FIXED FRAGMENT-SIZE PACKET TRANSMISSIONS WITH DISTRIBUTED REDUNDANCY OVER MULTIPATH FADING CHANNELS

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ABSTRACT

Standardized wireless transmissions include fixed-size fragments per packet. Relying on this structure, we develop two schemes for distributing redundancy across fragments, in order to improve performance of packet transmissions over frequency-selective fading channels. We prove that both schemes guarantee symbol detectability, which implies that they both enable the full multipath diversity. We test their relative merits, and compare them with competing alternatives using simulations.

1. INTRODUCTION

Block transmissions relying on linear redundant precoding with cyclic prefix (CP) or zero padding (ZP) guards have gained increasing interest recently for mitigating frequency-selective multipath effects; see e.g., [1, 2, 3] and references therein. Sufficient redundancy removes inter block interference (IBI), and facilitates (even blind) acquisition of channel state information at the receiver. It also leads to data efficient low-complexity linear equalizers (zero-forcing (ZF) or minimum mean-squared error (MMSE)) with guaranteed constellation-irrespective symbol detectability regardless of the zero locations of the underlying finite impulse response (FIR) channel [2]. Guaranteed symbol detectability implies full multipath diversity, and thus improved performance at moderate-high SNR [4, 5].

To take advantage of these benefits in e.g., ZP transmissions, the number of padded-zeros should be longer than the underlying FIR channel order. But in order to avoid severe bandwidth efficiency loss with long channels, this calls for longer block sizes, which in turn leads to higher decoding delay and decoding complexity. In many protocols however, such as the IEEE 802.11a, the fragment of a packet constitutes the size-invariant transmission unit, that is fixed *a priori*, and is not allowed to change depending on the realization of the random wireless fading channel. Selecting guard sizes for the longest possible channel is one approach, but it is certainly conservative. Instead, the approach pursued in this paper is to have fixed fragment size packet transmissions with redundancy distributed across fragments.

The two transmissions we develop utilize zero-padded information bearing fragments per packet, which enjoy benefits of ZP block transmissions, if the number of zeros is greater than the channel order. If not, we either transmit a controllable number of null fragments, or, we re-transmit a certain number of information bearing fragments. In other words, we either zero pad fragments per packet, or we cyclic prefix fragments per packet. Because either way the redundancy is distributed across fragments, we term our schemes distributed ZP (D-ZP) and distributed CP (D-CP), respectively. Both maintain the fragment size, but D-CP maintains also the fragment structure. They both guarantee symbol detectability irrespective of the constellation and the channel zero locations, which implies full multipath diversity. Simulation examples illustrate their relative merits, and compare them with competing alternatives.

2. MODELING AND PRELIMINARIES

We consider point-to-point wireless transmissions over time-flat but frequency-selective fading channels. At the transmitter, the information-bearing sequence $\{s(n)\}$ is parsed into blocks $\boldsymbol{s}(n) = [s(Mn), \ldots, s(Mn+M)]^T$ of size M. To mitigate the effects of frequency selective channels, we pad N_0 zeros at the end of each block to obtain zero-padded (ZP) transmitted fragments $\{\boldsymbol{u}(n)\}$ of size $N := M + N_0$, as in [3]. The ZP fragments can be described in matrix form as

$$\boldsymbol{u}(n) = \boldsymbol{T}_{zp}\boldsymbol{s}(n), \tag{1}$$

where the $(M+N_0)\times M$ zero-padding matrix $\boldsymbol{T}_{zp}$ is given by $\boldsymbol{T}_{zp} := [\boldsymbol{I}_M^T, \boldsymbol{0}_{N_0\times M}^T]^T$ with $\boldsymbol{I}_M$ denoting the identity matrix of size M, and $\boldsymbol{0}_{N_0\times M}$ the $N_0 \times M$ zero matrix.

