Admission Control Based Activity Detection in DS/CDMA Mobile Systems

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Abstract — **An on-line Bayesian based multiple hypotheses detection algorithm is used in the detection/isolation of a new user in a multiuser CDMA environment. The algorithm makes use of user arrival rate information available at the system admission controller. Comparison by simulation between this algorithm and the Matrix CUSUM shows that, when such information is available, the new algorithm that incorporates this information can achieve better performance than the non-Bayesian approach.**

I. Introduction

Multiuser detection (MUD) has been shown to be an important demodulation technique for use in direct sequence code division multiple access (DS/CDMA) systems. Though many MUD schemes have been proposed, their performance depends significantly on the assumptions made about the availability of other interferer parameters (for example, signature sequences and amplitudes) and on the complexity of the signal processors. For example, a change in user population in a mobile communication environment will degrade the performance of MUD if it cannot adapt quickly to take into account the new set of interferer parameters [5] (see also [2] for a good numerical example, where MUD suffers from catastrophic error after the entrance of a new user).

The acquisition of new user parameters in such situations has been studied in [1] using off-line methods. The authors in [5] have recently proposed a new on-line algorithm (the Matrix CUSUM) which is based on Nikiforov's generalized change detection algorithm in [4]. This new algorithm has the obvious on-line advantages over the off-line ones of [1] and its asymptotic optimality for the worst mean detection/isolation delay has been proven, These algorithms are designed to operate without prior knowledge of the change time in the network. However, in many applications some prior statistical information about entries and exits from the network is available at the system admission controller.

In this paper, we use a new on-line Bayesian admission control based detection algorithm that allows us to take into account the a priori probability of the change time given by the system admission controller. The algorithm is based on the generalized Shiryayev sequential probability ratio test (SSPRT) proposed by Durga and Speyer in [3]. Focusing on the detection of the entrace of a new user into the network, we also show how knowledge of the distribution of new user amplitude can be exploited in the SSPRT. Simulations for the case in which the amplitude of new user is known exactly and the case in which the amplitude is random with a known distribution, show that the SSPRT achieves superior performance to the Matrix CUSUM (in terms of fewer numbers of false alarms and misses, and shorter detection delay), especially in the latter case).

The paper is organized as follows. In Section II, a brief description of the signal model and the problem statement are presented. Section III summarizes the Matrix CUSUM algorithm. The SSPRT algorithm and its application in the cases of known new user amplitude and random amplitude with known distribution are presented in Section IV. Numerical results are given in Section V.

II. DS/CDMA Signal Model

Consider a synchronous binary DS/CDMA communication system with K active users transmitting through an additive white Gaussian noise channel. The received baseband signal during one symbol interval is passed through a chip-matched filter followed by a chip-rate sampler and is converted to an N vector of samples, where N denotes the processing gain (spreading factor). At the i-th received symbol we have:

$$r(i) = \mathbf{S}\mathbf{A}b(i) + \sigma n(i) \quad i = 1, 2, ... \tag{1}$$

where $\mathbf{S} = [s_1 s_2 ... s_K]$ is the matrix whose columns are the users normalized signature sequences (i.e., $s_k^T s_k = 1$), $b(i) = [b_1 b_2 ... b_K]$ is a vector of user data symbols ($b_k = \pm 1$), $\mathbf{A} = \text{diag}(A_1, A_2, ..., A_K)$ is the diagonal matrix of user received amplitudes, σ is the standard deviation of the noise samples, and $n(i)$ is a white Gaussian vector with mean zero and covariance matrix $\mathbf{I}_N$ (where $\mathbf{I}_N$ denotes the $N \times N$ identity matrix). It is assumed that there is a total of N linear independent signature sequences and $K < N$. Here and elsewhere x^T denotes the transpose of the matrix or vector x.

