

A PERFORMANCE STUDY OF DUAL-HOP TRANSMISSIONS WITH FIXED GAIN RELAYS

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ABSTRACT

This paper presents a study on the end-to-end performance of dual-hop wireless communication systems equipped with non-regenerative fixed gain relays and operating over flat Rayleigh fading channels. More specifically, it first derives generic closed-form expressions for the outage probability and the average probability of error when the relays have arbitrary fixed gains. It then proposes a specific fixed gain relay that benefits from the knowledge of the first hop's average fading power and compares its performance with previously proposed relay gains that in contrast require knowledge of the instantaneous channel state information of the first hop. Finally, the paper investigates the effect of the relay saturation on the performance of the systems under consideration. Numerical results show that non-regenerative systems with fixed gain relays have a comparable performance to non-regenerative systems with variable gain relays. These results also show that relay saturation of these systems results in a minimal loss in performance.

1. INTRODUCTION

Dual-hop transmission is a technique by which the channel from the source to the destination is split into two, possibly shorter, links using a relay. This scenario was originally encountered in bent-pipe satellites where the primary function of the spacecraft is to relay the uplink carrier into a downlink [1]. It is also common in various fixed microwave links to enable greater coverage without the need of large power at the transmitter. More recently, this concept has gained new actuality in the context of collaborative/cooperative wireless communication systems [2]-[7]. In this case, the key idea is that a mobile terminal relays a signal between the base station and a nearby mobile terminal when the direct link between the base station and the original mobile terminal is in deep fade. As a result, similar to the scenarios described above, signals from the source to the destination propagate through two hops/links in series.

Depending on the nature and complexity of the relays, dual-hop transmission systems can be classified into two main categories, namely, regenerative or non-regenerative systems. In regenerative systems, the relay fully decodes the signal that went through the first hop and retransmits the decoded version into the second hop. This is also referred to as decode-and-forward [3] or digital [5] relaying. On the other hand, non-regenerative systems use less complex relays that just amplify and forward the incoming signal without performing any sort of decoding. That is why it is sometimes referred to as amplify-and-forward [3] or analog [5] relaying. As a further categorization, relays in non-regenerative sys-

tems can in their turn be classified into two subcategories, namely, (i) channel state information (CSI)-assisted relays and (ii) "blind" relays. Non-regenerative systems with CSI-assisted relays use instantaneous CSI of the first hop to control the gain introduced by the relay and as a result fix the power of the retransmitted signal. In contrast, systems with "blind" relays do not need instantaneous CSI of the first hop at the relay but rather employ at these relays amplifiers with a fixed gain and consequently result in a signal with variable power at the relay output. Although systems with such kind of blind relays are not expected to perform as well as systems equipped with CSI-assisted relays, their low complexity and ease of deployment (together with their comparable performance as we will show later in this paper), make them attractive from a practical standpoint. While the exact end-to-end performance of systems employing CSI-assisted relays was extensively studied and compared with that of regenerative systems in [6, 7, 8, 9], the performance of non-regenerative systems with blind relays has not been investigated so far. In this paper, we look into this problem and focus on the case in which the two hops experience independent not necessarily identically distributed Rayleigh fading.

The remainder of this paper is organized as follows. Next section introduces the system and channel models under consideration. Section 3 presents closed-form expressions for the outage probability and average probability of error of these systems. Section 4 proposes a specific "semi-blind" relay and studies its performance. Finally, section 5 addresses a practical implementation issue related to the effect of saturation on the performance of systems with semi-blind relays.

2. SYSTEM AND CHANNEL MODELS

Consider the wireless communication system shown in Fig. 1. Here,

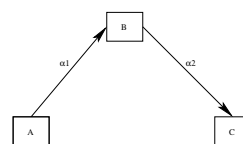


Fig. 1. A wireless communication system where terminal B is relaying the signal from terminal A to terminal C.

terminal A is communicating with terminal C through terminal B which acts as a relay. Assume that terminal A is transmitting a signal $s(t)$ which has an average power of \mathcal{E}_1 . The received signal at terminal B can be written as

$$r_b(t) = \alpha_1 s(t) + n_1(t), \quad (1)$$

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where α_1 is the fading amplitude of the channel between terminals A and B, and $n_1(t)$ is an additive white Gaussian noise (AWGN) signal with a power of N_{01} . The received signal is then multiplied by the gain of the relay at terminal B, G , and then retransmitted to terminal C. The received signal at terminal C can be written as

$$r_c(t) = \alpha_2 G(\alpha_1 s(t) + n_1(t)) + n_2(t), \quad (2)$$

where α_2 is the fading amplitude of the channel between terminals B and C, and $n_2(t)$ is an AWGN signal with power N_{02} . The overall signal-to-noise ratio (SNR) at the receiving end can then be written as

$$\gamma_{eq} = \frac{\frac{\mathcal{E}_1 \alpha_1^2}{N_{01}} \frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}}}. \quad (3)$$

