COOPERATIVE DIVERSITY THROUGH VIRTUAL ARRAYS IN MULTIHOP NETWORKS

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ABSTRACT

In this work we propose a multihop cellular network architecture which takes advantage of the cooperation among users to induce a diversity gain. Mobile terminals (MT) willing to cooperate share their data first, during a time slot reserved to MT-to-MT links, and then, in a successive time slot, they send their data to the base station (BS) through a *virtual* array of antennas, constituted by the antennas of the cooperating users. We derive the coding strategy, for such a virtual array, that maximizes the sum of the rates from the MT's to the BS, under the constraint of a given total available power. We assume at the beginning that the channels from the MT to the BS are perfectly known. This allows us to derive, for each MT, a closed form expression for the optimal power allocation, as a function of frequency. Then, we remove this assumption and we use a first order perturbation analysis to compute the loss resulting from imperfect channel knowledge.

1. INTRODUCTION

Cooperation diversity is not a novel idea and it constitutes a potential source of diversity made possible from proper cooperation among users in a wireless network. The basic theorems about relay channels were first established in [3]. The cooperation diversity idea was then suggested in a series of papers, like e.g. [7], [6], [4], where it was also proved the potential gain resulting from allowing users to share their data. In particular, in [7] it was shown how the capacity region of a multiple access channel can be enlarged thanks to the cooperation, whereas in [6] alternative cooperation strategies were analyzed and it was shown under which conditions the cooperation yields the desired diversity gain. In [6], the channel were supposed to be flat fading and users and relays sent their data to the BS adopting a time division strategy.

In this paper, we focus our attention on the uplink channel of a multihop cellular network, having a time-division duplexing (TDD) substrate. We consider transmission over wideband, frequency-selective channel and we optimize the use of the resources from each user according to a maximum sum-rate criterion, under a constraint of a given total power budget. We borrow the main ideas about the scenario from the so called Opportunity Driven Multiple Access (ODMA) system, which has been considered during the standardization of the third generation (3G) cellular system in Europe. In ODMA, the transmission frame is composed of: i) a time slot allocated to the forward link, between the base station (BS) and all Mobile Terminals (MT); ii) a time slot allocated to the reverse link, from the MT to the BS; and iii) a certain number of time slots dedicated to *direct* links among MT's. We assume that the direct links occur only between neighbor MT's and, in particular, the attenuation is low enough to consider these links error-free. This entails that the MT-to-MT link is established only if the information rate between two cooperating users is lower than the capacity of the link between them. Under such a condition, invoking Shannon's channel coding theorem, we assume that there is enough coding to guarantee a negligible error probability in the MT-to-MT link. More than one direct link can take place at the same time, in a given cell, provided that these links are sufficiently far apart so that the interference between them can be neglected. Of course, these are all assumptions which are not perfectly verified in practice. Nevertheless, these simplifications are important to derive a closed form expression for the optimal power allocation, as we will show in the next section. These expressions will be then used to assess the loss due to imperfect channel knowledge.

Once two or more MT's have shared their data using the direct link between them, in a successive time slot, they have the possibility to implement other hops towards other MT's or to send the shared data to the BS. During the time slot allocated to the reverse link, we do not impose any specific multiple access strategy, which will then result as part of our global optimization procedure. We will consider in Section 2 the case where the channel state information (CSI) is available with no error to all users, and then we will treat in Section 3 the more realistic scenario, where CSI is affected by inevitable estimation errors.

2. OPTIMAL TRANSMISSION STRATEGY WITH PERFECT CSI

Let us denote by the vector $\boldsymbol{s}(n)$ the set of M data shared by N_U users. The channels between the MT's and the BS are assumed to be frequency-selective time-invariant channels, modelled as FIR filters of maximum order L. We assume: **(a1)** no detection errors, so that $\boldsymbol{s}(n)$ contains only information symbols; **(a2)** perfect synchronization among the cooperative users. The vector $\boldsymbol{s}(n)$ is transmitted through the N_U antennas made available from the cooperation. We do not make any assumption about the multiple access strategy, which will then come out as a result of our optimization. We only assume a linear precoding strategy, so that the block $\boldsymbol{s}(n)$ to be sent to the transmit antenna of the k-th user is premultiplied by a tall $(N+L) \times M$ matrix $\bar{\boldsymbol{F}}_k$, with $N \geq M$. To avoid Inter-Block Interference (IBI) and simplify symbol detection at the receiver, we append a cyclic prefix (CP) of length L at the beginning of each transmitted block, by setting the first L rows of $\bar{\boldsymbol{F}}_k$ equal to the last L ones. In summary, the n-th block sent by the k-th antenna is $\bar{\boldsymbol{x}}_k(n) := [x_k(nN), \ldots, x_k(nN+N+L-1)]^T = \bar{\boldsymbol{F}}_k \boldsymbol{s}(n)$. For each information block $\boldsymbol{s}(n)$, the available average power is $P_T = \sum_{k=1}^{N_U} \mathrm{tr}(\boldsymbol{R}_{\mathrm{k}})$, where $\boldsymbol{R}_k$ is the covariance ma-