Our discrete-time baseband equivalent channel has order L, and is considered linear time-invariant over a number of fragments that comprise a packet. At the receiver, we assume perfect timing and carrier synchronization. We collect $N (= M + N_0)$ noisy samples in a $N \times 1$ received vector

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$\boldsymbol{x}(n)$. If the number N_0 of redundant zeros in each block is greater than or equal to the channel order L, i.e., $N_0 \geq L$, interblock interference (IBI) is removed, and we obtain,

$$\boldsymbol{x}(n) = \boldsymbol{H}_{tr}\boldsymbol{s}(n) + \boldsymbol{v}(n), \qquad (2)$$

where $\boldsymbol{H}_{tr}$ is a tall $N \times M$ (truncated) Toeplitz matrix with first column $[h(0), h(1), \ldots, h(L), \mathbf{0}^T]^T$ [3].

Although redundant zeros reduce the bandwidth efficiency to $M/(M + N_0)$, if $N_0 \geq L$, several benefits become available with ZP: i) low-complexity block-by-block processing with e.g., linear ZF or MMSE equalization, at the receiver [3]; ii) irrespective of the constellation, symbols can be detectable with linear equalizers regardless of the channel zero locations in the absence noise (symbol detectability) [4]; iii) full multipath diversity gain is enabled to enhance system performance (maximum diversity advantage) [4, 5]; iv) blind identification of the unknown channel becomes possible [2]. However, as soon as the guard interval is shorter than the channel order, i.e., $N_0 < L$, these properties may be lost. In a nutshell, the selection of M and N_0 affects: i) performance (by altering the diversity advantage); ii) bandwidth efficiency (by changing the ratio $M/(M + N_0)$; iii) blocking and decoding delay, as well as decoding complexity (via the frame size $N := M + N_0$).

The maximum order, call it $L_{\max}$, of a wireless propagation channel can be estimated experimentally. One may then select the number of redundant symbols $N_0 \geq L_{\max}$ to remove IBI of possible channel realizations. But this is a conservative approach, because it reduces bandwidth efficiency for channel realizations having order $L \ll L_{\max}$. This observation prompted us to consider transmissions with controlled redundancy, that can be added in a distributed fashion. The need for distributing the guard intervals, is well motivated for random fading channels since $L_{\max}$ varies with the propagation environment, which suggests adapting the amount of redundancy depending on the channel. At the same time, adhering to the standards calls for fixing the fragment structure: fragment size, number of information bearing symbols, and number of zeros per fragment. Notice that *a priori* fixed-size fragments, bound the selection of modulation and codeword sizes, but facilitate hardware implementation of communication protocols beyond the physical layer.

Our goal in this paper, is to develop *fixed fragment size packet* transmissions with *redundancy distributed* across fragments, capable of handling channels with long impulse response, while enjoying the performance benefits of ZP transmissions with low decoding complexity.

3. PACKETS WITH DISTRIBUTED REDUNDANCY

With an upper bound of the channel order available both at the transmitter and at the receiver, we consider packet transmissions with the short ($N \leq 10$) ZP fragments described in the previous section. Each packet consists of N_f information-bearing fragments, and N_r redundant fragments (packet guard intervals). Depending on the type of the packet guard interval (ZP or CP), we will develop two schemes in this section.

3.1 Packet transmissions with Distributed ZP (D-ZP)

Here the packet guard time comprises N_r null fragments, where $N_rN \geq L$. Each packet $\bar{\boldsymbol{u}}(n)$ has size $\bar{N} := (N_f + N_r)N$, and can be expressed as

$$\bar{\boldsymbol{u}}(n) = [\boldsymbol{u}^T(N_fn+1), \boldsymbol{u}^T(N_fn+2), \ldots, \boldsymbol{u}^T(N_fn+N_f), \underbrace{\mathbf{0}^T_{N\times 1}, \ldots, \mathbf{0}^T_{N\times 1}}_{N_r \text{ fragments}}]^T. \qquad (3)$$

It should be noted that although we express a set of fragments as a packet, the transmitter simply sends fragments successively, without being necessary to form $\bar{\boldsymbol{u}}(n)$; i.e., no blocking delay or buffering is required at the transmitter.