It is clear from (3) that the choice of the relay gain defines the equivalent end-to-end SNR of the two hops. In case of available instantaneous CSI at B, a gain of

$$G^2 = \frac{\mathcal{E}_2}{\mathcal{E}_1 \alpha_1^2 + N_{01}}, \quad (4)$$

where \mathcal{E}_2 is the power of the transmitted signal at the output of the relay, was proposed in [3]. The choice of this gain aims to invert the fading effect of the first channel while limiting the output power of the relay if the fading amplitude of the first hop, α_1 , is low. However, this CSI-assisted relay requires a continuous estimate of the channel fading amplitude which may make this choice of gain not always feasible from a practical point of view. Substituting (4) in (3) leads to γ_{eq1} given by

$$\gamma_{eq1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}, \quad (5)$$

where $\gamma_i = \mathcal{E}_i \alpha_i^2 / N_{0i}$, ($i = 1, 2$) is the per hop SNR. The performance of systems employing such relays over Rayleigh fading channels was first studied in [3] and then in [6, 7]. In addition, [8, 9] presented tight lower bounds on the performance of all CSI-assisted relays for Rayleigh and Nakagami fading channel, respectively.

In this paper, we are interested in studying the performance of the other class of non-regenerative systems, namely, those with blind relays. These relays introduce fixed gains to the received signal regardless of the fading amplitude on the first hop. Let $C = \mathcal{E}_2 / (G^2 N_{01})$, then

$$\gamma_{eq2} = \frac{\gamma_1 \gamma_2}{C + \gamma_2}, \quad (6)$$

where C is a constant for a fixed G . The two hops are assumed to be subject to independent not necessarily identically distributed Rayleigh fading. Hence, γ_1 and γ_2 are exponentially distributed with parameters $\bar{\gamma}_1 = \mathcal{E}_1 \Omega_1 / N_{01}$ and $\bar{\gamma}_2 = \mathcal{E}_2 \Omega_2 / N_{02}$ respectively, where $\Omega_i = \overline{\alpha_i^2}$ ($i = 1, 2$) is the average fading power on the i th hop. In the following section, we present generic formulas, in terms of C , for the outage probability and average probability of error of systems equipped with blind relays. Later, we focus on a specific relay gain and compare its performance to the one proposed in [3] and studied in [3, 6, 7]. In what follows we refer to γ_{eq2} as simply γ_{eq} .

3. PERFORMANCE ANALYSIS

3.1. Outage Probability

In noise limited systems, outage probability is defined as the probability that the instantaneous equivalent SNR, γ_{eq} , falls below a

predetermined protection ratio, γ_{th} . Consequently, outage probability is given by

$$P_{out} = P[\gamma_{eq} < \gamma_{th}] = P\left[\frac{\gamma_1 \gamma_2}{\gamma_2 + C} < \gamma_{th}\right], \quad (7)$$

which can be calculated as

$$\begin{aligned} P_{out} &= \int_0^\infty P\left[\frac{\gamma_1 \gamma_2}{\gamma_2 + C} < \gamma_{th} | \gamma_2\right] p_{\gamma_2}(\gamma_2) d\gamma_2 \\ &= \int_0^\infty \frac{1}{\bar{\gamma}_2} \left[1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_1} \left(1 + \frac{C}{\gamma}\right)}\right] e^{-\frac{\gamma}{\bar{\gamma}_2}} d\gamma \\ &= 1 - \frac{1}{\bar{\gamma}_2} e^{-\frac{\gamma_{th}}{\bar{\gamma}_1}} \int_0^\infty e^{-\frac{\gamma_{th} C}{\bar{\gamma}_1 \gamma} - \frac{\gamma}{\bar{\gamma}_2}} d\gamma. \end{aligned} \quad (8)$$

The integration in (8) is evaluated using [10, Eq. (3.324.1)] to yield

$$P_{out} = 1 - 2\sqrt{\frac{C\gamma_{th}}{\bar{\gamma}_1 \bar{\gamma}_2}} e^{-\left(\frac{\gamma_{th}}{\bar{\gamma}_1}\right)} K_1\left(2\sqrt{\frac{C\gamma_{th}}{\bar{\gamma}_1 \bar{\gamma}_2}}\right), \quad (9)$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind defined in [11, Eq. (9.6.22)].

