FREQUENCY-HOPPED NETWORK DIVERSITY MULTIPLE ACCESS FOR SEMI-AD-HOC WIRELESS NETWORKS

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ABSTRACT

We consider a wireless data network setup that provides scalability and robustness to node failures, while achieving high throughput and low mean delay. A semi-ad-hoc Frequency-Hopped (FH) Network Diversity Multiple Access (NDMA) scheme is proposed. In this scheme, nodes are dynamically organized into clusters, where each cluster operates under NDMA random access protocol with FH on a per packet slot basis using a cell-specific FH pattern. In a moderately loaded system, collisions between packets from different clusters are not negligible. In order to guard against such collisions, we propose NDMA with a fixed retransmission margin of M slots. For this system, we derive a sufficient condition for stability using a deterministic fluid arrivals model. We also analyze steady-state, and obtain useful formulas for approximating mean packet delay.

1. INTRODUCTION

Recent developments in wireless networking gravitate towards ad-hoc or semi-ad-hoc architectures, for scalability and robustness to node failures. This is of particular importance in military battlefield communications, where network survivability is a key issue. In semi-ad-hoc setup, nodes are organized into clusters, where lead node, that acts as a base station, is dynamically determined. Any node can act as a lead node, unlike in the classical cellular setup. To reduce interference among clusters, each cluster is assigned a pseudo-random frequency-hopping sequence. There is no synchronization among clusters. All nodes within a cluster are synchronized and use the same FH code.

To provide low-delay high-throughput data transfer, we are interested in applying improved random access schemes for multiplexing bursty sources within each cluster. In particular, we propose the use of Network-assisted Diversity Multiple Access (NDMA) [3], which achieves throughput close to 1 while having low mean delay over a wide range of offered loads.

The basic idea behind the NDMA protocol is that after a collision due to random access, the data is not discarded. Instead, immediate retransmissions of the same packets are requested using MAC layer, in order to generate diversity of the received packets. After K linear mixtures of the K collided packets is collected, a source separation method is applied to recover all packets. Thus, NDMA has throughput close to 1 and achieves collision resolution (CR) delay lower bound of any splitting algorithm for slotted ALOHA [3].

Organizing nodes into clusters provides flexibility of the network close to that of a pure ad-hoc architecture, while clusters using NDMA provide more efficient bandwidth and energy utilization than pure ad-hoc setup. Namely, NDMA resolves collisions without wasting slots. In pure ad-hoc networks, collisions waste slots, thereby decreasing throughput, and bandwidth and energy efficiency. In this paper, we extend NDMA analysis to cluster architecture to capitalize on its CR capability.

NDMA Protocol[3] Consider a discrete-time, slotted system with one lead node and J user-nodes synchronized to slot timing. Nodes have infinite-capacity buffers. Arrivals of packets are independent across nodes. At the beginning of each slot, a node transmits one packet if it is allowed to transmit and its buffer is nonempty.

Since listen-while-you-talk is not feasible in wireless environment, due to shadowing and fading, the lead node detects collisions and provides feedback to user-nodes. We assume 0/1/e feedback, that is available to all nodes at the beginning of each slot. Since clusters have small radius, feedback can be implemented through TDD or TD multiplexing. In the noiseless case, 0/1 feedback suffices: 0 allows all nodes to transmit, whereas 1 enables nodes that transmitted in the previous slot and disables all others. In the header of each packet, the originating node's signature (ID) is embedded. These IDs are mutually orthogonal. Assuming frequency-flat fading, the lead node can use matched filters to detect identities of the in-cluster nodes that collided, and determine the collision multiplicity. For a K-fold collision, $T = K$ successive transmissions are requested

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(initial collision and $K - 1$ retransmissions), so that the baseband-equivalent discrete-time model is

$$\boldsymbol{X}_{T\times N} = \boldsymbol{A}_{T\times K}\boldsymbol{S}_{K\times N} + \boldsymbol{W}_{T\times N} \tag{1}$$

where $\boldsymbol{X}$ is the received data, $\boldsymbol{A}$ is the mixing matrix, rows of $\boldsymbol{S}$ are N-symbol long data packets and $\boldsymbol{W}$ is AWGN. The channel is assumed constant over one slot, but different from slot to slot. Collision resolution is performed by finding the ML estimate of $\boldsymbol{A}$ based on the known ID sequences, followed by $\widehat{\mathbf{S}} = \boldsymbol{A}^{\dagger}_{ML}\boldsymbol{X}$.