A. Problem statement

We adopt the basic formulation of [5]. Let $M = N - K$ and $\mathbf{Z} = \{z_j\}_{j=1}^{M}$ denote the set of remaining signature sequences to be assigned to new users. The detection/isolation problem is formulated as detecting the appearance of a new users (detection) and deciding which element of $\mathbf{Z}$ has been assigned to that new user (isolation). Denote the unknown change time as α, Let $\mathcal{H}_j^\alpha$ be the hypothesis that the spreading code z_j is assigned to a new user at the change time α. Hence under $\mathcal{H}_j^\alpha$, $j = 1, 2, \ldots, M$, we have

$$r(i) = \begin{cases} \mathbf{SA}b(i) + \sigma n(i), & i = 1, 2, ..., \alpha - 1 \\ \mathbf{SA}b(i) + A_{K+1} b_{K+1}(i) z_j + \sigma n(i) & i = \alpha, \alpha + 1, ... \end{cases} \tag{2}$$

Define the composite hypothesis $\mathcal{H}_j = \cup_{\alpha=1}^{\infty} \mathcal{H}_j^\alpha$ for $j = 1, \ldots, M$ each corresponding to a different change type. Then we have a total of M hypotheses about the type of change, plus the null hypothesis $\mathcal{H}_0$ which represents the no change situation $\alpha = \infty$.

Unlike the situation in [5], we further assume that users arrive independently according to a Bernoulli process with probability p that a new user arrives at any time instant, where p is specified by the system admission controller. Therefore the mean time between arrivals is $\mathbf{E}[\alpha] = 1/p$. We do not treat the case in which users exit the network, although this situation can be treated in a straightforward modification of the approach developed here.

III. Matrix CUSUM

This section summarizes the likelihood based and minimum mean square error (MMSE) based Matrix CUSUM algorithms developed in [5]. These will be used for comparison with our Bayesian detection algorithm.

A. Matrix CUSUM

This algorithm compute the CUSUM statistic in a recursive fashion as follows. First, the score function is computed for each vector observation $x(i)$

$$g_i(j, 0) = \ln \frac{f_j(x(i))}{f_0(x(i))}, \quad 1 \le j \le M \tag{3}$$

where f_j is the probability density function (pdf) under $\mathcal{H}_j$. Secondly, we define $g_i \triangleq [g_i(1,0), g_i(2,0), ..., g_i(M,0)]^T$ and update the CUSUM matrix $\mathbf{T}_i$ as

$$\mathbf{T}_i = (\mathbf{T}_{i-1} + \mathbf{G}_i)^+, \quad \mathbf{T}_0 = \mathbf{0} \tag{4}$$

where $\mathbf{G}_i \triangleq g_i \mathbf{1}^T - \mathbf{1} g_i^T + \text{diag}(g_i)$ and $\mathbf{1}$ is an $M \times 1$ vector of all ones. The operation $(.)^+ \triangleq \max(., 0)$ is applied element-wise. After that, the vector Q_i containing the minimal element of each row of $\mathbf{T}_i$ is calculated. The algorithm stops as soon as any element of Q_i exceeds a threshold h and declares a change of type $\mathcal{H}_j$ where j is the index of the first element of q_i to exceed h.

B. Known amplitude case

The first step is to compute the noise column subspace $\mathbf{U}$ so that $\mathbf{U}^T\mathbf{U} = \mathbf{I}_M$ and $\mathbf{U}^T\mathbf{S} = \mathbf{0}$. Then, the vector observation $x(i)$ is defined as $x(i) \triangleq \mathbf{U}^T r(i)$. Also define $m(i) \triangleq \mathbf{U}^T n(i)$ and $w_j \triangleq \mathbf{U}^T z_j$. With the above definitions, we have the following change detection problem:

$$x(i) = \begin{cases} \sigma m(i) & i = 1, 2, ..., \alpha - 1 \\ A_{K+1} b_{K+1}(i) w_j + \sigma m(i) & i = \alpha, \alpha + 1, ... \end{cases} \tag{5}$$

Since the sequence $x(1), x(2), ..., x(\alpha - 1)$ are i.i.d with Gaussian pdf $f_0 \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$ and $x(\alpha), x(\alpha + 1), ...$ are i.i.d with Gaussian mixture pdf $f_j \sim \frac{1}{2}\mathcal{N}(A_{K+1}w_j, \sigma^2\mathbf{I}_M) + \frac{1}{2}\mathcal{N}(-A_{K+1}w_j, \sigma^2\mathbf{I}_M)$, the score function and thus CUSUM matrix in (3) and (4) can be readily obtained.