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trix of $\boldsymbol{x}_k$. We also assume, without any loss of generality, that the information symbols in the block $\boldsymbol{s}(n)$ are uncorrelated [1], with covariance matrix $\boldsymbol{R}_s = \sigma_s^2 \boldsymbol{I}_M$. At the receiver, the IBI is eliminated by simply discarding the first L samples of the received block, so that the $N \times 1$ IBI-free receive vector $\boldsymbol{y}(n)$ can be written as:

$$\begin{aligned} \boldsymbol{y}(n) &= \textstyle\sum_{k=1}^{N_U} \boldsymbol{H}_k \boldsymbol{F}_k \boldsymbol{s}(n) + \boldsymbol{\eta}(n) \\ &:= \boldsymbol{H}\boldsymbol{F}\boldsymbol{s}(n) + \boldsymbol{\eta}(n) := \boldsymbol{H}\boldsymbol{x}(n) + \boldsymbol{\eta}(n) \end{aligned} \quad (1)$$

where $\boldsymbol{F}_k$ is the $N \times M$ coding matrix given by the lower N rows of $\bar{\boldsymbol{F}}_k$; $\boldsymbol{H}_k$, thanks to the insertion of the CP, is an $N \times N$ circulant Toeplitz matrix with entries $H_k(i,j) = h_k((i-j) \mod N)$, where $h_k(n)$ is the channel impulse response between the k-th transmit antenna and the receiver; $\boldsymbol{\eta}(n)$ is white additive Gaussian noise with covariance matrix $\boldsymbol{R}_\eta \doteq \sigma_n^2 \boldsymbol{I}_N$; $\boldsymbol{H} := [\boldsymbol{H}_1, \ldots, \boldsymbol{H}_{N_U}]$; $\boldsymbol{F} := \left[\boldsymbol{F}_1^T, \ldots, \boldsymbol{F}_{N_U}^T\right]^T$ and $\boldsymbol{x}(n) := \boldsymbol{F}\boldsymbol{s}(n)$. The maximum sum of the achievable rates for the N_U users can be computed as the maximum mutual information $I(\boldsymbol{s};\boldsymbol{y})$ between $\boldsymbol{s}(n)$ and $\boldsymbol{y}(n)$. Assuming that all channels $\{\boldsymbol{H}_k\}_{k=1}^{N_U}$ are perfectly known to both transmitters and receiver, the mutual information $I(\boldsymbol{s};\boldsymbol{y})$ is [2]:

$$I(\boldsymbol{s};\boldsymbol{y}) = \frac{1}{N} \log \left| \boldsymbol{I}_N + \boldsymbol{H}\boldsymbol{R}_x \boldsymbol{H}^H \boldsymbol{R}_\eta^{-1} \right|, \quad (2)$$

where $\boldsymbol{R}_x := E\{(\boldsymbol{F}\boldsymbol{s})(\boldsymbol{F}\boldsymbol{s})^H\} = \sigma_s^2 \boldsymbol{F}\boldsymbol{F}^H$.

The optimal coding strategy requires then the search for the matrices $\{\boldsymbol{F}_k\}_{k=1}^{N_U}$ that, for a given set of channels $\{\boldsymbol{H}_k\}_{k=1}^{N_U}$ and under the power constraint $P_T = \mathrm{tr}\{\sigma_s^2 \boldsymbol{F}\boldsymbol{F}^{\mathrm{H}}\}$, maximizes the information rate (2). This problem is mathematically equivalent to the problem of maximum information in a multiple-input/single-output (MISO) channel. In [1] we have derived a closed form solution for such a problem. We briefly recall the main results of [1], which will then used in next section to derive the performance of our system.

