OPTIMAL AND SUB-OPTIMAL TRAINING SEQUENCE DESIGN FOR TWO-DIMENSIONAL MAXIMUM-LIKELIHOOD CHANNEL ESTIMATOR FOR SINGLE-CARRIER AND MULTICARRIER QUASISYNCHRONOUS CDMA SYSTEMS

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ABSTRACT

This paper proposes a unified two-dimensional maximum-likelihood multipath channel estimator for both multi-carrier and single-carrier CDMA systems with cyclic prefix. The conditions for the optimal training sequence and the lower bound for the performance of this estimator are obtained by solving an optimisation problem. Optimal and sub-optimal training sequences are designed from several classes of sequences with ideal autocorrelation and cross correlation properties. Furthermore, the complexity of this channel estimator will be reduced considerably by applying the optimal training sequence. The simulation result demonstrates that this channel estimator combined with optimal or sub-optimal sequence can give near single-user channel estimation performance even for fully loaded systems.

1. INTRODUCTION

Recently, various combinations of CDMA and OFDM (Orthogonal Frequency Division Multiplexing) such as MC-CDMA [1], SCRIP-CDMA [2] and CP-CDMA [3] are suggested for broadband wireless communications. Due to the frequency diversity offered by OFDM, they are more robust to multi-path fading channel than conventional DS-CDMA. Different data detection or equalization methods are proposed for these schemes ([1]-[3]).

For all these schemes, adaptive channel identification is necessary at the receiver due to the time-varying nature of the wireless communication channel. One standard method for the channel identification is to send a short training sequence as the pilot signal. By knowing the input and output of the system in the presence of additive noise, the system response can be estimated in time domain. However, time domain channel estimation for high-speed systems may involve noise enhancement and convergence problems when implemented with adaptive algorithms such as LMS (Least Mean Square) and RLS (Recursive Least Square). Another method is to estimate the channel in the frequency domain by DFT [4]. However, this estimation method cannot be applied directly to multi-user cases. Here, a general two-dimensional maximum-likelihood (ML) channel estimator is proposed for all systems. This estimator estimates the time domain channel response from the received signal in frequency domain. Moreover, the optimal training sequence problem for this channel estimation is addressed as a constrained optimization problem. Optimal and sub-optimal training sequences are derived from sequences such as the *park-park-song-suehiro* sequence [5], *Periodic root-of-unit optimal* (PRUO) sequence [6] and *Schroeder* sequence [7].

The other parts of this paper are organized as follows. In section 2 a unified system model is presented. Section 3 demonstrates the proposed two-dimensional ML channel estimator. The conditions for optimal training sequence and the lower bound for channel estimation performance are shown in section 4 together with optimal and sub-optimal training sequences. The simulation result is demonstrated in section 5.

2. UNIFIED QS-CDMA SYSTEMS

We consider a system model for uplink transmission of a quasi-synchronous multiuser CDMA system over a multipath channel for the studies. In the downlink transmission, due to the common pilot signal for all users, the model is simpler and similar with the single user case of the uplink transmission. This model can be generalized to various systems such as MC-CDMA [1], SCRIP-CDMA [2], or CP-CDMA [3].

2.1. Single-carrier CDMA with Cyclic Prefix (CP)

Consider the uplink transmission in a single cell SC-CDMA system with *K* users. Because of the timing errors, (the relative timing delay between users) the base station will receive the data stream from each user asynchronously. The timing errors for all users are assumed to be less than the length of the CP. Assuming packet transmission, the n^{th} training block for the i^{th} user to be transmitted can be described as follows:

$$\mathbf{P}_i(n) = [p_{i,0}(n), p_{i,1}(n), \ldots\ldots, p_{i,N-1}(n)]^T$$

CP is inserted before transmission. Each user's signal passed through a multipath channel with the following characteristics:

$$h_k(t) = \sum_{l=1}^{L} h_k(l)\delta(t - \tau_{k,l}) \qquad (1)$$

where L is the number of paths. $h_k(l)$ and $\tau_{k,l}$ are the i^{th} complex path gain and delay for user k. The corresponding channel vector is

$$\mathbf{h}_k(n) = [h_{k,0}(n), h_{k,1}(n), \ldots\ldots, h_{k,L-1}(n)]^T \qquad (2)$$