If there is a collision with packets from another cluster during one slot, then K-fold diversity is not achieved after K-1 retransmissions. One more retransmission must be requested and the contaminated slot must be discarded. Note that due to lack of synchronization between clusters, packet(s) from another cluster cannot be detected using the IDs. In order to guard against such collisions, we propose NDMA protocol with a fixed retransmission margin of M slots (NDMA-M), i.e., for a K-fold collision, a total of $K - 1 + M$ retransmissions will be requested. Then, the CR algorithm can detect and discard contaminated slots, if there is any, by fitting the model in (1) with $T = K+M$ and using the known orthogonal IDs of the users within the cluster; followed by energy detection on the rows of the residual $\widehat{\mathbf{W}}$. Up to M collision slots can be discarded this way; the choice of M depends on the system-wide collision probability, which is a function of the number of frequency bins and the number of active clusters in the network.

Due to small cluster size (J on the order of 10), packet IDs are short and the associated overhead negligible. Also, the complexity of CR algorithm is small due to small cluster size ($K \leq J$). Due to FH, the channel frequency response is sampled at a different point in each slot. Therefore, the complex channel gain of a node is different from slot to slot which guarantees that $\boldsymbol{A}$ is full column rank w.p. 1.

For the multiple access system within a cluster, we derive conditions for stability in the noiseless case, under a deterministic fluid traffic model.

2. STABILITY ANALYSIS

We use the following definition of stability.

Definition 1 *The system is stable if every queue remains bounded for all time, irrespective of (finite) initial conditions.*

To find a sufficient condition for stability we use a deterministic fluid model for packet arrivals, first suggested by Cruz [1]. In this model, the number of packet arrivals to the jth queue over the time interval $[s, t)$, denoted by $n_j(s, t)$, satisfies the following deterministic constraints along each sample path:

$$n_j(s,t) \leq \lambda_j(t-s) + \sigma_j, \tag{2}$$

where λ_j is the long-term average arrival rate of queue j

$$\lambda_j = \lim_{t\to\infty} \frac{n_j(0,t)}{t}, \tag{3}$$

and $0 \leq \sigma_j < \infty$ is a measure of burstiness [1]. Time is measured in slots, so that the variables s, t are integer. Note that this approach conforms to a leaky bucket rate control mechanism.

The deterministic fluid arrivals model captures the essence of the NDMA-M protocol, without distractions due to intricate asymptotic probabilistic behavior that arise under Poisson arrivals.

Theorem 1 *The NDMA-M system is stable if*

$$\sum_{j=1}^{J} \lambda_j + M \max_{j=1}^{J}(\lambda_j) < 1. \tag{4}$$

In the special case of $M = 1$, we have proven that if (4) is satisfied, then every queue in the system will empty out in finite time irrespective of initial conditions [2]. Therefore, every queue will empty out infinitely often as time tends to infinity. This implies that the state (backlog) of every queue remains bounded for all time, irrespective of initial conditions. The proof for $M > 1$ is a generalization of the proof for $M = 1$ [2].

3. STEADY-STATE ANALYSIS

Assume deterministic fluid arrivals model (2) and assume that all queues are stable (condition (4) is satisfied). It follows that all queues remain bounded for all time. Then long-term sample averages of queue lengths exist. Let $E[.]$ denote the time-averaging operator.

Let $\mathbf{s}(m) := [s_1(m), \ldots, s_J(m)]^T$ denote the vector of queue lengths at the beginning of slot m. Define CR *epoch* as a sequence of consecutive slots required for CR (initial transmission and successive retransmissions). Let $l(k)$ denote the length of the kth epoch, where $1 \leq l(k) \leq J + M$. Note that the behavior of the service portion of the NDMA-M system is determined by the state of queues at the beginnings of epochs. Therefore, it is convenient to consider queue lengths at the beginning of CR epoch k, $\mathbf{s}(k) := [s_1(k), \ldots, s_J(k)]^T$. Let $\tau_k(j) := I\{s_j(k) = 0\}$, where $I\{.\}$ is the indicator function, indicate that queue j is empty at the beginning of epoch k. We define $P_{e,j}$ as the long-term fraction of the number of epochs that queue j starts empty

$$P_{e,j} := \lim_{k\to\infty} \frac{\sum_{h=0}^{k} \tau_h(j)}{k} = E[\tau_k(j)]. \tag{5}$$