At the receiver, we collect $N_f + N_r$ fragments in a $\bar{N} \times 1$ vector $\bar{\boldsymbol{x}}(n)$ that can be expressed as

$$\bar{\boldsymbol{x}}(n) = \bar{\boldsymbol{H}}_0\bar{\boldsymbol{u}}(n) + \bar{\boldsymbol{H}}_1\bar{\boldsymbol{u}}(n-1) + \bar{\boldsymbol{v}}(n), \qquad (4)$$

where $\bar{\boldsymbol{H}}_0$ and $\bar{\boldsymbol{H}}_1$ are $\bar{N} \times \bar{N}$ square Toeplitz channel convolution matrices with first column $[h(0), \ldots, h(L), \mathbf{0}^T]^T$ first row $[h(0), \mathbf{0}^T]$, and with first row $[\mathbf{0}^T, h(L), \ldots, h(1)]$ last column $[h(1), \ldots, h(L), \mathbf{0}^T]^T$, respectively; and $\bar{\boldsymbol{v}}(n)$ is a zero-mean additive noise.

Since the last $N_0 + N_rN$ entries of $\bar{\boldsymbol{u}}(n)$ are zero, IBI caused by channels up to order $L_{\max} = N_0 + N_rN$ is removed. With the IBI removed, (4) reduces to a form similar to (2), but with the packet dimensionality $\bar{N}$ replacing the fragment size N. The channel mixing matrix $\boldsymbol{H}_{tr}$ is also replaced by $\bar{\boldsymbol{H}}_0$. But similar to $\boldsymbol{H}_{tr}$, the matrix $\bar{\boldsymbol{H}}_0$ is always full rank for any channel up to order $L_{\max} = N_0 + N_rN$, thanks to the $N_0 + N_rN$ padded zeros. It thus follows readily that this D-ZP scheme inherits the symbol detectability and performance properties (maximum multipath diversity and coding gains) that have been established in [2, 5].

With regards to its bandwidth efficiency, $\mathcal{E}_{dzp}$, it suffices to observe that each $\bar{N} \times 1$ packet $\bar{\boldsymbol{u}}(n)$ in (3) contains N_fM information-bearing symbols. Hence,

$$\mathcal{E}_{dzp} := [M/(M + N_0)][N_f/(N_f + N_r)]. \qquad (5)$$

The first factor $M/(M + N_0)$ is the bandwidth efficiency without sending null fragments; i.e., the bandwidth efficiency of the original ZP fragment, while the second factor arises due to the transmission of the null fragments.

Like any ZP block transmission, depending on complexity versus performance tradeoffs, D-ZP can be decoded using: linear zero-forcing (ZF) or minimum mean-square error (MMSE) equalization; nonlinear decision-feedback equalization (DFE) [1]; or, maximum-likelihood (ML) demodulation. Instead of exhaustive search, ML optimum decoding of D-ZP transmissions can be implemented using the Viterbi Algorithm (VA) [4]. In all cases, decoding complexity depends on the block size $\bar{N}$, but unlike the VA whose complexity is constellation dependent, ZF and MMSE alternatives have relatively low complexity, $\mathcal{O}(\bar{N}^2)$ per block, irrespective of the underlying constellation. In summary, for D-ZP transmissions we have established:

Proposition 1: *If the aggregate redundancy satisfies $N_r N \geq L$, then D-ZP packet transmissions enable the maximum multipath diversity and guarantee symbol detectability. By decoding D-ZP packets (as opposed to ZP fragments) we can accommodate channels of order greater than the fragment guard time, at the expense of increased decoding delay and complexity, and reduced bandwidth efficiency by a factor $N_f/(N_f + N_r)$.*

At this point, one may wonder how for the same bandwidth efficiency, D-ZP compares with a non-distributed ZP packet transmission, which pads all $N_0 + N_r N_0$ zeros at the end of the packet. Although we do not currently have a rigorous proof, our simulations will indicate that D-ZP outperforms its non-distributed counterpart.