At the receiver, after removing the CP of the received signal and applying fast Fourier transform (FFT), the received signal in frequency domain can be represented as follows:

$$\mathbf{R}_i(n) = diag(\mathbf{F} * \mathbf{T}_i^n * \mathbf{P}_i(n)) * \mathbf{F}_L * \mathbf{h}_i(n) + \hat{\mathbf{A}}(n) \qquad (3)$$

where **F and $\mathbf{F_L}$** are FFT matrices which satisfies the following conditions:

$$[\mathbf{F}]_{n,m} = e^{-j2\pi(n-1)(m-1)/N} \qquad (1 \le n \le N, 1 \le m \le N) \qquad (4)$$

$$[\mathbf{F_L}]_{n,m} = e^{-j2\pi(n-1)(m-1)/N} \qquad (1 \le n \le N, 1 \le m \le L)$$

The function *diag(x)* generates a diagonal matrix with the vector x as the diagonal elements. $\hat{\mathbf{A}}(n)$ is the additive white noise vector in frequency domain and $\mathbf{T}_i^n$ is a N x N matrix with

$$\mathbf{T}_i^n(p,q) = \begin{cases} 1, (p-q=\tau_i/T_c \;\; or \;\; q-p=N-\tau_i/T_c) \\ 0, (others) \end{cases} \qquad (5)$$

where τ_i is the timing error for the i^{th} user and T_c is the sampling interval. If there is no timing errors, the system is synchronous and the matrix $\mathbf{T}_i^n$ will become an identity matrix. Define a new matrix $\mathbf{X}_i(n)$ such that

$$\mathbf{X}_i(n) = diag(\mathbf{F} * \mathbf{T}_i^n * \mathbf{P}_i(n)) * \mathbf{F}_L \qquad (6)$$

Eq (3) can be reformed as

$$\mathbf{R}_i(n) = \mathbf{X}_i(n) * \mathbf{h}_i(n) + \hat{\mathbf{A}}(n) \qquad (7)$$

Suppose there are K users, the signal received in the frequency domain will be the summation of all users signal described in Eq (7), which can be described as follows:

$$\mathbf{R}(n) = \sum_{i=1}^{K} \mathbf{R}_i(n) = \mathbf{X}(n) * \mathbf{h}(n) + \mathbf{A}(n) \qquad (8)$$

where $\mathbf{X}(n) = [\mathbf{X}_1(n), \mathbf{X}_2(n), ...\mathbf{X}_K(n)]$ (9)

$$\mathbf{h}(n) = [\mathbf{h}_1(n)^T, \mathbf{h}_2(n)^T, ..., \mathbf{h}_K(n)^T]^T \qquad (10)$$

2.2 Multicarrier CDMA

In this paper, we assume the training sequence is generated in the time domain for MC-CDMA system. Hence, the same expression as Equ (8) can be used. If the training sequence is generated in the frequency domain, the logic applied in the section 2.1 can be applied to it as well. However, there is a multicarrier modulation for the chip blocks to be transmitted, which corresponds to an inverse fast Fourier transform (IFFT). The alternative matrix for MC-CDMA system corresponds to Equ (6) of SC-CDMA will become

$$\mathbf{X}_i(n) = diag(\mathbf{F} * \mathbf{T}_i^n * \mathbf{F}^H * \mathbf{P}_i(n)) * \mathbf{F}_L \qquad (11)$$

Hence, a similar expression as in Equ (8) can be obtained.

From these discussions, it is obvious that a unified model can be proposed for both the MC-CDMA and SC-CDMA system with CP. As a result, generalized channel estimation and optimal pilot design schemes can be designed as given below.

3. TWO-DIMENSIONAL ML CHANNEL ESTIMATOR

To make the expression simpler the index n is omitted in the following sections. The noise in Equ (8) is considered as additive white Gaussian. The ML estimation of the time domain channel response is equivalent to the simple least square (LS) estimation which can be described as follows:

$$\hat{\mathbf{h}} = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{R} \qquad (12)$$

and the mean squared error (MSE) for the estimation is

$$MSE = \frac{1}{KL} E[\| \mathbf{h} - \hat{\mathbf{h}} \|^2] = \frac{\sigma_w^2 tr((\mathbf{X}^H \mathbf{X})^{-1})}{KL} \qquad (13)$$

where σ_w^2 is the noise power, L is the length of paths, K is the number of users and *tr(.)* is the trace operation defined as the sum of the diagonal element of the matrix. After some standard algebraic operations, we can obtain the following relation.