We want to find a relation between $P_{e,j}$ and the vector of average arrival rates $(\lambda_1, \ldots, \lambda_J)$, for $j = 1, \ldots, J$. Using this relation we can approximate the probability mass function (PMF) of CR epoch length. This is used in approximating mean delay for each queue (see Section 4). In the steady-state, there must be balance between the average number of incoming and outgoing packets of each queue. Writing this on a per-average-epoch basis, we obtain

$$\lambda_j E[l] = 1 - P_{e,j},\ \text{for } j = 1,\ldots,J, \\ \text{and hence } E[l]\sum_{j=1}^{J}\lambda_j = \sum_{j=1}^{J}(1-P_{e,j}). \tag{6}$$

Note that the average number of transmitted packets per epoch by queue j is $(1-P_{e,j})$, because if a queue is not empty at the beginning of an epoch, it transmits exactly one packet during that epoch. Denoting the number of active queues during the kth epoch as $a(k)$, we have

$$E[a] = \sum_{j=1}^{J}(1-P_{e,j}). \tag{7}$$

According to the NDMA-M protocol, the relation between epoch length and the number of active queues is

$$l(k) = \begin{cases} 1 & a(k) = 0 \\ a(k)+M & a(k) > 0 \end{cases} \tag{8}$$

Averaging (8), we obtain

$$E[l] = E[a] + M - (M-1)P(\mathbf{s}(k) = \mathbf{0}), \tag{9}$$

where $\mathbf{s}(k) = \mathbf{0}$ denotes the all-queues-empty state. Define $\tau_k(1,\ldots,J) := I\{\mathbf{s}(k) = \mathbf{0}\}$, so that $P(\mathbf{s}(k) = \mathbf{0}) := E[\tau_k(1,\ldots,J)]$ denotes the long-term fraction of the number of idle epochs – those that begin when all the queues are empty.

To make the analysis tractable, we approximate $P(\mathbf{s}(k) = \mathbf{0})$ with $\prod_{j=1}^{J} P_{e,j}$. Let $\mathbf{P}_e = (P_{e,1},\ldots,P_{e,J})$. Substituting (6) and (7) into (9), and using the above approximation, it follows that $P_{e,j}$, for $j = 1,\ldots,J$ satisfy

$$\boldsymbol{\mathcal{P}}_j(\mathbf{P}_e) = 0,\ \text{where} \\ \boldsymbol{\mathcal{P}}_j(\mathbf{P}_e) = -\lambda_j(M-1)\prod_{i=1}^{J} P_{e,i} + zP_{e,j} - z + M\lambda_j, \\ \text{and } z = (1-\sum_{i=1}^{J}\lambda_i). \tag{10}$$

The uniqueness of solution of the above system of equations, is established in the following proposition.

Proposition 1 $P_{e,j}$ *has a unique solution in* $(0,1), \forall j$, *if*

$$\sum_{i=1}^{J}\lambda_i + M\max_{i=1}^{J}(\lambda_i) < 1$$

Note that this condition is the same as (4). Also, note that in the noiseless case, throughput is equal to the offered load, $\sum_{i=1}^{J}\lambda_i$, if the system is stable. Therefore, the above condition yields maximum stable throughput for approximate NDMA-M system (10). Proof of the proposition is given in the appendix.

4. DELAY

Average delay is approximated by modeling each queue as an M/G/1 queue with server vacation, where service time of queue j is equal to the length of an epoch in which queue j transmits a packet – relevant epoch – denoted $l_{rel,j}$, and vacation time is equal to the length of an epoch in which queue j is idle – irrelevant epoch – $l_{irr,j}$. For $j = 1,\ldots,J$, we have [3],[2]

$$D_j = E[l_{rel,j}] + \frac{\lambda_j E[l_{rel,j}^2]}{2(1-\lambda_j E[l_{rel,j}])} + \frac{E[l_{irr,j}^2]}{2E[l_{irr,j}]}, \tag{11}$$

where j denotes the index of the queue of interest. The first and second moments of $l_{rel,j}$ and $l_{irr,j}$ depend on the steady-state behavior of all queues. Let $I_{(h)}(k) := 1\{s_{(h)}(k) > 0\}$. Define $I(k) = [I_{(1)}(k),\ldots,I_{(J)}(k)]^T$ and

$$S_{rel,t}(j) := \{I \mid t = \sum_{h=1,h\neq j}^{J} I_{(h)},\ and\ I_{(j)} = 1\}, \\ S_{irr,t}(j) := \{I \mid t = \sum_{h=1,h\neq j}^{J} I_{(h)},\ and\ I_{(j)} = 0\}.$$