3.2 Packet Transmissions with Distributed CP

Instead of padding N_r null fragments as in the previous subsection, following the N_f ZP information bearing fragments, we pad in a circular fashion the first N_r (of the N_f) fragments. Specifically, the nth packet now has the form:

$$\bar{\boldsymbol{u}}(n) = [\boldsymbol{u}^T(N_f n+1), \ldots, \boldsymbol{u}^T(N_f n + N_f), \underbrace{\boldsymbol{u}^T(N_f n+1), \ldots, \boldsymbol{u}^T(N_f n + N_r)}_{N_r \text{ fragments}}]^T. \quad (6)$$

Every N_f fragments in this scheme, that we naturally term D-CP, we simply re-transmit N_r of them. Different from D-ZP, and similar to the CP removal that takes place at an OFDM receiver, we remove the first $N_r N$ entries from each received packet. This operation removes IBI from (4), provided that we select N, N_r to satisfy: $N_r N \geq L$ [3].

As far as transmission rate, it is clear that D-CP has bandwidth efficiency identical to that of D-ZP; i.e., $\mathcal{E}_{dcp} = \mathcal{E}_{dzp}$. A nice feature of D-CP, not available in D-ZP, is that D-CP transmits fragments of the same structure. This in turn allows for easy adaptation of the transmitter to possible changes in N_f, N_r.

With regards to performance, thinking along uncoded OFDM lines, one would be tempted to infer that D-CP does not guarantee constellation and channel irrespective symbol detectability, and thus it does not enable the full multipath diversity. Interestingly, we can establish (but omit the proof) that D-CP enjoys these nice features as well, provided that our fragment and packet parameters are chosen to satisfy:

$$N_f N_0 \geq L, \quad N_r N \geq L. \quad (7)$$

Proposition 2: *Under (7), D-CP guarantees constellation and channel irrespective symbol detectability (and thus full multipath diversity) for any FIR channel up to order $\min(N_f N_0, N_r N)$.*

Comparing D-CP with D-ZP for the same (N_f, N_r), we observe that D-ZP guarantees symbol detectability for longer channel orders (by N_0). In addition, D-CP is not as energy efficient as D-ZP, since we allocate $N_r/(N_f + N_r)$ percent of the transmit-power per packet to re-transmitting N_r fragments. This is the price we pay in D-CP for maintaining the fragment structure. The decoding options we outlined for D-ZP, apply also for D-CP, and their delay and complexity increase as (N_f, N_r) increase. The packet size (and thus bandwidth efficiency) depends not only on the maximum channel order L, but also on the channel coherence time. For D-CP to satisfy (7) for channels with relatively short coherence time, increasing N_0 also reduces the bandwidth efficiency.

Two remarks are now in order:

Remark 1: When a feedback channel is available from the receiver to the transmitter, or during a time-division duplex session, both D-ZP and D-CP can adapt their (N_f, N_r) values, depending on the channel state information (channel order and/or SNR) in order to strike desirable performance-rate tradeoffs. This is an interesting future direction, but goes beyond the scope of this paper.

Remark 2: A related transmission with ZP sub-blocks was considered in a vector OFDM context in [6]. Different from our D-CP scheme, ZP sub-blocks in [6] are grouped into a super-block, and are modulated by IFFT before transmission. The received super-blocks are FFT processed, but they are decoded on a sub-block basis, which reduces decoding complexity. Although not mentioned in [6], this is nothing but the single user counterpart of the generalized multicarrier CDMA scheme introduced earlier in [3] for multiuser communications. Unlike [3] and D-CP though, the single user scheme in [6], neither leads to constant modulus transmissions, nor it ensures symbol detectability (and thus maximum multipath diversity).

4. NUMERICAL EXAMPLES

We generated 10^3 Rayleigh distributed channels of order $L = 9$, having complex zero-mean Gaussian taps with exponential power profile: $E\{|h(l)|^2\} = \exp(-l)$, for $l \in [0, L+1]$. For BPSK, we computed the BER analytically when hard-decoding was used at the ZF equalizer output, and then we averaged BER over the randomly generated channels. As a benchmark, we also implemented ML decoding of ZP block transmissions using Viterbi's Algorithm (VA), and computed the resulting BER via Monte-Carlo simulations.

