OPTIMAL SYMBOL MAPPING DIVERSITY FOR MULTIPLE PACKET TRANSMISSIONS

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ABSTRACT

In this paper we present a simple, but effective method of creating and exploiting diversity from packet retransmissions in systems that employ nonbinary modulations such as PSK and QAM. This diversity results from differing the symbol mapping for each packet retransmission. By developing a general framework for evaluating the bit error rate (BER) upper bound with multiple transmissions, a criterion to obtain optimal symbol (re)mappings is attained for memoryless AWGN channels. The optimal adaptation scheme reduces to solutions of the Quadratic Assignment Problem (QAP). Symbol mapping adaptation only requires a small increase in receiver complexity but provides very substantial BER gains.

1. INTRODUCTION

In many communication systems, if errors remain after error correction when processing a transmitted data packet, a frame error is declared and a request for retransmssion is made to the transmitter. In systems equipped with this Automatic Repeat reQuest (ARQ) mechanism, various approaches have been proposed for both packet combining and creating diversity among retransmissions. For example, Chase developed a maximum-likelihood combining scheme for an arbitrary number of packets [1]. Harvey and Wicker proposed several ARQ strategies, including an approach where soft-decoded codewords from the multiple packet retransmissions are combined into a single soft codeword [2]. Others, including Hagenauer, Rowitch, and Milstein, have developed schemes involving rate-compatible codes, where retransmitted copies of a packet are each uniquely punctured to improve throughput [3, 4]. Stuber and Narayanan developed an ARQ receiver for turbo codes where the extrinsic information from previous packets is reused [5]. Recent works in [6, 7, 8] introduced approaches which exploit diversity created by retransmissions through intersymbol interference (ISI) channels.

The purpose of this paper is to present a simple retransmission scheme for systems that employ higher-order modulations such as PSK or QAM in AWGN channels. By varying the bit-to-symbol mapping for each retransmission, the diversity is enhanced among M transmissions. We first develop an upper bound for the BER when M transmissions are made using M distinct symbol mappings. From this upper bound, an optimization problem is formulated to determine the mappings the produce the minimal BER. An iterative solution to this problem leads to finding M solutions to M instances of the Quadratic Assignment Problem (QAP). We present results found for several constellation types that illustrate the effectiveness of symbol mapping diversity among retransmissions.

*Supported in part by NSF grants CCR-0196364 and ECS-0121469 and Accel Partners, Inc.

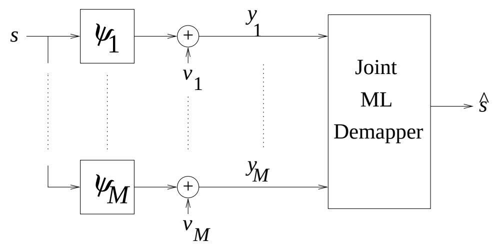

Fig. 1. Mapping diversity transmitter.

2. MAPPING DIVERSITY

Let us provide an overview of a mapping diversity retransmission system, with M denoting the number of packet transmissions. We begin with a set $\mathcal{C}$ of real or complex valued points that represent the points of a signal constellation, e.g. 16QAM. Given a packet of bits, consecutive groups of $\log_2 |\mathcal{C}|$ bits (s represents the decimal equivalent of these bits) are assigned to symbols in $\mathcal{C}$ via a symbol mapping function $\psi : \{0, 1, \ldots, |\mathcal{C}| - 1\} \rightarrow \mathcal{C}$. With M transmissions of a packet, we define M symbol mapping functions $\psi_1, \ldots, \psi_M$. Using different mappings enhances the diversity across multiple transmissions.

Fig. 1 shows a group of bits s that are ultimately transmitted M times (via M transmissions of a packet); the transmitter sends symbols $\psi_1[s], \ldots, \psi_M[s]$. The receiver obtains samples $y_m = \psi_m[s] + v_m$, where $m = 1, \ldots, M$ and $v_m = v_{m,r} + jv_{m,i}$ is a complex Gaussian random variable with $v_{m,r}$ and $v_{m,i}$ each zero-mean with variance σ_v^2. We assume that $v_1, \ldots, v_m$ are independent. From $y_1, \ldots, y_M$, the receiver decides that bits $\hat{s}$ were transmitted according to the maximum likelihood (ML) rule

$$\min_{\hat{s}=0,1,\ldots,|\mathcal{C}|-1} \sum_{m=1}^{M} |y_m - \psi_m[\hat{s}]|^2. \tag{1}$$

Note that the number of states $|\mathcal{C}|$ remains unchanged regardless of the number of transmissions made. In fact, the complexity of the receiver only grows linearly with M.