Here, $S_{rel,t}$ and $S_{irr,t}$ are the sets of all permutations of t active queues excluding queue j, during a relevant or irrelevant epoch of queue j, respectively. The relation between epoch length, b, and the number of active queues, t, excluding queue j, for relevant epochs is: if $b = 1$, then $t = 0$; if $b > M$, then $t = b-(M+1)$; for irrelevant epochs: if $b \geq M$, then $t = b - M$. Finally, assuming independence of queues, the PMFs of the jth queue relevant and irrelevant epochs are:

$$P(l_{rel,j} = b) = \sum_{I\in S_{rel,t}(j)}\prod_{h=1,h\neq j}^{J}(1-P_{e,h})^{I_{(h)}}P_{e,h}^{1-I_{(h)}} \\ P(l_{irr,j} = b) = \sum_{I\in S_{irr,t}(j)}\prod_{h=1,h\neq j}^{J}(1-P_{e,h})^{I_{(h)}}P_{e,h}^{1-I_{(h)}}$$

5. SIMULATION RESULTS

We simulated a cluster with $J = 6$ queues, where each of the queues 1–3 and 4–6 has average arrival rate λ_1 and $\lambda_2 = 1.5\lambda_1$, respectively. This ratio remains constant as the sum of arrival rates changes. The number of extra retransmissions is $M = 2$. Fig. 1 shows $P_{e,j}$ for all queues, against total offered load. Note the accuracy of approximation at low loads and near the stability boundary. Fig. 2 plots D_j for all queues against total offered load and compares it with the delay characteristic of first-come-first-serve (FCFS) splitting algorithm for slotted ALOHA under the same scenario. Actual delay performance has a tolerable mismatch with analytic performance, due to M/G/1 with server vacations and epoch length PMF approximations.

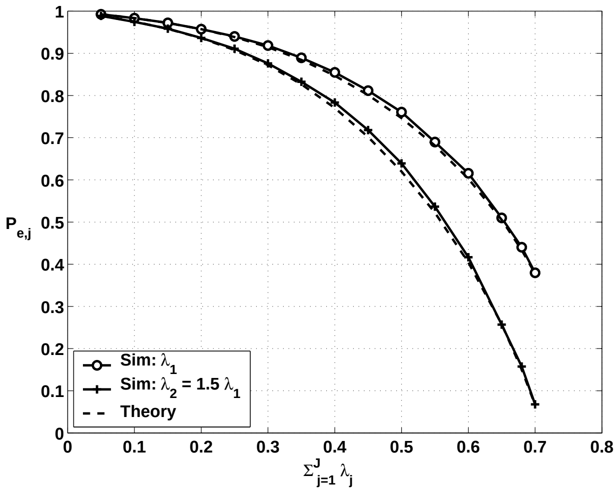

Fig. 1. $P_{e,j}$ vs. total load: 6 users, 2 classes

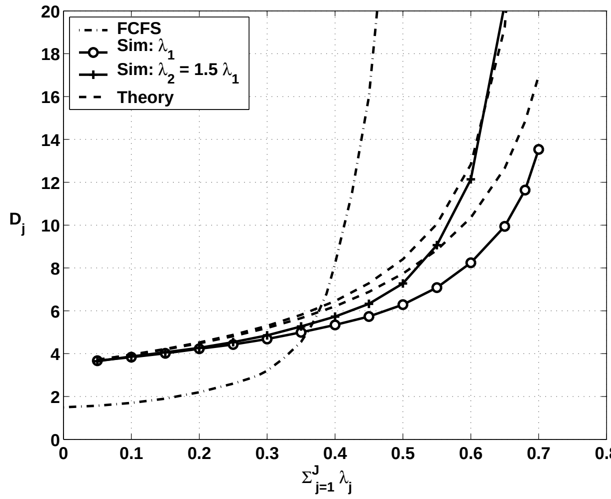

Fig. 2. Delay vs. total load: 6 users, 2 classes

6. CONCLUSIONS

We have proposed the use of the NDMA protocol, which has throughput close to 1 and low delay characteristic over a wide range of offered loads, in clustered semi-ad-hoc networks. Analysis of the NDMA protocol is extended to the case with the retransmission margin of M slots. A sufficient condition for stability of NDMA-M system under deterministic fluid arrivals model is provided. An NDMA-M model is proposed, which serves for determining the maximum stable throughput, and for evaluating mean packet delay. Simulations show that the analysis provides good approximation of the steady-state distribution and delay performance. All results apply to asymmetric systems.