3.2. Average Probability of Error

In order to get the moment generating function (MGF) of γ_{eq} , $\mathcal{M}_{\gamma_{eq}}(s) = \mathbf{E}(e^{-s\gamma_{eq}})$, where $\mathbf{E}(\cdot)$ denote the statistical average operator, we need first to get the probability density function (PDF) of γ_{eq} . This PDF can be found by taking the derivative of (9) with respect to γ_{th} , yielding

$$\begin{aligned} p_{\gamma}(\gamma) &= \frac{2}{\bar{\gamma}_1} e^{-\left(\frac{\gamma}{\bar{\gamma}_1}\right)} \left[\sqrt{\frac{C\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}} K_1\left(2\sqrt{\frac{C\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}}\right) \right. \\ &\quad \left. + \frac{C}{\bar{\gamma}_2} K_0\left(2\sqrt{\frac{C\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}}\right) \right], \end{aligned} \quad (10)$$

where $K_0(\cdot)$ is the zero order modified Bessel function of the second kind defined in [11, Eq. (9.6.21)] and where we used the expression for the derivative of the Bessel function, given in [10, Eq. (8.486.12)] to get the desired result in (10). The MGF of γ_{eq} can now be calculated as

$$\begin{aligned} \mathcal{M}_{\gamma_{eq}}(s) &= \int_0^\infty \frac{2}{\bar{\gamma}_1} e^{-\left(\frac{\gamma}{\bar{\gamma}_1}\right)} \left[\sqrt{\frac{C\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}} K_1\left(2\sqrt{\frac{C\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}}\right) \right. \\ &\quad \left. + \frac{C}{\bar{\gamma}_2} K_0\left(2\sqrt{\frac{C\gamma}{\bar{\gamma}_1 \bar{\gamma}_2}}\right) \right] e^{-s\gamma} d\gamma. \end{aligned} \quad (11)$$

The integral in (11) can be evaluated with the help of [10, Eq. (6.643.3)] yielding an expression in terms of the Whittaker function which can be further simplified using identities [11, Eqs. (13.1.33), (13.6.28), (13.6.30)] and finally identity [10, Eq. (6.5.19)] to yield the compact closed-form expression

$$\mathcal{M}_{\gamma_{eq}}(s) = \frac{1}{(\bar{\gamma}_1 s + 1)} + \frac{C\bar{\gamma}_1 s e^{\left(\frac{C}{\bar{\gamma}_2(\bar{\gamma}_1 s + 1)}\right)}}{\bar{\gamma}_2(\bar{\gamma}_1 s + 1)^2} E_1\left(\frac{C}{\bar{\gamma}_2(\bar{\gamma}_1 s + 1)}\right), \quad (12)$$

where $E_1(\cdot)$ is the exponential integral function defined in [11, Eq. (5.1.1)].

Having the MGF of γ_{eq} in closed form as given in (12) and using the MGF-based approach for the performance evaluation of digital modulations over fading channels [12] allows to obtain the

average bit and symbol error rate for a wide variety of M -ary modulations (such as M -ary phase shift keying (M -PSK), M -ary differential phase shift keying (M -DPSK), and M -ary quadrature amplitude modulation (M -QAM)). For example, the average bit error rate (BER) of binary differential phase-shift keying (DPSK) is well known to be given by $P_b(E) = \frac{1}{2} \mathcal{M}_{\gamma_{\text{eq}}}(1)$.

4. SYSTEMS WITH SEMI-BLIND RELAYS

The performance analysis presented in the previous section applies to blind relays with arbitrary fixed gains. In this section, we still consider fixed gain relays and we still assume that these relays do not have access to instantaneous CSI of the first hop. However, we assume that they have statistical CSI about the first hop and have in particular knowledge of the average fading power Ω_1 which changes slowly (relative to α_1) and as such does not imply continuous monitoring of the channel (as it is the case in CSI-assisted relays). We call these relays “semi-blind” and we study in what follows their performance in comparison with that of CSI-assisted relays [3] as per (4).