$$\mathbf{X}^H \mathbf{X} = \hat{\mathbf{P}}^H \hat{\mathbf{P}} \qquad (14)$$

where $\hat{\mathbf{P}}$ is a N x $L*K$ matrix which can be described as follows:

$$\hat{\mathbf{P}} = \begin{bmatrix} \bar{p}_1(1) & .. & \bar{p}_1(N-L+2) & .. & \bar{p}_K(1) & .. & \bar{p}_K(N-L+2) \\ \bar{p}_1(2) & .. & \bar{p}_1(N-L+3) & .. & \bar{p}_K(2) & .. & \bar{p}_K(N-L+3) \\ ... & .. & ... & .. & ... & .. & \\ \bar{p}_1(N) & .. & \bar{p}_1(N-L+1) & .. & \bar{p}_K(N) & .. & \bar{p}_K(N-L+1)) \end{bmatrix} \qquad (15)$$

$\bar{p}_k(i)$ is the i^{th} element of the vector $\bar{\mathbf{P}}_k$ which satisfies the following conditions.

$$\bar{\mathbf{P}}_i = \mathbf{T}_i * \mathbf{P}_i \qquad (16)$$

$\bar{\mathbf{P}}_i$ is the sequence after some circular shifts of the original training sequence $\mathbf{P}_i$.

4. OPTIMAL AND SUB-OPTIMAL TRAINING SEQUENCE

It is well known that the optimal training sequence minimizing the MSE should satisfy $\mathbf{X}^H \mathbf{X} \propto \mathbf{I}$ where $\mathbf{I}$ is the identity matrix. It can be shown by matrix inequalities or by matched filter. A simple proof is given in the following paragraph by treating the original problem as a constrained optimization problem and applying a necessary condition under some mild assumptions (KKT Conditions) [8]. Assume the energy of the training sequence from all the users are E, which can be defined as follows:

$$E = \sum_{k=1}^{K} \sum_{j=1}^{N} p_k^2(j) \qquad (17)$$

Let $\{\lambda_m : 1 \le m \le LK\}$ stands for the eigenvalues of the matrix $\hat{\mathbf{P}}^H \hat{\mathbf{P}}$, which are sure to be non-negative. The optimal training sequence sets should be the solutions of the following optimization problems.

Minimize $tr((\hat{\mathbf{P}}^H\hat{\mathbf{P}})^{-1}) = \sum_{m=1}^{KL}\frac{1}{\lambda_m}$ (18)

Subject to $tr(\hat{\mathbf{P}}^H\hat{\mathbf{P}}) = \sum_{m=1}^{KL}\lambda_m = \sum_{k=1}^{K}\sum_{j=1}^{N}p_k^2(j) = E$ (19)

$\lambda_m \geq 0, \quad 1 \leq m \leq KL$

The KKT condition [8] for this constrained optimization problem can be described as the follows:

$$-\begin{bmatrix}\frac{1}{\lambda_1^2}\\ \ldots\ldots\\ \frac{1}{\lambda_{KL}^2}\end{bmatrix} - \sum_{m=1}^{KL}u_m\begin{bmatrix}1\\ \ldots\ldots\\ 1\end{bmatrix} + v\begin{bmatrix}1\\ \ldots\ldots\\ 1\end{bmatrix} = \mathbf{0} \quad (20)$$

$$u_m \geq 0 \qquad u_m\lambda_m = 0 \quad 1 \leq m \leq KL$$

Obviously, $\lambda_m \neq 0$. Hence, from Equ (19) and (20) it is easy to obtain the solutions:

$$\lambda_m = \frac{E}{KL} \qquad 1 \leq m \leq KL \quad (21)$$

Because $\hat{\mathbf{P}}^H\hat{\mathbf{P}} = (\hat{\mathbf{P}}^H\hat{\mathbf{P}})^H$

There is $\hat{\mathbf{P}}^H\hat{\mathbf{P}} = \mathbf{U}^H\mathbf{\Lambda}\mathbf{U}$ where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix consists of the eigenvalues of the matrix $\hat{\mathbf{P}}^H\hat{\mathbf{P}}$. Due to Equ (21) there is

$$\mathbf{\Lambda} = \frac{E}{KL}\mathbf{I}\,. \quad (22)$$

From the previous discussion, it is shown that the optimal training sequences for this two-dimensional ML channel estimator should satisfy the following condition.

$$\mathbf{X}^H\mathbf{X} = \hat{\mathbf{P}}^H\hat{\mathbf{P}} = \frac{E}{KL}\mathbf{I} \quad (23)$$

The alternative lower bound of MSE or MMSE (Minimum Mean Square Error) can be described as the follows.

$$MMSE = \frac{\min(\sigma_w^2 tr((\mathbf{X}^H\mathbf{X})^{-1}))}{KL} = \frac{\sigma_w^2 KL}{E} \quad (24)$$

Moreover, it is very easy to reach the following conclusions from Equ, (17) and Equ. (23).