3. OPTIMAL SYMBOL MAPPINGS

If we intend to transmit the same group of bits M times, we want the optimal mappping functions $\psi_1, \ldots, \psi_M$. We define optimality as minimizing the bit error rate (BER) when the symbols are corrupted by additive white Gaussian noise (AWGN).

3.1. BER Upper Bound

In order to optimize the BER, we first derive a BER expression for multiple transmissions. Our general error (ϵ) expression is

$$\Pr[\epsilon] = \sum_{s=0}^{|\mathcal{C}|-1} \Pr[s] \Pr[\hat{s} \neq s|s]. \tag{2}$$

The function $\Pr[s]$ denotes the *a priori* probability that s is transmitted. The error detection probability $\Pr[\hat{s} \neq s|s]$ is difficult to obtain. We apply the union bound, which states that

$$\Pr[\hat{s} \neq s|s] \leq \sum_{\substack{k=0\\k\neq s}}^{|\mathcal{C}|-1} \Pr[\alpha_M(k) < \alpha_M(s)|s], \tag{3}$$

with $\alpha_M(s)$ the minimization metric in (1). This allows us to deal with the pairwise error probability (PEP) $\Pr[\alpha_M(k) < \alpha_M(s)|s]$, the probability that k is detected when s is transmitted.

To obtain the BER upper bound, we need to account for the number of bit errors caused by a detection error. We define a function $B[s,k]$ to be

$$B[s,k] = \frac{\text{number of differing bits between } s \text{ and } k}{\log_2 |\mathcal{C}|}.$$

Including $B[s,k]$ and (3) into (2) leads to a BER upper bound of

$$\sum_{s=0}^{|\mathcal{C}|-1} \sum_{\substack{k=0\\k\neq s}}^{|\mathcal{C}|-1} \Pr[s] B[s,k] \Pr[\alpha_M(k) < \alpha_M(s)|s]. \tag{4}$$

We now need to define the PEP. Using the ML criterion in (1), $\Pr[\alpha_M(k) < \alpha_M(s)|s]$ becomes

$$\Pr\left\{ \sum_{m=1}^{M} |y_m - \psi_m[k]|^2 < \sum_{m=1}^{M} |y_m - \psi_m[s]|^2 \,\middle|\, s,k \right\}. \tag{5}$$

Because $y_m = \psi_m[s] + v_m$, we reduce (5) to an equivalent probability expression

$$\Pr\left\{ \sum_{m=1}^{M} d^2[\psi_m[s], \psi_m[k]] + 2d[\psi_m[s], \psi_m[k]] v_{m,r} < 0 \,\middle|\, s,k \right\} \tag{6}$$

where $d[a,b]$ is the Euclidean distance between points a and b (in $\mathcal{C}$). With the independence assumption on the Gaussian noise variable v_m, we simplify (6) to derive the PEP expression

$$\Pr[\alpha_M(k) < \alpha_M(s)|s] = Q\left(\sqrt{\frac{1}{4\sigma_v^2} \sum_{m=1}^{M} d^2[\psi_m[s], \psi_m[k]]} \right), \tag{7}$$

and $Q(\cdot)$ is the well-known Gaussian integral. Substituting (7) into (4) provides us with a BER upper bound.

3.2. Optimization Criterion

Our problem is to determine the M optimal symbol mappings $\psi_1, \ldots, \psi_M$ that minimize the BER upper bound in (4). This optimization is stated as

$$\min_{\psi_1,\ldots,\psi_M \in \psi} \sum_{s=0}^{|\mathcal{C}|-1} \sum_{\substack{k=0\\k\neq s}}^{|\mathcal{C}|-1} f\left[s, \mathbf{a}, k, \mathbf{b}\right], \tag{8}$$

