ROBUST POWER CONTROL FOR CDMA NETWORKS SUBJECT TO MODELIZATION ERRORS

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ABSTRACT

In this paper, we present a robust decentralized method for jointly performing channel estimation and closed loop power control for the reverse link of CDMA networks. Our method, based on linear quadratic control systems theory and Kalman filtering, does not require any training symbols for channel or Signal to Interference Ratio (SIR) estimation. The main interest of this new scheme is that it improves the performance of current SIR based power control techniques while avoiding the problem of power escalation, which is often observed in current systems. In this paper, we also provide an analysis on the robustness of this method towards errors on the control command and imperfect call admission control.

1. INTRODUCTION

Up-link Power Control (PC) is a crucial element in CDMA multi user systems. In order to maximize capacity and Quality of Service, all Mobile Station (MS) transmissions should be received at the Base Station (BS) with equal power. Since CDMA systems are interference limited, much work concentrates on the Signal to Interference Ratio (SIR) as a measure to control MS powers [1].

In theory, assuming perfect knowledge of the SIR, SIR based PC outperforms signal strength based PC in interference limited systems such as CDMA. However, SIR based PC is associated with 2 major drawbacks. Firstly, SIR is difficult to estimate accurately [2]. Current systems, such as UMTS address this problem using a pilot channel dedicated to SIR estimation, which obviously reduces the overall capacity of the network. On the other hand, centralized schemes can determine SIR more accurately than distributed systems [1]. However, they are difficult to implement due to their high computational complexity.

The other major downfall of SIR based PC is the problem of power escalation. SIR, as the name suggests, is a ratio between signal and interference. As one MS increases its power to compensate for interference from other MS's, its signal interferes more on all other MS's which will in turn increase their transmit power. Instability and power escalation (also defined as positive feedback) can result while the SIR for each MS remains the same. This is particularly prevalent when the system is operating at or near the capacity limit. Therefore, an SIR based PC scheme should be used in conjunction with a perfect call admission control mechanism, which is very difficult to guarantee in real systems.

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Zhang [3] addresses the power escalation problem with a joint signal strength and SIR based PC scheme that compares both quantities to thresholds and adjusts power in the MS with a simple adaptive step size algorithm. This approach may stop the escalation problem but it still does not minimize the MS transmit power and it is not robust to control command errors. Qain and Gajic [4] address the power escalation problem by applying stochastic control systems theory to SIR estimation and PC. However, this method is not suitable for tracking channel variations due to the mobile speed.

In this paper, we present a robust decentralized method for jointly performing channel estimation and close loop power control for the reverse link of CDMA networks. We base our approach on optimal control systems theory. The novelty of our approach is that while it aims at maintaining the SIR of each MS close to the SIR targets, its implementation does not rely on the actual calculation of the SIR. In other words, our approach takes *implicitly* into account the interference component of each signal but is not affected by positive feedback.

Another important feature is that our proposed method does not rely on any pilot or training sequence, thus increasing the system capacity. The general structure of this algorithm is similar to the one already implemented in the IS95 system. It is therefore very easy to implement in practical systems. Finally, being an adaptive method, it allows taking into account fading characteristics of wireless channels. In this paper, we also address the issue of robustness towards modelization errors, namely, call admission control imperfections and errors in the transmission of the control command. We show that our method is not subject to instability but also recovers very quickly from such imperfections.

The paper is organized as follows: we first present the problem formulation and the general linear quadratic gain controller. Then, the channel estimation is addressed using a Kalman filter. Finally, we show the impact of modelization errors on the proposed method and confirm our results by simulations.

2. NOTATION AND ASSUMPTIONS

In this paper, we use the following notations and assumptions:

- $p_k(t)$is the transmit power of mobile user k during slot t.
- $\Gamma_k(t)$ is the squared absolute value of the average (over slot t) of the up-link channel gain (for user k).
- $P_k(t) = p_k(t)\Gamma_k(t)$is the power of the signal, received at the base station, after despreading for user k.

- σ^2 is the variance of the thermal noise process (modeled as a zero mean Gaussian random variable).
- We use a simple Matched Filter Correlation Receiver for demodulation. The system is chip synchronous.