	complexity(ZF)	delay	bandwidth eff.
D-ZP	63	63	2/3
C-CP	63	63	2/3
ZP (a)	27	27	2/3
ZP (b)	63	63	6/7
[6]	9	63×2	2/3

Table 1. Comparison of simulated systems

We first considered D-ZP and D-CP transmissions with $(M, N_0) = (7, 2)$, which can accommodate symbol codewords of length $M = 7$, and can handle IBI for channels up to order 2. As our channels had order $L = 9$, the guard time in this case has insufficient length to remove the IBI. We thus selected $(N_f, N_r) = (6, 1)$, which imply bandwidth efficiency $(7/9)(6/7) = 2/3$, and a packet size of $(N_f + N_r)(M + N_0) = 7 \cdot 9 = 63$. We compared our D-ZP and D-CP transmissions with the vector precoded OFDM scheme in [6], for identical frame size, and bandwidth efficiency. We also tested two more ZP-based block transmissions that ensure channel-irrespective symbol-detectability using linear ZF equalization for channels up to order 9:
(a) one having $(M, N_0) = (18, 9)$, packet size $M + N_0 = 27$, and bandwidth efficiency $M/(M + 9) = 2/3$, identical to those of D-ZP and D-CP transmissions; and
(b) one having $(M, N_0) = (54, 9)$, and packet size $M + N_0 = 63$, which causes decoding delay equal to a packet in the D-ZP and D-CP transmissions, but enjoys higher bandwidth efficiency $54/63 = 6/7$.

Table 1 compares simulated systems in terms of: approximate ZF equalizer complexity per symbol, blocking delay at the transmitter, decoding delay at the receiver, and bandwidth efficiency. We underscore, that [6] requires extra FFTs to implement OFDM that are not needed in the other systems. Fig. 1 depicts BER as a function of E_b/N_0. Among all schemes using low-complexity ZF equalization, D-ZP exhibits the best performance. A slight performance loss of D-CP and vector OFDM is due to the energy loss corresponding to the CP (recall that for a fixed energy per packet, the energy saved from ZP guards implies increased energy of the information bearing symbols).

Clearly, the D-ZP and D-CP schemes exhibit better performance than ZP (b) at the expense of rate reduction. However, it is interesting to note that with a low-complexity ZF equalizer, they outperform ZP (a) that has identical bandwidth efficiency. We believe that the reason behind this improvement, is the fact that D-ZP transmissions offer a better conditioned channel matrix than ZP (a), and this leads to a larger coding gain (recall that they both enable the maximum multipath diversity gain since they both guarantee symbol detectability [4]).

Equally interesting is the fact that D-ZP with ZF equalization comes close (less than 2dB in the practical SNR range) to the benchmark ML performance of ZP (a) with VA decoding. The latter requires about $2^{10} \approx 10^3$ computations per symbol, while D-ZP with ZF equalization only 63. We expect this gap to narrow with channel coding. But even without coding, the potential of D-ZP can be appreciated further if one considers channels with long impulse response: indeed, for channels with, say $L > 5$, one does not need to collect the maximum diversity gain (here $G_d^{\max} = L + 1 = 6$), simply because this gain will show up in the BER for SNR values ($>$ 30dB) well beyond the practical range. Thus, low-complexity high-performance linear equalizers (ZF or MMSE) are well motivated in such cases.

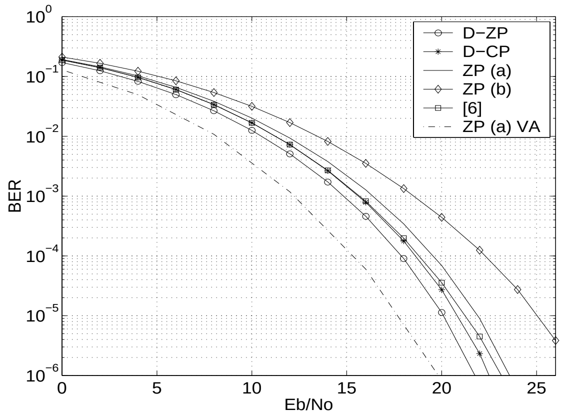

Fig. 1. BER comparisons

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