$$\mathbf{a} = [a_1, \ldots, a_M]^T = [\psi_1[s], \ldots, \psi_M[s]]^T,$$
$$\mathbf{b} = [b_1, \ldots, b_M]^T = [\psi_1[k], \ldots, \psi_M[k]]^T,$$

with ψ denoting the set of mappings. The cost $f\left[s, \mathbf{a}, k, \mathbf{b}\right]$ is the pairwise BER that results by mapping s to symbols $\mathbf{a}$ and k to symbols $\mathbf{b}$ across the M mappings,

$$f\left[s, \mathbf{a}, k, \mathbf{b}\right] = \Pr[s] B[s,k] Q\left(\sqrt{\frac{1}{4\sigma_v^2} \sum_{m=1}^{M} d^2[a_m, b_m]} \right).$$

With $|\psi| = |\mathcal{C}|!$, (8) becomes a massive combinatorial optimization problem whose solution space contains $(|\mathcal{C}|!)^M$ possible solutions. Thus, we propose a simpler, and probably sub-optimal, iterative solution by computing mapping M from the previous $M-1$ mappings. We assume that the first $M-1$ mappings have already been determined and we work towards obtaining only the M^{th} mapping, ψ_M. Our optimization problem then simplifies to

$$\min_{\psi_M \in \psi} \sum_{s=0}^{|\mathcal{C}|-1} \sum_{\substack{k=0\\k\neq s}}^{|\mathcal{C}|-1} g\left[s, \psi_M[s], k, \psi_M[k]\right], \tag{9}$$

where $g[s,a,k,b]$ is the pairwise BER that results by mapping s to symbol a and k to symbol b in the M^{th} mapping,

$$g[s,a,k,b] = \Pr[s] B[s,k] Q\left(\sqrt{\frac{1}{4\sigma_v^2} \left(h[s,k] + d^2[a,b] \right)} \right),$$

$$h[s,k] = \sum_{m=1}^{M-1} d^2[\psi_m[s], \psi_m[k]].$$

This solution is optimal for ARQ-type applications which have a secondary objective to minimize the number of transmissions (mappings) needed to achieve a desired BER. In other words, it is desirable to choose mapping M without relying on future (re)transmissions.

Though still computationally difficult, (9) falls into a category of combinatorial optimzation problems commonly referred to as the Quadratic Assignment Problem (QAP).

3.3. The Quadratic Assignment Problem

The QAP is one of the most difficult and extensively studied problems in optimization. It was first introduced in 1957 to model the assignment of N economic facilities to N physical locations [9]; a more general version was published in 1963 [10]. Using the facilities-location analogy, we have the costs of assigning facility s to location a and facility k to location b, denoted by $c[s,a,k,b]$. The objective is to minimize the total cost by choosing the assignment $\psi : \{0, 1, \ldots, N-1\} \rightarrow \{0, 1, \ldots, N-1\}$ that satisfies

$$\min_{\psi \in \psi} \sum_{s=0}^{N-1} \sum_{k=0}^{N-1} c[s, \psi[s], k, \psi[k]],$$

where ψ is the set of all possible assignments. Clearly, (9) is an instance of the QAP. Yet to fully apply the QAP, we need to define the cost $g[s, a, s, a]$ of assigning s to symbol a. This cost is zero, since it does not exist in the BER upper bound.

Most exact solutions to the QAP involve a branch-and-bound search. Typically, lower bounds for the QAP are computationally expensive ($O(N^5)$) and generally not very tight. Recent work by Hahn and Grant proposed an efficient lower bounding technique and its inclusion in a branch-and-bound scheme [11].

4. RESULTS

We now present results found by optimizing (9) for several constellation types, beginning with 16QAM. Fig. 2 contains the optimal 16QAM symbol mappings for 2, 3, and 4 transmissions with an E_b/N_0 of 6 dB for each transmission. Notice that the first mapping is a Gray mapping, which is intuitively pleasing. The subsequent mappings are not Gray mappings; bits that are assigned to adjacent symbols in one mapping are assigned symbols that are spaced further apart in subsequent mappings. This again is intuitively satisfying, since (7) implies that increasing the combined distance (across all mappings) between two bit groups reduces their PEP. Thus, bits assigned to largely spaced symbols can "afford" to be assigned to adjacent symbols in subsequent mappings.