3. PROBLEM FORMULATION

Power control will be performed in order to achieve $SIR_k(t) = \beta$, where β is the SIR target (common to all users). It has been shown in [5] that this is realized if the receive powers for all users are equal to P^* defined as:

$$P^* = \frac{\beta\sigma^2}{1 - \frac{\beta(K-)}{N}} \tag{1}$$

where K is the total number of users in the system and N the spreading gain. The aim of this paper is to find appropriate power control commands such that the received power for all users approaches P^*.

Let us denote by $w_k(t)$ the average over slot t of the power of the CDMA signal despread by the spreading sequence of user k. $w_k(t)$ is written as:

$$w_k(t) = P_k(t) + \frac{1}{N}\sum_{j\neq k} P_j(t) + \sigma^2 + v_k(t) \tag{2}$$

Here, $v_k(t)$ is the measurement noise due to the limited number of samples involved in the average operation. Note that power control is usually performed on the logarithmic scale. In [5], we have shown that it is possible to express $w_k(t)$ on the logarithmic scale using the general notation $X_{dB}(t) = 10\log_{10}(X(t))$:

$$w_{dB_k}(t) = P_{dB_k}(t) + \lambda^2 + n_k(t) \tag{3}$$

where $\lambda^2 = 10\log_{10}(1 + \frac{1}{\beta})$ and $n_k(t)$ is the measurement noise which comprises effects from imperfect average, imperfect power control on all users and arithmetical approximations that arise from the conversion from linear to logarithmic scale. Our simulations show that this measurement noise is zero mean.

Our aim is use Linear Quadratic Gaussian (LQG) control theory in order to design the **control command** $u_k(t)$ which updates the transmit power according to,

$$p_{dB_k}(t+1) = p_{dB_k}(t) + u_k(t) \tag{4}$$

so that the receive power is close to P^* and the control command minimized.

4. THE LINEAR QUADRATIC CONTROLLER

From now on, for simplification, we will omit the subscripts k and dB, bearing in mind that all quantities are expressed in dBs and that each user performs the same operations in a decentralized way.

In this section, we assume that the channel gain $\Gamma(t)$ is exactly known to the base station. In the next section, we will explain how to estimate these quantities. The aim of this section is to design the control command $u(t)$ in an LQG framework, i.e. which minimizes the following linear quadratic cost function:

$$J = E\{\lim_{N\to\infty}\sum_{t=0}^{N} q_c\|P(t) - P^*\|^2 + r_c\|u(t)\|^2\} \tag{5}$$

where q_c and r_c are quantities to be determined. A discussion on the choice of these two parameters can be found in [5].

It is well know that a static feedback law determined by the solution to an Algebraic Ricatti Equation (ARE) gives the solution to this problem.

In other words, the optimal control is given by $u(t) = \alpha' P(t) + \alpha P^*$, where

$$\alpha' = -(P^{(1)} + r_c)^{-1}P^{(1)} \tag{6}$$

$$\alpha = -(P^{(1)} + r_c)^{-1}P^{(2)} \tag{7}$$

with $P^{(1)}$ given by the solution to the ARE:

$$P^{(1)} = P^{(1)} - P^{(1)}(P^{(1)} + r_c)^{-1}P^{(1)} + q_c \tag{8}$$

and $P^{(2)}$ obtained by:

$$P^{(2)} = \frac{q_c}{\alpha'} = -\frac{q_c(P^{(1)} + r_c)}{P^{(1)}} \tag{9}$$

We can easily see that $\alpha = \frac{q_c}{P^{(1)}} = -\alpha'$ and that $P^{(1)}$ is computed as:

$$P^{(1)} = \frac{1 + \sqrt{1 + 4r_c}}{2} \tag{10}$$

Finally, the command $u(t)$ is expressed as

$$u(t) = -\alpha(p(t) + \Gamma(t) - P^*) \tag{11}$$

In general, q_c is set to 1 since it is only the ratio between r_c and q_c which matters. It can be shown that when r_c tends to 0, the LQG performs a *Loop Transfer Recovery* which is useful when the system is faced with modelization errors, as we will see in the last section of this paper. However, the drawback of choosing r_c too small is that the control command is no longer minimized which can be an issue.