7. APPENDIX

Proof (Proposition 1) Cases for $M = 0$ and $M = 1$ are proven in [2]. Consider $M > 1$ and assume $0 < \lambda_1 \leq \ldots \leq \lambda_J$. Let $c_j = \frac{\lambda_j}{\lambda_J}$ ($c_j \in (0,1]$). Multiplying $\mathcal{P}_J(\mathbf{P}_e) = 0$ by c_j and subtracting the result from $\mathcal{P}_j(\mathbf{P}_e) = 0$ for $j = 1, \ldots, J-1$ we have a system equivalent to (10)

$$P_{e,j} = c_j P_{e,J} + (1 - c_j), \text{ for } j = 1, \ldots, J-1$$
$$-\lambda_J (M-1) \prod_{i=1}^{J} P_{e,i} + z P_{e,J} - z + M\lambda_J = 0,$$

Hence, if $P_{e,J} \in (0,1)$, then $P_{e,j} \in (0,1)$ for all j. Using equations 1 to $J-1$ of the above system, one can express $\mathcal{P}_J(\mathbf{P}_e)$ as $\mathcal{P}_J(P_{e,J})$ with $(c_1, \ldots, c_J)$ as parameters. Thus, it suffices to prove that $\mathcal{P}_J(P_{e,J})$ has exactly one root $P_{e,J} \in (0,1)$ if (4) holds. Given $(\lambda_1, \ldots, \lambda_J)$, if (4) is satisfied, we have

$$\lim_{P_{e,J}\to 0} \mathcal{P}_J(P_{e,J}) = \sum_{i=1}^{J} \lambda_i + M\lambda_J - 1 < 0,$$
$$\lim_{P_{e,J}\to 1} \mathcal{P}_J(P_{e,J}) = \lambda_J > 0. \qquad (12)$$

Therefore, there is at least one root in $(0,1)$. In addition,

$$\mathcal{P}'_J(P_{e,J}) = z - \lambda_J (M-1) \sum_{i=1}^{J} c_i \prod_{g=1, g\neq i}^{J} [c_g P_{e,J} + (1 - c_g)]$$

$$\lim_{P_{e,J}\to 0} \mathcal{P}'_J(P_{e,J}) \geq 1 - \sum_{i=1}^{J} \lambda_i - (M-1)\lambda_J > 0$$
$$\lim_{P_{e,J}\to 1} \mathcal{P}'_J(P_{e,J}) = 1 - M(\textstyle\sum_{i=1}^{J} \lambda_i).$$

The last result is inconclusive. We proceed with $\mathcal{P}''_J(P_{e,J})$.

$$\mathcal{P}''_J(P_{e,J}) = -\lambda_J (M-1) \textstyle\sum_{i=1}^{J} c_i \sum_{g=1, g\neq i}^{J} c_g \times$$
$$\times \textstyle\prod_{d=1, d\neq g, d\neq i}^{J} [c_d P_{e,J} + (1 - c_d)] < 0, \; P_{e,J} \in (0,1).$$

Therefore, if $\lim_{P_{e,J}\to 1} \mathcal{P}'_J(P_{e,J}) > 0$ then $\mathcal{P}'_J(P_{e,J}) > 0$, for $P_{e,J} \in (0,1)$, and hence $\mathcal{P}_J(P_{e,J})$ has exactly one root in $(0,1)$. Otherwise, if $\lim_{P_{e,J}\to 1} \mathcal{P}'_J(P_{e,J}) \leq 0$, then $\mathcal{P}'_J(P_{e,J})$ has exactly one root in $(0,1)$. Then $\mathcal{P}_J(P_{e,J})$, for $P_{e,J} \in (0,1)$, is concave (it has exactly one peak). This and (12) implies that $\mathcal{P}_J(P_{e,J})$ has exactly one root for $P_{e,J} \in (0,1)$ in the latter case. Hence $\mathcal{P}_J(P_{e,J})$ always has exactly one root for $P_{e,J} \in (0,1)$.

8. REFERENCES

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