THROUGHPUT MAXIMIZATION WITH BUFFER CONSTRAINTS IN BROADBAND OFDMA NETWORKS

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ABSTRACT

This paper presents a radio resource allocation scheme for OFDMA-based wireless broadband networks. The problem of maximizing the base-station packet throughput subject to individual users' outage probability constraints is formulated. The proposed algorithm assumes a finite buffer for the arrival packets and allocates the radio resource based on users' channel characteristics and traffic patterns. By performing the radio resource allocation in two steps, namely bandwidth allocation and channel assignment, admission control is realized with low complexity. Simulations show that the algorithm yields significant lower outage probability and higher throughput compared to existing multiple access methods.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a form of modulation that presents the type of parallelism most suitable for DSP implementation. Moreover, adaptive modulation [1] can be used to significantly improve the performance of OFDM system over frequency selective channel. One way of applying OFDM to a multi-user scenario is through OFDM-TDMA or OFDM-CDMA [2], where different users are allocated different time slots or different subsets of orthogonal codes. Alternatively, people divide the total bandwidth into traffic channels (one or a cluster of OFDM subcarriers) so that multiple access can be accommodated in an orthogonal frequency division multiple access (OFDMA) fashion, e.g. IEEE 802.16a. A salient feature of OFDMA over OFDM-TDMA and OFDM-CDMA is its capability of avoiding "null" traffic channels (due to deep fading and narrowband interference). Since a null traffic channel for one user may still be favorable to other users, such can be accomplished by judicious subcarriers allocation.

Optimum channel allocation in OFDMA is an NP-hard problem that is fundamentally difficult to tackle. In practice, additional constraints, e.g., individual user's rate requirements, further complicate the problem. Please refer to [4] and [5] for details. In both papers, non-iterative approaches are developed to offer near-optimum performance with manageable computations. Despite their significance, none of these approaches consider the packet nature of the traffic at the base station, where buffer overflow is one of the important causes of the traffic outage and throughput degradation. For example, if a user is assigned with a deep faded channel, its packets are likely to get stuck in the buffer while certain mechanism (e.g. ARQ) trying to recover lost packets over the air. In this case its buffer will eventually overflow. Once an overflow occurs, packet losses can only be recovered within high level protocols (e.g. TCP), which will significantly degrades the system performance.

The objective of this paper is to develop a radio resource allocation strategy that addresses the buffer overflow problem in base station of OFDMA systems. Towards this end, a constrained optimization problem is formulated and is addressed with a two-step algorithm.

The remainder of the presentation is organized as follows. In Section 2, the system model is described and the constrained optimization problem is formulated. Section 3 derives the two-step algorithm. In Section 4, simulation results against the performance of conventional multiple access techniques are analyzed, and finally, the paper is summarized in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider an OFDMA system with M users and N traffic channels at the base station. Define one OFDM symbol transmission time as a time slot. Packets coming into user m's buffer is modeled as a Poisson process with independent rate λ_m packets per slot. For all practical purposes, a finite buffer of size d_m packets is allocated to user m. The channel conditions associated with users are characterized by an SNR matrix $\mathbf{V}_{\mathrm{M}\times\mathrm{N}}$ with entries v_{mn} denoting the SNR level seen by the m^{th} user on channel n.

The following assumptions are invoked for the ensuing analysis:

- Perfect knowledge of users' channels.
- Centralized traffic channel allocation at the base station.
- Adaptive modulation on each traffic channel.

Assumption 1 is reasonable for fixed or portable applications (e.g. IEEE 802.16a) with static or semi-static channels. The radio resource allocation algorithm is implemented at the base station and users are notified of the traffic channel assignment through a multiple access protocol. Users then employ adaptive modulation on each traffic channel assigned to them according to the channel SNRs.

Using adaptive modulation, the maximum number of bits channel n can transmit by user m (denoted as u_{mn}) per slot is expressed in terms of v_{mn} with a certain function $u_{mn} = f(v_{mn})$[1][4]. In the following analysis, we use u_{mn} normalized by packet length. As a result, the SNR matrix V is now fully characterized by a transmission rate matrix $U_{M\times N}$ with entries u_{mn}.