Note that in practice, a quantized version of $u(t)$ is used. Indeed, once the power control command is computed at the base station, it is quantized, encoded and the resulting bits are transmitted on the feedback channel to the mobile station. The base station knows this quantization error. Therefore, all the calculations that follow are still valid in a quantized framework when we assume that the BS takes into account the known quantization errors in the calculations.

We can see that the control command depends on the channel gain $\Gamma(t)$. However, in practice, $\Gamma(t)$ is not known at the base station and needs to be estimated using the observation process $w_{dB}(t)$. In the next section, we propose to estimate the channel gain using the Kalman Filter.

5. KALMAN FILTERING ESTIMATION OF THE CHANNEL GAIN

In this section, we assume that due to the Doppler effects, the channel coefficients are correlated in time. We therefore model the (unknown) channel gain on the logarithmic scale as an Auto Regressive (AR) process as $\Gamma(t) = [\Gamma(t-1)\ \Gamma(t-2)\ \cdots\ \Gamma(t-L)]\mathbf{h} + b(t)$, where $\mathbf{h} = [h_1\ \ h_2\ \ \cdots\ \ h_L]^t$ is supposed to be known at the base station. Using this time dependency, we can easily write a state equation for vector $\underline{\Gamma}(t) = [\Gamma(t)\ \Gamma(t-1)\ \cdots\Gamma(t-L+1)]$ as follows:

$$\underline{\Gamma}(t) = A\underline{\Gamma}(t-1) + [b(t)\ 0\ \cdots\ 0]^t \tag{12}$$

where

$$A = \begin{bmatrix} h_1 & h_2 & \cdots & h_L \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \quad (13)$$

In [6], we have shown that

$$w(t) = V_1^T \hat{\underline{\Gamma}}(t-1) + \Gamma(t) + P^* + \lambda^2 + n(t) \quad (14)$$

where $\mathbf{V}_1^T = -\alpha[1 \ (1-\alpha) \cdots (1-\alpha)^{L-1}]$. This leads to the following observation equation

$$w(t) = C\underline{\Gamma}(t) + P^* + \lambda^2 + n'(t) \quad (15)$$

where $n'(t)$ comprises of the estimation error on $\Gamma(t-1)$. We can check by simulation (as shown in figure 1) that $n'(t)$ is a zero mean, white Gaussian process. In this paper we do not provide a theoretical proof for this, however such an analysis could be obtained from the results on propagation of modelization errors, which is provided in the next section. Using the state equation 12 and the observation equation 15, we can easily derive the Kalman filter estimate of state $\underline{\Gamma}(t)$, i.e. $\hat{\underline{\Gamma}}(t|t)$:

$$\hat{\underline{\Gamma}}(t|t) = \mathbf{FA}\hat{\underline{\Gamma}}(t-1|t-1) + \mathbf{M}(w(t) - P^* - \lambda^2) \quad (16)$$

where F and M are obtained by solving Riccati equations as shown in [6].

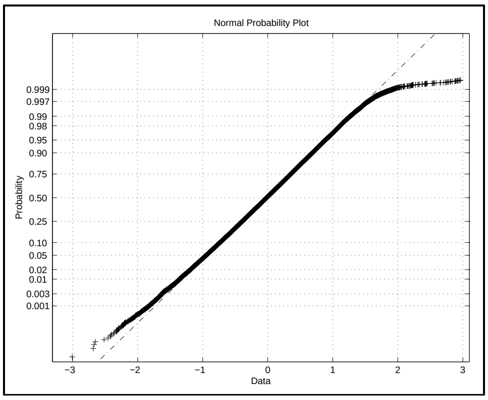

Fig. 1. Error Probability Analysis: n'(t)

6. ERROR ANALYSIS

One might argue that expressing $p(t)$ in the observation equation as a function of the channel gains is not necessary: indeed, $p(t)$ is in theory known at the base station where the channel gain estimate takes place. However, we choose this formulation for two reasons: firstly, $p(t)$ effectively depends on the channel gains. Therefore, using this a-priori information will improve the Kalman filter performance. Second, once the control command is computed, it is quantized, encoded and the coded bits are sent on a forward control channel. This control channel does not have error protection. Therefore, the control command bits are subject to errors and we cannot assume that the base station knows exactly the transmit power. It is instead better to write $p(t)$ as a function of the channel gains perturbed by errors taken into account in the term $n(t)$.