Fig. 3 contains the upper bounds achieved by the mappings in Fig. 2, as well as the upper bounds achieved by ML combining, where the first optimal mapping is used for the subsequent $M - 1$ transmissions ($\psi_1 = \ldots = \psi_M$).

Fig. 4 illustrates the tightness of the BER upper bounds when compared to simulated BER curves for 16QAM. Clearly, these bounds are essentially equal to the actual BER for values below 10^{-2}. Figs. 5-8 present the BER upper bounds obtained in optimizing (9) for $M = 1, \ldots, 4$ transmissions using constellation types 16PSK, 16PAM, 8PSK, and 8PAM, respectively. In nearly all cases, the optimization for $M = 1$ produced the expected Gray mapping, with the exception of cases with very low E_b/N_0 values.

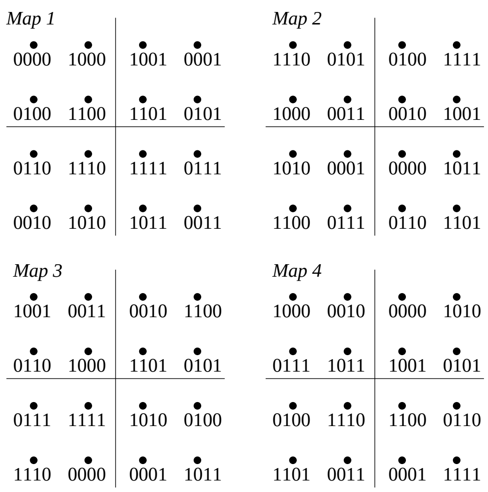

Fig. 2. Symbol mappings of 16QAM.

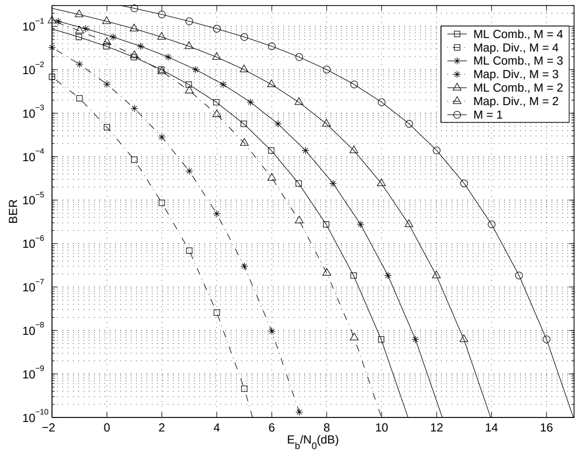

Fig. 3. BER upper bounds for symbol mappings of 16QAM.

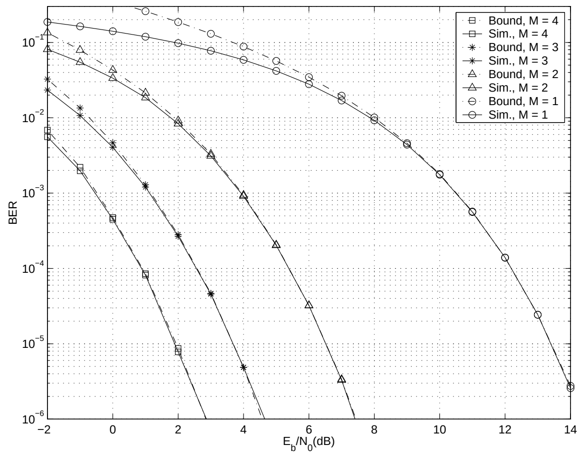

Fig. 4. Simulated BERs vs. upper bounds for 16QAM mappings.

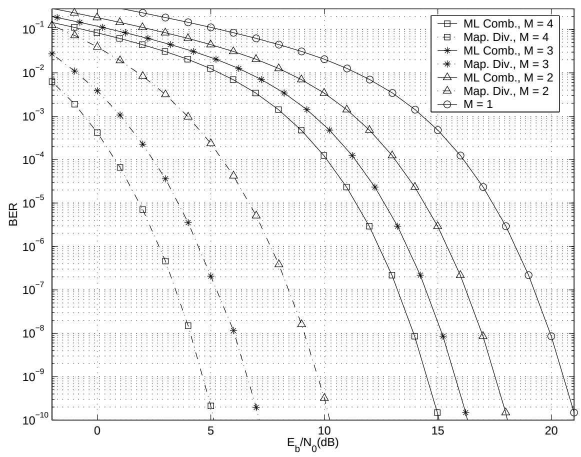

Fig. 5. BER upper bounds for symbol mappings of 16PSK.