The basic of [1], interpreted in the context of multihop cooperating networks, are the following.

R.1 The optimal coding strategy for each user is OFDM, with proper power/bit allocation across the sub-carriers.

R.2 Denoting by $H_j(k)$ the value assumed by the j-th channel transfer function over the k-th frequency bin, and by $|\Phi_j(k)|^2$ the power allocated on the k-th subcarrier, from the transmit antenna of the j-th user, the power $|\Phi_j(k)|^2$ is either null or equal to

$$|\Phi_j(k)|^2 = \frac{|H_j(k)|^2}{\sum_{j=1}^{N_U} |H_j(k)|^2} \left(\mathcal{K} - \frac{\sigma_n^2/\sigma_s^2}{\sum_{j=1}^{N_U} |H_j(k)|^2} \right)^+, \quad (3)$$

for $k \in \mathcal{I}_u$, $j \in [1,\ N_U]$, where the set of indices $\mathcal{I}_u$ and the constant $\mathcal{K}$ are such that the average power constraint is satisfied [1], i.e. $\mathrm{tr}\{\boldsymbol{R}_\mathrm{x}\} = \sigma_\mathrm{s}^2 \sum_{\mathrm{j}=1}^{\mathrm{N_U}} \sum_{\mathrm{k}\in\mathcal{I}_\mathrm{u}} |\Phi_\mathrm{j}(\mathrm{k})|^2 = \mathrm{P_T}$.

R.3 If one antenna, let us say the antenna of the j-th user, does not allocate any power over the k-th sub-carrier, and all channels are not null at $z_k = e^{j2\pi k/N}$, then the antennas of all other users do not put any power over the k-th sub-carrier. This implies that, except for the cases of channels having zeros exactly at $z_k = e^{j2\pi k/N}$, with k integer, all the antennas transmit over the same portion of the available spectrum.

R.4 Since (3) gives indications only about the square modulus of $\Phi_j(k)$, we have complete freedom to choose the phase of $\Phi_j(k)$. We exploit this possibility by choosing, $\forall k \in \mathcal{I}_u$,

$$\Phi_j(k) = \frac{H_j^*(k)}{\sqrt{\sum_{j=1}^{N_U} |H_j(k)|^2}} \sqrt{\left(\mathcal{K} - \frac{\sigma_n^2/\sigma_s^2}{\sum_{j=1}^{N_U} |H_j(k)|^2} \right)^+}. \quad (4)$$

In this way, the received k-th symbol, for $k \in \mathcal{I}_u$, in the n-th block is [2]

$$y_k(n) = \sum_{j=1}^{N_U} H_j(k)\Phi_j(k) s_k(n) \quad (5)$$

$$\approx \sqrt{\mathcal{K} \sum_{j=1}^{N_U} |H_j(k)|^2 - \frac{\sigma_n^2}{\sigma_s^2}}\; s_k(n). \quad (6)$$

Therefore, the signal-to-noise ratio (SNR) at the receiver, on the k-th sub-carrier is

$$SNR(k) = \frac{\mathcal{K} \sum_{j=1}^{N_U} |H_j(k)|^2 \sigma_s^2 - \sigma_n^2}{\sigma_n^2} \approx \mathcal{K} \sum_{j=1}^{N_U} |H_j(k)|^2 \frac{\sigma_s^2}{\sigma_n^2}.$$

This shows that, within the limit of validity of the last approximation, i.e. when the noise is negligible with respect to the sum of the square moduli, *the system maximizing mutual information is equivalent to a maximal ratio combining scheme*, at least for the sub-carriers effectively used by the system. Besides being a generalization of the water-filling formula known for the SISO case, which is a particular case of (3) for $N_U = 1$, (3) has an interesting physical interpretation, as detailed in the following remarks.

Remark 1. If we exclude the limiting case where the channels have zeros exactly over the set of complex points $\{z_k := e^{j2\pi k/N};\ k = 0, 1, \ldots, N-1\}$, the criterion for allocating a nonnull power over the k-th sub-carrier depends on the *sum* of the square moduli of *all* the channel transfer functions. If a subcarrier is discarded for one channel, it is discarded for all the channels. This means that *all users transmit over the same portion of the spectrum*. In other words, with respect to the absence of cooperation, it is less likely that a sub-carrier is not used because, to discard a sub-carrier it is necessary that the channels of all (most of the) users have a high attenuation over that sub-carrier. The only exception to the above statement is represented by the case where some antenna has a zero exactly in the set $\{z_k := \exp j2\pi k/N, k = 0, 1, \ldots, N-1\}$. In such a case, the other antennas are allowed to allocate power over the k-th sub-carrier.