Corollary 1: .For the uplink transmission of both multiuser CP-CDMA and MC-CDMA systems, the optimal training sequences should satisfy the following conditions. 1) The autocorrelation function of the training sequence for a user should be zero over a shifted interval from one chip to a specific position of that user. The maximum shifted position for zero autocorrelation is the sum of the synchronization error of that user and the maximum delay spread. 2) The cross-correlation function between the training sequences of users should be zero over a shifted interval from one chip to the sum of the maximum delay spread and the possible maximum synchronization error of all the users.

In the downlink transmission, the common pilot can be used [3]. Because of the perfect synchronization between users, there is no interference between users as in the uplink cases. It leads to the following conclusion.

Corollary 2: For the downlink transmission of both multiuser SC-CDMA and MC-CDMA systems with CP, the autocorrelation function of the common pilot sequence for a user should be zero over a shifted interval from one chip to the maximum delay spread.

These two corollaries demonstrate that the requirements of optimal training sequences highly depend on the maximum delay spread and the synchronization errors. If the system is perfectly synchronous and the channel is single path, simple orthogonal sequence such as the Wash-Hadmard codes can be applied as optimal training sequences. Therefore, special optimal training sequence can be designed to satisfy systems with different requirements. Here, we consider a more general case where the channel is asynchronous with severe multipath fading. The optimal and sub-optimal training sequences are obtained from three classes of sequences called *park-park-song-suehiro* sequence [5], *Periodic Root of Unit Optimal (PRUO)* Sequence [6] are binary *Schroeder sequence* [7] , which have ideal autocorrelation and/or cross-correlation properties,

4.1 Park-Park-Song-Sehiro (PS) Sequence

The PS sequence is demonstrated in [5], which has zero autocorrelation except at periodic intervals and zero cross correlation in a set of sequences. Moreover, it is shown in [5] that the PS sequence can be obtained by using only integer sums and modular techniques, which make it more suitable for practical application. If PS sequence is used as training sequence, in order to make them to be optimal, both corollary (1) and (2) leads to the following results:

$$\max_{1\leq k\leq K}(\tau_k/T_c) < T_p \quad (25)$$

where τ_k denotes the timing error between users, T_c is the sampling interval and T_p stands for the periodic intervals of the PS sequence[5].

4.2 Periodic Root-of -Unit Optimal (PRUO) Sequence

Complex optimal sequence is shown in [6], which has ideal autocorrelation property. By selecting the parameters properly, PRUO sequence with ideal cyclic autocorrelation property can be generated. Denote the sequence $\mathbf{Z} = [Z_1,\ldots,Z_N]$ as one PRUO sequence. For the uplink transmission, the training sequence for each user can be obtained by circularly shifting the original PRUO sequence which can be described as follows:

$$Z^1 = Z \quad (26)$$
$$Z^{k+1} = f(Z^k) = [Z^k_{N-S+1},\ldots,Z^k_{N,\ldots}Z^k_1\ldots Z^k_{N-S+2}] \quad 1 \leq k \leq K-1$$

where Z^k denotes the training sequence for k^{th} user. If $\max_{1\leq k\leq K}(\tau_k/T_c) + L < s = \lfloor N/K \rfloor$, where $\lfloor\bullet\rfloor$ denotes the largest integer number not exceeding $\bullet$, these training sequences are sure to satisfy corollary (1). Furthermore, the constant envelop of these sequences allows more efficient utilization of the transmitter power amplifier, which makes the PRUO sequence more suitable for practical implementation. For the downlink transmission, this sequence can be directly applied as the common pilot because of its ideal autocorrelation property.