Let $\mathbf{X}_{\mathrm{M\times N}}$ be a channel assignment index matrix with entries $x_{mn},\ x_{mn} \in \{0,1\}$, in which $x_{mn} = 1$ indicates that the n^{th} channel been assigned to the m^{th} user and $x_{mn} = 0$ otherwise. Given the index matrix $\mathbf{X}$ and the transmission rate matrix $\mathbf{U}$, user m's packet processing rate μ_m is defined as its maximum transmission rate using all the traffic channels assigned to it:

$$\mu_m = \sum_{n=1}^{N} u_{mn} x_{mn} = \sum_{n\in\Phi_m} u_{mn} \tag{1}$$

where $\Phi_m = \{n : x_{mn} = 1, 1 \le n \le N\}$ denotes the traffic channel set assigned to user m with a total of N_m traffic channels in it.

For an OFDM system, the transmission interval between any two consecutive symbols is a predetermined constant, thus for user m, the packet arrival and the packets transmission process can be modeled as a $M/D/1/d_m$ queue with arrival rate λ_m, processing rate μ_m and buffer size d_m. We define the packet outage probability P_m as the blocking probability of an incoming packet when the buffer is full. The exact analysis of $M/D/1/d_m$ buffer occupancy and blocking probability involves recurrent solving of the mean busy period in an $M/D/1/d_m$ queue, and cannot be expressed in closed-form. According to [6] that studies several approximations for $M/G/1/d_m$ systems, Gelenbe's formula is the most accurate and robust one:

$$P_m = \frac{\lambda_m(\mu_m - \lambda_m)e^{-2\frac{(\mu_m-\lambda_m)(d_m-1)}{\lambda_m a^2+\mu_m s^2}}}{\mu_m^2 - \lambda_m^2 e^{-2\frac{(\mu_m-\lambda_m)(d_m-1)}{\lambda_m a^2+\mu_m s^2}}} \tag{2}$$

where a^2 and s^2 are the squared coefficients of variations of the arrival and service processes, respectively. In case of the $M/D/1/d_m$ system, $a^2 = 1$ and $s^2 = 0$. In the following analysis, we use this closed-form formula for the outage probability. With the outage probability, the throughput, i.e., actual load that joins the queue of user m, can be expressed as:

$$\eta_m = \lambda_m(1 - P_m). \tag{3}$$

With closed-form approximations for the outage probability and the traffic throughput, both η_m and P_m become direct functions of the allocation index matrix X.

With the model and approximations above, we seek to find a radio resource allocation algorithm so that the total throughput can be maximized while satisfying the outage probability requirement for each user. Incorporating the fact that no OFDMA traffic channel can be shared by more than one users at the same time, the optimization problem is mathematically formulated as follows:

$$\begin{aligned} &Max\ \eta_{total} = \sum_{m=1}^{M} \eta_m \\ &subject\ to\ 1)\ P_m \le O_m,\ m = 1,2,...,M \\ &\qquad 2) \sum_{m=1}^{M} x_{mn} = 1,\ n = 1,2,...,N. \end{aligned} \tag{4}$$

where O_m denotes the outage requirement of user m.

3. RESOURCE ALLOCATION STRATEGY

In this section, we present a radio resource allocation strategy that is divided into two steps: bandwidth allocation and channel assignment. The steps address different aspects of the optimization problem stated in (4).

1. *Bandwidth Allocation (BA)*: to decide the number of traffic channels assigned to each user.

2. *Channel Assignment (CA)*: to determine the specific traffic channel set associated with each user.

Based on this algorithm, an admission control mechanism becomes readily available.

3.1. Bandwidth Allocation (BA)

Based on the first constraint in (4), the following inequality must be satisfied:

$$\frac{\lambda_m(\mu_m - \lambda_m)e^{-2\frac{(\mu_m-\lambda_m)(d_m-1)}{\lambda_m}}}{\mu_m^2 - \lambda_m^2 e^{-2\frac{(\mu_m-\lambda_m)(d_m-1)}{\lambda_m}}} \le O_m.$$

Assume θ_m is the processing rate of user m that satisfies the following equation:

$$\frac{\lambda_m(\theta_m - \lambda_m)e^{-2\frac{(\theta_m-\lambda_m)(d_m-1)}{\lambda_m}}}{\theta_m^2 - \lambda_m^2 e^{-2\frac{(\theta_m-\lambda_m)(d_m-1)}{\lambda_m}}} = O_m.$$

It is easy to prove that $\frac{\partial P_m}{\partial \mu_m} < 0$, therefore, $P_m \leq O_m$ for $\forall \mu_m \geq \theta_m$ and then the first constraint in (4) becomes:

$$\mu_m \geq \theta_m. \tag{5}$$

It is difficult to give a closed-form expression of θ_m in terms of λ_m, O_m and d_m. In practice a numerical solution can be easily obtained.