Remark 2. If a nonnull power is allocated over the k-th sub-carrier, each antenna uses a portion of such a power equal to its *relative weight*, quantified by the ratio $|H_j(k)|^2 / \sum_{j=1}^{N_U} |H_j(k)|^2$.

Remark 3. Equation (4) shows that the amplitude distribution across the *virtual* transmit array performs a beamforming, which is generally different for each sub-carrier, whose amplitude tapering depends on the SNR and on the global available power. Asymptotically, for N_U going to infinity, assuming that the channels between each transmit antenna and the receiver are realizations of a set of independent ergodic processes and that the coefficients

[1] Any correlation of $\boldsymbol{s}(n)$ could in fact be taken into account by including a proper whitening matrix in $\boldsymbol{F}_k$.

[2] The second line of (6) is an approximation of the first line, valid when all the arguments of the square root in (4) are strictly positive.

of each channel $h_j(k)$ are also the outcomes of independent random variables, the summations $\sum_{j=1}^{N_U} |H_j(k)|^2$ tend to a value independent of k. Hence, asymptotically, the maximum mutual information distribution tends to a classical beamforming, with $\Phi_j(k) = cH_j^*(k)$, where c is a constant dictated only by the available average power.

3. DIVERSITY LOSS DUE TO IMPERFECT CSI

In practice, there is an inevitable error in the channels' estimate. Let us denote by $\hat{H}_j(k)$ the estimated channel gain over the k-th frequency bin of the j-th link and by $\epsilon_j(k) := \hat{H}_j(k) - H_j(k)$ the resulting error. Substituting $H_j(k)$ with $\hat{H}_j(k)$ in (4) we obtain an erroneous power distribution, which we denote by $\hat{\Phi}_j(k)$. We wish to assess now the impact of such errors on the system performance. Substituting $\Phi_j(k)$ with $\hat{\Phi}_j(k)$ in (6), and adopting a first order Taylor's series expansion, as a function of the errors $\epsilon_j(k)$, we can approximate the received symbol, over the k-th sub-channel, as

$$\begin{aligned} \hat{y}_k(n) &\approx \sqrt{\mathcal{K}\sum_{j=1}^{N_U} |H_j(k)|^2 - \frac{\sigma_n^2}{\sigma_s^2}}\; s_k(n) \\ &- \sqrt{\mathcal{K} - \frac{\sigma_n^2/\sigma_s^2}{\sum_{j=1}^{N_U}|H_j(k)|^2}}\; \frac{\sum_{j=1}^{N_U} H_j^*(k)\epsilon_j(k)}{\sqrt{\sum_{j=1}^{N_U}|H_j(k)|^2}}\; s_k(n) \\ &+ w_k(n), \end{aligned} \quad (7)$$

where $w_k(n)$ indicates the additive noise over the k-th sub-carrier. The second term on the right-hand side represents the interference induced by the channel error. Even though this disturbance is symbol-dependent, its multiplication by the error terms $\epsilon_j(k)$, which are uncorrelated from the symbols, gives rise to an interference term uncorrelated (although not independent) from the symbols. We can define the Signal-to Interference plus Noise ratio (SINR) at the receiver, on the k-th sub-carrier, as the ratio between the power of the first term on the right-hand side of (7) and the sum of the power of the other two contributions. Using (7), the result is

$$SNIR(k) = \frac{\mathcal{K}\sum_{j=1}^{N_U}|H_j(k)|^2\sigma_s^2 - \sigma_n^2}{\left(\mathcal{K} - \frac{\sigma_n^2}{\sigma_s^2\sum_{j=1}^{N_U}|H_j(k)|^2}\right)\sigma_e^2\sigma_s^2 + \sigma_n^2}. \quad (8)$$

Equation (8) is interesting because it shows that at high SNR, i.e. when the ratio σ_n^2/σ_s^2 can be neglected, the SNIR has a floor, which represents the signal-to-interference ratio (SIR), equal to

$$SIR(k) \approx \frac{\sum_{j=1}^{N_U}|H_j(k)|^2}{\sigma_e^2}. \quad (9)$$

Conversely, when the channel estimation error is negligible, i.e. when we can consider $\sigma_e = 0$ in (8), we have a signal-to-noise ratio (SNR) equal to

$$SNR(k) \approx \frac{\mathcal{K}\sum_{j=1}^{N_U}|H_j(k)|^2}{\sigma_n^2} - 1. \quad (10)$$

Both equations (9) and (10) show that, at least for low estimation errors, where the first order perturbation analysis is valid, transmitting with N_U antennas provides a diversity gain of N_U over both the SNR and the SIR.