The relay gain in the semi-blind scenario is chosen such as

$$G^2 = \mathbf{E} \left[\frac{\mathcal{E}_2}{\mathcal{E}_1 \alpha_1^2 + N_{01}} \right]. \quad (13)$$

This way, both relays in (4) and (13) consume the same power on average. For Rayleigh fading, G^2 in (13) can be shown to be given by

$$G^2 = \frac{\mathcal{E}_2}{\mathcal{E}_1 \Omega_1} e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right). \quad (14)$$

Consequently, C is given by

$$C = \frac{\bar{\gamma}_1}{e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right)}, \quad (15)$$

which when substituted back in (6) results in an equivalent end-to-end SNR of

$$\gamma_{\text{eq}} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{\bar{\gamma}_1}{e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right)}}. \quad (16)$$

Fig. 2 compares the outage probability performance of a dual-hop non-regenerative system employing a fixed gain relay as per (14) with that of an equivalent (in terms of average power consumption) system with a relay as per (4), whose exact performance was derived in [6]. The figure also plots, as a benchmark, the performance of the more complex regenerative systems studied in [8]. For the medium to large average SNR region, systems with variable gain relays (4) outperform those with fixed gain relays (14). However, the surprising and interesting result is that the gap in performance is not as much as one would expect in comparison with the difference in implementation and complexity of both relays. Note also that systems with fixed gain relays can even slightly outperform systems with variable gain relays at low average SNR. This is due to the fact that the variable gain relay has a gain floor of \mathcal{E}_2/N_0 when α_1 is too small, which is a relatively frequent event in the low average SNR region. Finally, note that as γ_{th} is increased, the range of average SNR in which fixed gain relays outperform variable gain relays extends to the right. Fig. 3 illustrates a similar comparison but from the average BER of DPSK perspective, where the performance results for variable gain relays were given in [7]. It is clear again that the gap in performance is very small for all ranges of average SNR.

5. EFFECT OF RELAY SATURATION

Up to this point, we assumed that the relay under study amplifies the signal by multiplying it with a fixed gain, regardless of the magnitude of the input signal. Obviously, under this mode of operation, if the first hop is only slightly faded then the relay amplifier may go into saturation. Although this problem may not be an issue when relaying is used over severely faded first hops, the loss in performance due to saturation has to be quantified especially in the context of cooperative/collaborative diversity where per design first hops are often picked such as good channel conditions exist between the transmitter and the relay. As such, assume that the relay has a maximum output power of $K\mathcal{E}_2$. Hence, a fixed gain as per (13) can be used as long as

$$G^2(\mathcal{E}_1 \alpha_1^2 + N_{01}) \leq K\mathcal{E}_2. \quad (17)$$

If the fading conditions of the first hop are such as (17) is violated, the output power is clipped at $K\mathcal{E}_2$, and as a result the relay reduces its gain. Mathematically speaking, this is equivalent to using, when (17) is violated, the relay gain (4) with a fixed output power of $K\mathcal{E}_2$. It is straightforward to transform the constraint (17) into a threshold on the SNR of the first hop above which clipping takes place. Consequently, the modified relay gain G_s is given by

$$G_s^2 = \begin{cases} \frac{\mathcal{E}_2}{\mathcal{E}_1 \Omega_1} e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right) & \gamma_1 < \mathcal{T} \\ \frac{\mathcal{E}_2}{\mathcal{E}_1 \alpha_1^2 + N_0} & \gamma_1 > \mathcal{T} \end{cases} \quad (18)$$

where the SNR threshold \mathcal{T} is given by

$$\mathcal{T} = \frac{K\bar{\gamma}_1}{e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right)} - 1, \quad (19)$$

and the resulting equivalent SNR can be shown to be given by

$$\gamma_{\text{eq}} = \begin{cases} \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{\bar{\gamma}_1}{e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right)}} & \gamma_1 < \mathcal{T} \\ \frac{K\gamma_1 \gamma_2}{\gamma_1 + K\gamma_2 + 1} & \gamma_1 > \mathcal{T} \end{cases} \quad (20)$$

Using this definition for the equivalent SNR, and assuming that $\gamma_{\text{th}} < \mathcal{T}$, the outage probability can be calculated in a similar fashion to (8) yielding

$$P_{\text{out}} = 1 - \frac{1}{\gamma_1} e^{-\frac{\gamma_{\text{th}}}{\gamma_1}} \int_0^{T-\gamma_{\text{th}}} e^{-\frac{\gamma_{\text{th}} C}{\gamma_2 \gamma} - \frac{\gamma}{\gamma_1}} d\gamma \\ - \frac{1}{\gamma_1} e^{-\frac{\gamma_{\text{th}}}{K\gamma_2} - \frac{\gamma_{\text{th}}}{\gamma_1}} \int_{T-\gamma_{\text{th}}}^{\infty} e^{-\frac{\gamma_{\text{th}}(\gamma_{\text{th}}+1)}{K\gamma_2 \gamma} - \frac{\gamma}{\gamma_1}} d\gamma. \quad (21)$$