The following proposition establishes a direct relationship between the number of traffic channels needed for the m^{th} user in order to satisfy its outage probability requirement and its traffic and channel conditions.

Proposition 1 *Let $N_m = \lceil \theta_m/\alpha_m \rceil$, where $\alpha_m = \sqrt[N]{\Pi_{n=1}^{N} u_{mn}}$ denotes the geometric mean of user m's transmission rate over all the channels. By allocating no less than N_m traffic channels to user m, its outage probability requirement can be satisfied almost surely.*

Proof. Define β_m as the geometric mean of user m's transmission rate over the channel set assigned to it: $\beta_m = \left(\Pi_{n=1}^{N} (u_{mn})^{x_{mn}}\right)^{1/N_m}$. Since the arithmetic mean is larger than or equal to the geometric mean,

$$\mu_m = \sum_{n=1}^{N} u_{mn} x_{mn} \geq N_m \left(\Pi_{n=1}^{N} (u_{mn})^{x_{mn}}\right)^{1/N_m} = N_m \beta_m. \tag{6}$$

If the right side of (6) is larger than or equal to θ_m, then (5) can be satisfied and so does the first constraint in (4). However at this stage, we have not decided on which channel set to assign to which user. We can replace β_m with α_m in view that the channel assignment (CA) procedure will likely yield a subset of channels for user m no worse than its average condition. As a result,

$$\mu_m \geq N_m \beta_m \geq N_m \alpha_m \geq \theta_m. \tag{7}$$

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To summarize, BA procedure is described as following:
For $m = 1 : M$ **do**
1) $\alpha_m \leftarrow \left(\Pi_{n=1}^{N} (\mu_{mn})\right)^{1/N}$;
2) $\theta_m \leftarrow P^{-1}(\lambda_m, O_m, d_m)$;
3) $N_m = \lceil \theta_m/\alpha_m \rceil$.
end for

As a result, the first constraint in (4) now becomes a restriction on the number of traffic channels assigned to users:

$$\sum_{n=1}^{N} x_{mn} \geq N_m, \; m = 1, 2...M. \tag{8}$$

Admission control: With the knowledge of bandwidth (number of traffic channels) required for each user after BA step, admission control can then be easily implemented. Let $Nr = \sum_{m=1}^{M} N_m$, the number of channels required from all the accessing users. Without loss of generality, let us assume $N_1 \leq N_2 \leq ... \leq N_M$. If $Nr > N$, i.e., there is not enough traffic channels for the assignment, one simple approach is to drop user M which requires the most bandwidth and recalculate Nr until the bandwidth requirement for each user can be satisfied.

3.2. Channel Assignment (CA)

After the BA step, we obtain the number of traffic channels assigned to each user. The remaining problem is to determine the exact set of traffic channels for them so that the total throughput at the base station is maximized. Combining (4) and (8), the optimization problem is restated as follows:

$$Max \sum_{m=1}^{M} \eta_m$$

$$subject\ to \quad 1) \sum_{n=1}^{N} x_{mn} \geq N_m \; m = 1, 2, ..., M$$

$$2) \sum_{m=1}^{M} x_{mn} = 1, \; n = 1, 2, ..., N. \tag{9}$$

Here we derive a Throughput Greedy Maximization (TGM) algorithm for channel assignment, the key strategy is to find a user for each channel so that its impact on the total throughput increase is maximized. TGM algorithm is based on greedy sensing, and is carried out without iterations. Specifically, the channel is assigned to the user that can increase the throughput to the maximum amount. We should keep in mind that in assigning the first Nr traffic channels, after user m get N_m traffic channels, it can not have more until all the users get the bandwidth they need to meet their outage probability requirements. After all the users' bandwidth requirements are satisfied, the remaining channels are assigned to users with no bandwidth constraint. The TGM algorithm is described as follows:

For each traffic channel $n = 1 : N$ **do**
$G_m \leftarrow \eta(\lambda_m, \mu_m + u_{mn}, d) - \eta(\lambda_m, \mu_m, d) \; \forall m$
$m^* \leftarrow \arg\max_m G_m$;
  while($sizeof(\Phi_{m^*}) \geq N_{m^*}$ & $n \leq Nr$)
    $G_{m^*} = -\infty$;
    $m^* \leftarrow \arg\max_m G_m$;
  end while
$\mu_{m^*} \leftarrow \mu_{m^*} + u_{m^*n}$;
$\Phi_{m^*} \leftarrow \Phi_{m^*} \cup \{n\}$.
end for

4. SIMULATION RESULTS

A 128−subcarrier OFDMA system is studied over Rayleigh fading channel of $1M$ bandwidth. Each subcarrier repre-

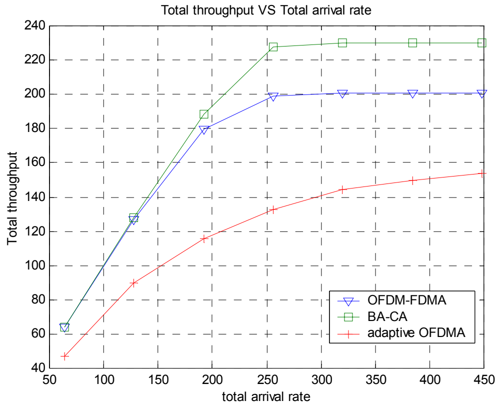

Fig. 1. Throughput vs. arrival rate.

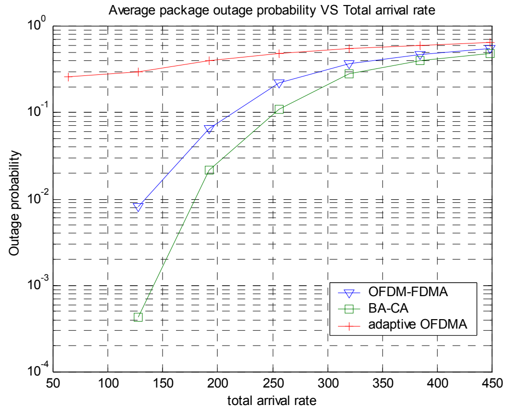

Fig. 2. Outage probability vs. arrival rate.

sents a traffic channel. The traffic channels are shared by 64 users and each user has an uncorrelated channel profile with average SNR $3dB$. The buffer size allocated to each user is set to be 20 packet length. The described two-step algorithm (BA-CA) is compared with OFDM-FDMA (equal bandwidth among users and random channel assignment) and adptive OFDMA [1] which assigns each channel to the user who has the highest SNR on it. Fig.1 and Fig.2 compare the throughput and outage probability respectively.

For all the simulated traffics, the two-step algorithm (BA-CA) algorithm yields higher throughput and lower outage probability than both OFDM-FDMA and adaptive OFDMA, with adaptive OFDMA being the worst of the three. The worst performance of adaptive OFDMA can be explained by its channel assignment criteria that assigns the channel to the user with the highest SNR level: the outage probability is almost one for those users who get few channels and almost zero for those users who get more than enough channels, resulting in unbalanced queue lengths and outage probabilities among users. The situation is even worse with unbalanced traffics. On the contrary, BA-CA utilizes the traffic information as well as channel condition to distributes radio resources intelligently, thus gives the best performance over the existing methods.

5. CONCLUSION

In this paper, a radio resource allocation algorithm for OFDMA system has been derived based on users' traffic and channel statistic information. The main idea of the scheme is to first perform the bandwidth allocation to satisfy the outage requirement for each user, and then the traffic throughput is maximized through a TGM algorithm developed in this paper. By dividing the radio resource allocation into two steps, admission control can be easily realized without iteration. Simulations show that the algorithm yields significantly higher throughput and lower outage probability compared to existing methods.

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