Finally, it is interesting to check the performance in terms of average BER. We have assumed, for simplicity, a fixed constellation, namely a QPSK constellation, even though this is clearly not the optimal choice, in terms of information rate. In the absence of channel estimation errors, i.e. when $\sigma_e^2 = 0$, the BER corresponding to a given channel realization and averaged on the used sub-carriers is

$$BER(\boldsymbol{h}_1, \ldots, \boldsymbol{h}_{N_U}) = \frac{1}{|\mathcal{I}_u|}\sum_{k\in\mathcal{I}_u} BER_k(\boldsymbol{h}_1, \ldots, \boldsymbol{h}_{N_U}), \quad (11)$$

where $BER_k(\boldsymbol{h}_1, \ldots, \boldsymbol{h}_{N_U}) = 2p_e(k) - p_e^2(k)$, with $p_e(k) = 0.5\, erfc\left(\sqrt{0.5SNIR(k)}\right)$, $SNIR(k)$ is given by (8) for $\sigma_e^2 = 0$ and $|\mathcal{I}_u|$ denotes the cardinality of the set I_u. In the presence of channel estimation errors, we can still use (11) with the SNIR as defined in (8), but only as a first order approximation, because of the approximations made to derive (8) and also because the interference term is not independent of the symbols. Furthermore, using (8) within (11) is equivalent to consider the interference term, which should act as a bias, like an additive Gaussian noise. Therefore, (11) should only be taken as an approximate expression for the BER. Nevertheless, we will show in the next section that such an approximation provides a good fit with the simulation results.

4. NUMERICAL RESULTS

4.1. Diversity gain

We report in Figs. 1 and 2 the behavior of the BER, averaged over $1,000$ independent channel realizations. The channel are simulated as FIR filters of order $L = 5$, with taps generated as i.i.d. zero-mean complex Gaussian random variables, with unit variance. We have computed the BER by simulation, incorporating the errors in the channel knowledge. We have supposed to have an estimate of the channel impulse response $\hat{h}_k(n) = h_k(n) + \delta_k(n)$, where the errors $\delta_k(n)$ are i.i.d. zero mean, Gaussian complex random variables, with variance σ_δ^2. The channel transfer functions $\hat{H}_k(l)$ are then computed by taking the N-point FFT of $\hat{h}_k(n)$. The result is a set of values $\hat{H}_k(l) = H_k(l) + \epsilon_k(l)$, where the random variables $\epsilon_k(l)$ are i.i.d. complex Gaussian random variables with zero mean and variance $\sigma_e^2 = (L+1)\sigma_\delta^2$. In particular, in Fig. 1, we report the average BER vs. the ratio σ_s^2/σ_n^2, for different values of the error-to-signal ratio, defined as σ_e^2/σ_s^2. The number of cooperating users is $N_U = 4$ and the error-to-signal ratio is equal to $-20, -10$, or -5 dB. Solid lines refer to the theoretical formula (11), whereas the dotted line refers to simulations. We can see that, in spite of the approximations, the theoretical curve is able to predict the simulated values very closely. In particular, the theoretical curve is able to predict the floor induced by the channel errors.

To evaluate the effect of increasing the number of cooperating users, in Fig.2, we report the average BER, again as a function of σ_s^2/σ_n^2, setting σ_e^2/σ_s^2 equal to -10 dB, and using $N_U = 1, 2$, and 4 users. We can clearly see the gain resulting from cooperation among the users, also in the presence of channels' estimate errors.