As a double check, it is clear that (21) reduces to (9) when the threshold \mathcal{T} goes to infinity, where outage probability in that case can be evaluated in closed-form as per (9). One point to note here is that with the gain definition in (18), the overall average power consumed is less than that using (14) since less power is used during clipping. In order to have a fair comparison and investigation on the effect of saturation, the average power consumption for the equivalent gain in (18) has to be found in terms of the parameter K , and then this value can be used to set up an equivalent fixed gain relay. The average of (18) can be shown to be given by

$$\mathbf{E}(G_s^2) = \frac{\mathcal{E}_2 e^{\frac{1}{\gamma_1}}}{\mathcal{E}_1 \Omega_1} \left[E_1\left(\frac{1}{\gamma_1}\right) \left[1 - e^{-\frac{\mathcal{T}}{\gamma_1}}\right] + E_1\left(\frac{K}{e^{\frac{1}{\gamma_1}} E_1\left(\frac{1}{\gamma_1}\right)}\right) \right], \quad (22)$$

which when equated to the average power consumption of the relay in (14) results in a transcendental equation in K given by

$$E_1\left(\frac{1}{\bar{\gamma}_1}\right) e^{-\left(\frac{K}{e^{\frac{1}{\bar{\gamma}_1}} E_1\left(\frac{1}{\bar{\gamma}_1}\right)} - \frac{1}{\bar{\gamma}_1}\right)} = E_1\left(\frac{K}{e^{\frac{1}{\bar{\gamma}_1}} E_1\left(\frac{1}{\bar{\gamma}_1}\right)}\right). \quad (23)$$

Now the values of K resulting from solving (23) are used to construct an equivalent fixed gain relay with a gain given by

$$G^2 = \frac{K\mathcal{E}_2}{\mathcal{E}_1\Omega_1} e^{\frac{1}{\bar{\gamma}_1}} E_1\left(\frac{1}{\bar{\gamma}_1}\right), \quad (24)$$

where K is solution of (23) and as such is different for different values of $\bar{\gamma}_1$.

Fig. 4 compares the outage probability of a “practical” relay which uses clipping with an equivalent fixed gain relay. It is clear from the figure that the loss in performance is not only minimal but is also affecting only the low average SNR ranges. This is due to the fact that the proposed semi blind relay has a larger gain (relative to $\bar{\gamma}_1$) in these ranges and this makes clipping a more probable event.

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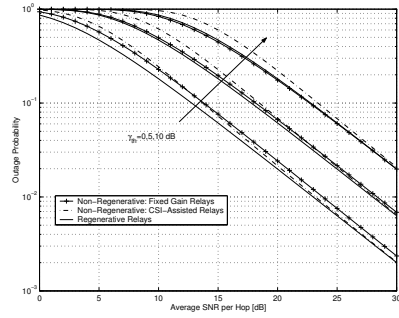


Fig. 2. Outage probability of a dual-hop system for different relay configurations.

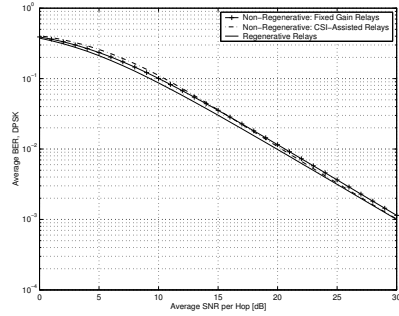


Fig. 3. Average BER of a dual-hop system for different relay configurations.

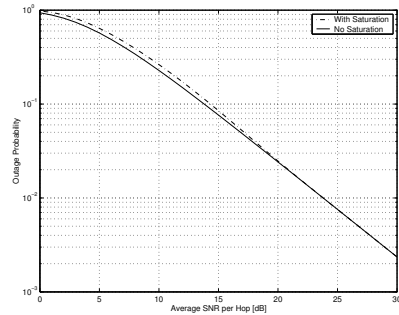


Fig. 4. Effect of relay saturation on the performance of non-regenerative systems with fixed gain relays, $\gamma_{th} = 0$ dB.