4.3 Schroeder sequence

If binary training sequence is desired in the practical system, the Schroeder sequence [7] can be considered as a sub-optimal sequence of PRUO sequence. Schroeder sequence with arbitrary length w can be generated according to the following methods:

$$b_i = 1 - 2 * \lfloor i^2 / w \rfloor_{\text{mod } 2} \qquad 1 \le i \le w \qquad (27)$$

The training sequence for each user in the uplink transmission can be obtained by the same rule described in Equ (26).

The proposed two-dimensional ML channel estimator with optimal training sequence is very efficient and simple for practical implementation. Because $(\mathbf{X}^H \mathbf{X})^{-1} \propto \mathbf{I}$, no matrix storage or matrix inverse operation is needed. Moreover, because of the special properties that have been described in Equ (6) and Equ (11) $\mathbf{X}^H \mathbf{R}$ can be efficiently computed using FFT and some simple multiplications.

5. SIMULATION STUDIES

The simulation system assumes packet transmission of data. Each packet consists of 64 blocks. In each packet, 4 blocks are used as the pilot blocks while the others as the data blocks being transmitted. The number of subcarriers is selected as 512 and the processing gain is 16. The chip rate is assumed as 3.84 Mcps. The channel model is a 25-path slow fading Rayleigh channel (the vehicular mobility is 36 kmph). Relative timing delays in uplink transmission between the users are less than six chips, which satisfies Equ (25) and Equ (26). The simulation results are presented in the terms of MSE and BER (Bit Error Rate).

Fig.1 presents the MSE performance of this ML channel estimator under fully loaded system using different training sequences. It is shown that the optimal training sequences can reach the analytical lower bound (MMSE) of this estimator. Sub-optimal Schroeder sequence performs in a close manner compared to the optimal training sequence with a short in performance of about 1 dB.

In Fig. 2 the BER performance for the downlink transmission of multi-user (=16) SC-CDMA and MC-CDMA systems with CP are shown. Minimum Mean Squared Error Combing (MMSEC) is evaluated for estimated channel based on different pilot signals because it can eliminate the multiple access interference (MAI). For the optimal training sequence (PRUO or PS) and the Schroeder sequence, the BER performance of both systems degrades only a little compared with the result from the ideal channel estimation. The BER performance of the multi-user uplink transmission is not presented because it depends on the multi-user detection (MUD) schemes we apply.

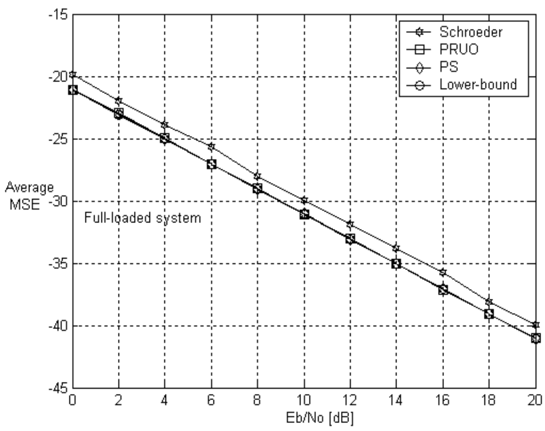

Fig.1 MSE estimation performance of ML channel estimator using different training sequences for fully-loaded system

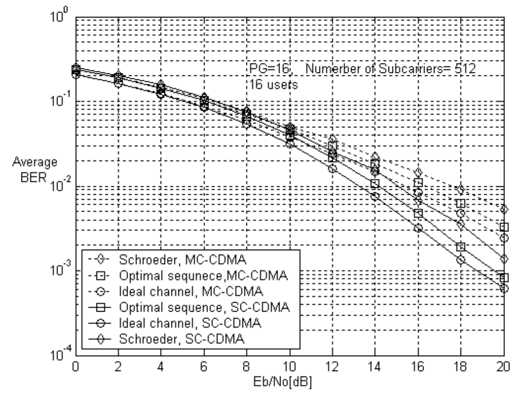

Fig.2 BER performance using different training sequences for fully-loaded SC-CDMA and MC-CDMA systems

6. CONCLUSION

This paper investigates the optimal training sequence design problem for a unified two-dimensional ML multipath channel estimator. The requirements for the optimal training sequence of this channel estimator are discovered by solving a constrained optimization problem. Optimal and sub-optimal training sequences are obtained from PS sequence, PRUO sequence and binary Schroeder sequence. The simulation results show that this channel estimator with optimal training sequence can reach single-user channel estimation performance with a considerable reduction in system complexity.

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