_h - mL$ columns if we combine those columns in $\mathcal{H}$ that are corresponding to the same symbols. Denote the combined channel matrix as $\tilde{\mathcal{H}}$.

Proposition 3. With proper M and $m > L_h/L$, the corresponding channel matrix $\tilde{\mathcal{H}}$ is full column rank.

Proof. The matrix $\tilde{\mathcal{H}}$ has M rows and at most $M + L_h - mL$ columns. In addition, considering the Toeplitz structure of the channel matrix $\mathcal{H}$, it is easy to find that the matrix $\tilde{\mathcal{H}}$ can be arranged in a block upper-triangular form with h_0 on the main diagonal. Therefore, it is full column rank. □

Let the correlation matrix be

$$\mathbf{R}_M = E\{\mathbf{x}_M(nP)\mathbf{x}_M^H(nP)\} = \tilde{\mathcal{H}}\tilde{\mathcal{H}}^H. \tag{21}$$

We can select any adjacent N columns in $\tilde{\mathcal{H}}$, e.g., the $(D, \cdots, D+N-1)$th columns with some proper delay parameter D. We denote them as M dimensional columns $\mathbf{c}_d$, $d = 0, \cdots, N-1$. In fact, they can be represented by the estimated channel coefficients (13) as obtained in Section 4.1. Hence we can use them as constraints.

We define a bank of N constrained MOE optimization problems as

$$\mathbf{f}_d = \text{argmin } \ \mathbf{f}_d^H \mathbf{R}_M \mathbf{f}_d, \quad s.t., \ \ \mathbf{f}_d^H \mathbf{c}_d = 1, \tag{22}$$

for all $d = 0, \cdots, N-1$. Then similarly as analyzed in Section 3.2, the closed form solution of (22) can be obtained from the fact that $\mathbf{f}_d$ is the generalized eigenvector of the matrix pencil $(\mathbf{R}_M, \mathbf{c}_d\mathbf{c}_d^H)$. Because $\tilde{\mathcal{H}}$ is full column rank, $\mathbf{f}_d$ is zero-forcing in noiseless case, i.e., $\mathbf{f}_d^H \tilde{\mathcal{H}} = \mathbf{e}_{D+d}$. On the other hand, in noisy case or if we choose $m < L_h/L$ to reduce complexity, the equalizer $\mathbf{f}_d$ becomes the MMSE solution [7].

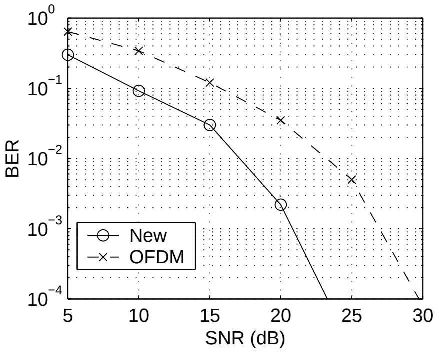

Fig. 2. Compare the proposed MOE blind equalizer and the OFDM receiver in systems with sufficient cyclic prefix.

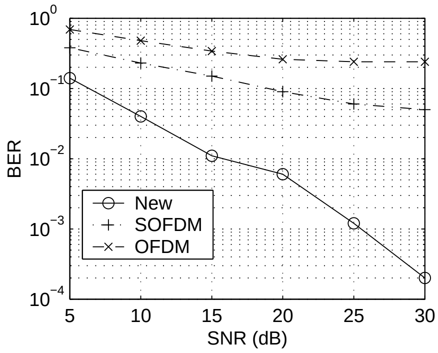

Fig. 3. Performance in systems with insufficient cyclic prefix.

The block of N data symbols can be estimated by the equalizer bank $\mathbf{f}_d$ as

$$\hat{u}_{nP-D-d} = \mathbf{f}_d^H \mathbf{x}_M(nP), \quad d = 0, \cdots, N-1. \quad (23)$$

Then FFT can be used to re-construct the transmitted symbols s_n in OFDM systems.

Similarly as discussed in Section 3.2, in noisy case, we can use $\mathbf{y}_M(nP)$ instead of $\mathbf{x}_M(nP)$. In addition, because the estimation of constraints $\{\mathbf{c}_d\}$ and the equalizer bank $\{\mathbf{f}_d\}$ can all be conveniently implemented as LMS adaptive algorithms, the computational complexity is $O(NM)$. Furthermore, with highly parallel implementation of the filterbank (each with complexity $O(M)$), the complexity will still be comparable to traditional FFT based methods.

5. SIMULATIONS

In this section, we use simulations to study the performance of the proposed algorithms in Sections 3 and 4, and to compare them with some existing algorithms, specifically, differentially encoded OFDM systems where blind detection can be performed without channel estimation, and OFDM with channel shortening techniques [5] which we denote as SOFDM. We used the bit error rate (BER) to evaluate the performance of equalizations. The transmitted symbols s_n are QPSK and differentially encoded. We used the average of 100 Monte Carlo runs to evaluate BER. The channel was chosen as $\mathbf{h} = [-1.28 - j0.301, -0.282 + j0.562, 0.031 - j0.211, 0.106 + j1.164]$ with channel length $L_h = 3$.

First, we compare the new algorithm in Section 3 with OFDM detector when the cyclic prefix length is sufficient $L = L_h$. We chose $N = 16$ for both transmissions. We used $M = L$ for channel estimation, and $M = N = 16$ for MOE equalization. 500 blocks are used for channel and equalizer estimation. The BER is compared in Fig. 2. The proposed method has better performance. The reason might be that symbols from all subcarriers are treated fairly in our method, whereas in OFDM and SOFDM, some subcarriers may have lower SNR compared with others.

Second, we compare the new algorithm in Section 4 against OFDM and SOFDM [5], considering an insufficient cyclic prefix length $L = 2$. To achieve similar transmission efficiency as in the first example, we chose $N = 8$. We used $K = 5$ for channel estimation and $M = 28$ for MOE equalization. 1000 data blocks are used for constraint and equalizer estimation. For SOFDM, we used a 10-tap filter to shorten the channel, where we used the known channel to obtain its optimal performance. As shown in Fig. 3, the new MOE algorithm can still achieve sufficiently low BER, whereas the OFDM and SOFDM receivers fail.

6. CONCLUSION

In this paper, we present a new blind receiver for systems with cyclic prefix such as OFDM. Constrained minimum-output-energy optimization is used to derive blind equalizers. The new method has computational complexity comparable to the traditional OFDM receivers, whereas its performance outperforms the latter. More important, the new method works even when the length of the cyclic prefix is smaller than the channel length. Hence smaller cyclic prefix and smaller block size can be used to enhance system efficiency and to reduce complexity.

7. REFERENCES

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