4.2. Rate gain

If the BS has only one antenna, the cooperative system can give an advantage only in terms of diversity, but not in terms of rate. However, if the BS has, let us say, N_R receive antennas, if the number

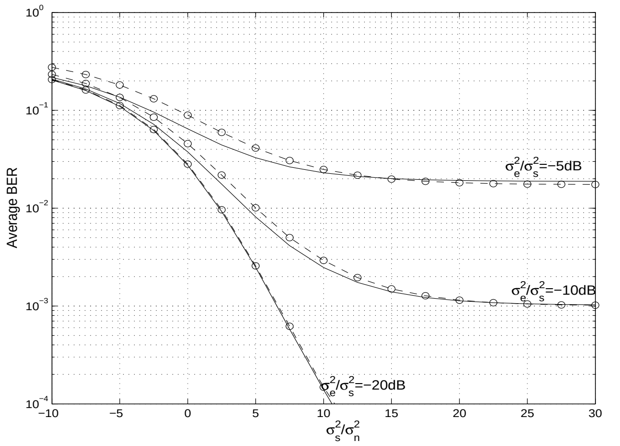

Fig. 1. Average BER vs. σ_s^2/σ_n^2, for $\sigma_e^2/\sigma_s^2 = -5dB, -10dB, -20dB$ and $N_U = 4$..

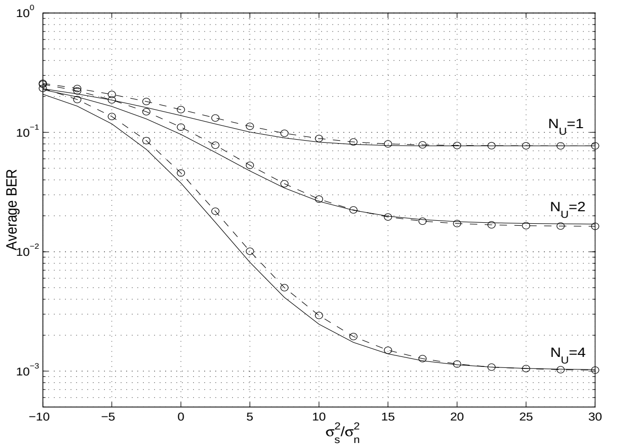

Fig. 2. Average BER vs. σ_s^2/σ_n^2, for $\sigma_e^2/\sigma_s^2 = -10dB$ and $N_U = 1, 2, 4$.

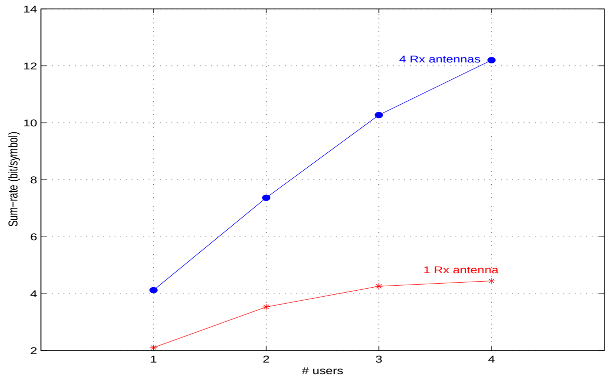

Fig. 3. Average Maximum Sum-Rate (bit/symbol) vs. the number of cooperating users

of users is N_U, perfect cooperation would induce a *virtual* MIMO channel. If the channels can be supposed to be independent, there is a potential increase of capacity, using the same arguments established, for example, in [5] for MIMO wireless systems. Clearly, the situation in our case is much more complicated because we have assumed frequency selective channels and we need perfect synchronization among the users. As a numerical result, we report in Fig. 3 the maximum sum-rate, expressed in bits/symbol, obtained averaging over $2,000$ independent channel realizations, as a function of the number of cooperating users. The two curves refer to the case of 1 receive antenna ('$*$'), 4 receive antennas ('o') and $N_U = 1, 2, 3, 4$. The channel is frequency selective with order $L = 4$ and CSI is available with no error to all users. We can clearly observe the rate increase due to perfect cooperation among the users.

5. CONCLUSION

In summary, cooperation among users at the physical layer level can yield a considerable gain both in terms of diversity and capacity. In this paper, we have considered a simple case where the exchange of data among users occurs without errors. Furthermore, different users are perfectly synchronous. This simplification allowed us to derive a closed form expression for the optimal coding and for the performance loss resulting from imperfect knowledge of the channel. The formulas are interesting because they have a clear physical justification. In particular, we showed that optimal joint coding leads to a combined beamforming/water-filling technique. We are currently investigating the effect of interferences and decision errors in the links between cooperating users.

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