In this section, we concentrate our analysis on how various errors on the feedback channel are handled by the combination of the Kalman filter and the LQG. We will see that our proposed scheme quickly recover from such errors.

Let us assume that an error on the feedback channel occurs at time t. For sake of simplicity, we will assume that the channel estimation error is negligible up to this time (i.e $\hat{\underline{\Gamma}}(t-1) = \underline{\Gamma}(t-1)$). Therefore, using equation 14 the transmit power at time t can be written as:

$$p(t) = \mathbf{V}_1^T \underline{\Gamma}(t-1) + P^* + \epsilon(t) \quad (17)$$

Note that $\epsilon(t)$ could also be seen as an error on the channel gain estimate. Therefore, the observation $w(t)$ will be written as:

$$w(t) = C\Gamma(t) + \epsilon(t) + P^* + \lambda^2 + n(t)$$

and the Kalman filter estimate of state $\underline{\Gamma}(t)$ will be computed as:

$$\hat{\Gamma}(t|t) = (\mathbf{I} - \mathbf{MC})\mathbf{A}\underline{\Gamma}(t-1) + \mathbf{M}(C\underline{\Gamma}(t) + \epsilon(t)) \quad (18)$$

It is therefore obvious that the estimation error due to the feedback channel error is expressed as $\mathbf{M}\epsilon(t)$. The estimate of the channel gain at time t is then $\hat{\Gamma}(t|t) = \Gamma(t) + M(1)\epsilon(t)$, where $M(1)$ is the first component of vector $\mathbf{M}$.

Let us now evaluate $u(t+1)$, the control command calculated at time $t+1$. Note that the base station computes $u(t+1)$ using not only the estimate of the channel gain but also what it believes to be the transmit power. Because the BS is not aware of the error on the feedback channel, it uses $p(t) - \epsilon(t)$ as the transmit power. Using equation 17, we have: $p(t) - \epsilon(t) = \mathbf{V}_1^T \underline{\Gamma}(t-1) + P^*$ and therefore, $u(t+1)$ is computed as:

$$\begin{aligned} u(t+1) &= -\alpha(\Gamma(\hat{t}|t) + p(t) - \epsilon(t) - P^*) && (19) \\ &= -\alpha(\Gamma(t) + M(1)\epsilon(t) + \mathbf{V}_1^T \underline{\Gamma}(t-1)) && (20) \end{aligned}$$

which yields for the transmit power at time $t+1$:

$$\begin{aligned} p(t+1) &= p(t) - \alpha(\Gamma(t) + M(1)\epsilon(t) + \mathbf{V}_1^T \underline{\Gamma}(t-1)) && (21) \\ &= -\alpha\Gamma(t) + (1-\alpha)\mathbf{V}_1^T \underline{\Gamma}(t-1) + P^* \\ &\quad +(1 - M(1)\alpha)\epsilon(t) && (22) \end{aligned}$$

Therefore, the error on the transmit power at time $t+1$ due to the error on the feedback channel at time t is expressed as:

$$\epsilon(t+1) = (1 - \alpha M(1))\epsilon(t)$$

is obviously smaller than 1 which confirms that the system is stable towards errors and it decreases as α increases. Given equation 10 it can be easily seen that if r_c is chosen small, α will be close to 1 which minimizes the error propagation due to errors on the feedback channel. This confirms our first analysis on the choice of r_c.