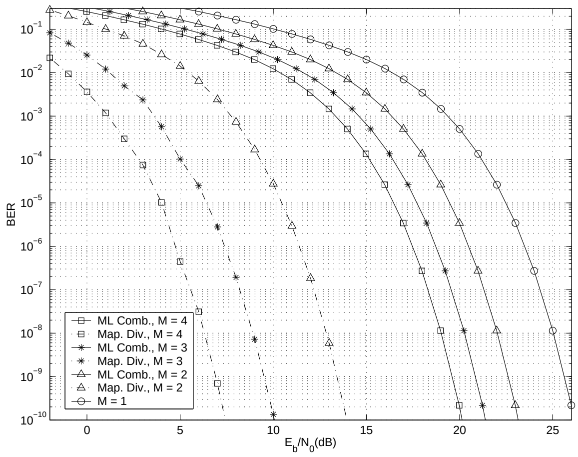

Fig. 6. BER upper bounds for symbol mappings of 16PAM.

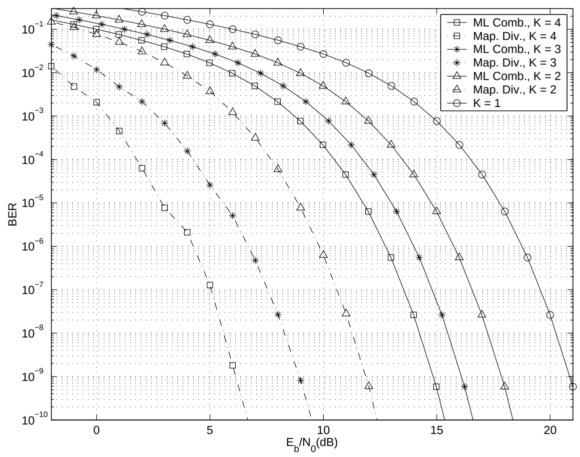

Fig. 8. BER upper bounds for symbol mappings of 8PAM.

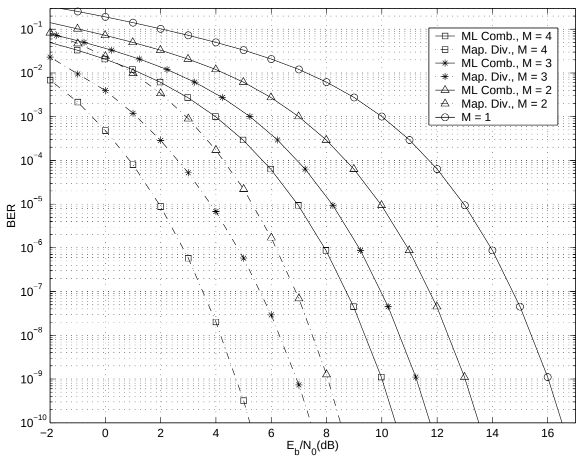

Fig. 7. BER upper bounds for symbol mappings of 8PSK.

These figures indicate that the mapping diversity gains are actually quite substantial. For example, two transmissions using mapping diversity provide approximate gains of 4, 7.5, 9, 5, and 6 dB over ML combining for 16QAM, 16PSK, 16PAM, 8PSK, and 8PAM, respectively. For all constellation types, two mapping diversity transmissions are superior to four ML combining transmissions. One can view these multiple transmissions as rate-$1/M$ codes with considerable coding gains.

5. CONCLUDING REMARKS

In conclusion, this paper presented a method for enhancing diversity among packet retransmissions by adapting symbol mapping. This method requires a minimal increase in transmitter complexity while receiver complexity only grows linearly with the number of transmissions. A general framework was introduced for finding optimal symbol mappings that minimize BER. This framework ultimately simplifies into iterative solutions of the Quadratic Assignment Problem. Results revealed that mapping diversity leads to quite substantial gains (up to 9 dB) for several constellation types. It would be of interest to extend this diversity to several other scenarios including coded transmissions, transmissions through ISI channels, and space-time coded transmissions.

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