7. SIMULATION RESULTS

We develop a simulation environment, which corresponds to the proposed UMTS guidelines. We operate at a continuous data rate of 60Kbps and we assumed a processing gain of 64. The power control command operates at a rate of $1.5kHz$ All users' signals pass through a fast fading Rayleigh channel. We use 3-bit quantized PC commands and optimized step sizes for both our proposed method and the IS-95 power control device. All figures show the theoretical channel gains and transmit powers (smooth curves) as well as the estimate of the channel gain and the experimental transmit powers.

The first simulation illustrates the good robustness of our proposed method to modelization errors: we have chosen a scenario where the call admission control is imperfect and allows a new user to enter the system which causes the network to momentarily operate above its capacity. In this case, the IS95 becomes unstable: all users try to increase their power simultaneously to compensate for the interference caused by the new user, as shown in figure 2. Our proposed method (figure 3) recovers very quickly from this perturbation and shows very good stability properties since the experimental transmit power remains close to the theoretical one. We also note an increase in BER as the new user is admitted.

The second simulation shown in figure 4 illustrates our error analysis provided in section 6. One can see that the error on the transmit power caused by the feedback channel error occurring at frame number 125 is only obvious during one frame. One can also observe a bias persisting on the Kalman estimate of the channel. This can be explained by the fact that out method allows the Kalman filter estimate to "absorb" the errors on the transmit power which is a very interesting feature.

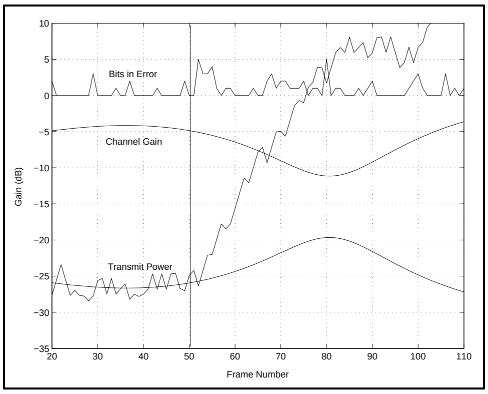

Fig. 2. IS95 method with imperfect call admission control

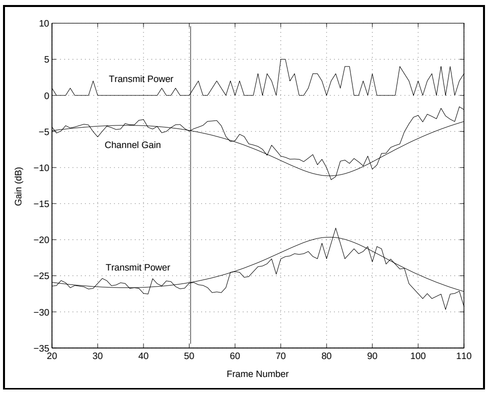

Fig. 3. Proposed method with imperfect call admission

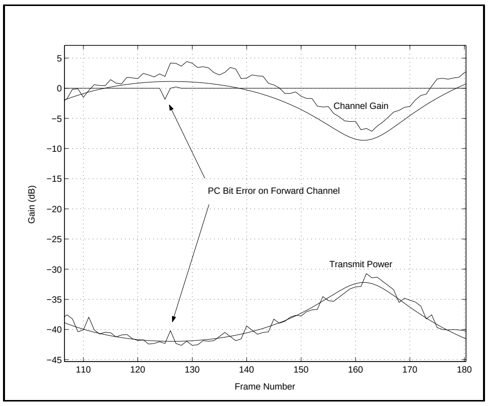

Fig. 4. Proposed method with error on feedback channel

8. CONCLUSION

In this paper, we have proposed a new scheme for power control in a CDMA system, based on a LQG controller and using Kalman filtering for channel estimation purposes. The main feature of this method is that it ensures that the SIR targets for each user are respected without the usual drawback of SIR based power control techniques, i.e. positive feedback. Our method does not require any SIR estimation, which is usually difficult to accurately perform without training or pilot symbols. We have shown in this paper that this algorithm has good robustness properties towards system imperfections such as call admission control failures or errors on the control command. More analysis similar to the one provided in section 6 is currently being done to investigate the impact of Kalman filter estimation errors and imperfect power control on the performance of this algorithm.

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