C. MMSE-based Matrix CUSUM

Because of the unavailability of exact knowledge about amplitude of new user A_{K+1} in practice, authors in [5] replaced the score function in (3) by the following

$$g_i(j, 0) = (v_j^T r(i))^2 - v_j^T \mathbf{C} v_j, \quad 1 \le j \le M \tag{6}$$

where $\mathbf{C}$ is the autocorrelation matrix of the received signal before change and given b

$$\mathbf{C} \triangleq \mathbf{E}\{r(i) r(i)^T\} = \mathbf{SA}^2\mathbf{S}^T + \sigma^2 \mathbf{I}_N \tag{7}$$

and v_j is MMSE detector for the new user

$$v_j = \frac{\mathbf{C}^{-1} z_j}{z_j^T \mathbf{C}^{-1} z_j} \tag{8}$$

The CUSUM matrix can be now computed using (4).

IV. Admission control based activity detection

In this section, we present the SSPRT algorithm which uses apriori knowlege of the arrival rate of new users from the admission controller. We present two detection algorithms: The first is for the detection of a new user for the case when the amplitude is known. The second algorithm considers the case when the amplitude is a random variable with known Rayleigh distribution. We use Rayleigh distribution as it is a commonly used model for a mobile environment, the extension to other distributions is straight forward.

A. SSPRT in known amplitude case

By showing that the posterior probabilities of hypotheses $\mathcal{H}_l$ where $l = 0, 1, 2..., M$ can be computed recursively, the authors in [3] have formulated the optimal decision algorithm as a dynamic programming problem and derived the explicit method for interpreting the threshold selection. Firstly, the posterior probabilities of hypotheses $\mathcal{H}_l$ are updated iteratively by

$$\begin{aligned} F_{i+1,l} &= \frac{\phi_{i,l}.f_l(x(i+1))}{\sum_{l=0}^{M} \phi_{i,l}.f_l(x(i+1))} \quad 0 \le l \le M \\ F_{0,l} &= \pi_l \\ \phi_{i,j} &= F_{i,j} + p_j(1 - F_{i,j}) \quad 1 \le j \le M \\ \phi_{i,0} &= \prod_j (1 - \phi_{i,j}) \end{aligned} \tag{9}$$

where π_l is initial probability of $\mathcal{H}_l$, p_j is a priori probability of occurrence of $\mathcal{H}_j$ (note that $\sum_j p_j = p$). After putting (9) in a dynamic programming formulation, the optimal decision rule, that minimizes total expected cost, becomes

- Announce a change $\mathcal{H}_j$ if $F_{i,j} \ge F_{T_i,j}$
- Take another observation otherwise

where $F_{T_i,j}$ is the optimal threshold at time i. Due to the complexity in evaluating the optimal thresholds, it was shown in [3] that the limiting stationary threshold F_j can be used as the following has been proved $F_{T_0,j} \le F_{T_1,j} \le ... \le F_j = \frac{R_j}{R_j + D_j}$ where R_j and D_j are the cost of false decision and the cost of delay by taking another measurement respectively. Thus the choice of threshold becomes the setting the costs R_j, D_j which depends on the specific applications.

The application of SSPRT in the case of known new user amplitude is straight forward, using the same vector observation x and pdf as in the subsection III.B. and (9)

B. SSPRT in random amplitude case

Here we assume that the new user amplitude is not known but its a priori Rayleigh distribution is known. For this case, we also use the statistic x in (5) and pdf $f_0 \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$. The other pdf f_j can be derived using

$$f_j(x) = \int_0^\infty f_j(x \mid a) f_A(a) da \quad 1 \le j \le M \tag{10}$$

where $f_j(x \mid a) \sim \frac{1}{2}\mathcal{N}(aw_j, \sigma^2 \mathbf{I}_M) + \frac{1}{2}\mathcal{N}(-aw_j, \sigma^2 \mathbf{I}_M)$ is the conditional pdf given the amplitude of new user is $a \ge 0$ and $f_A(a)$ is pdf of new user amplitude.

The pdf $f_A(a)$ of the Rayleigh distribution is

$$f_A(a) = \frac{a}{\sigma_a^2} \exp(-\frac{a^2}{2\sigma_a^2}) \tag{11}$$

Placing (11) into (10) we obtain $f_j \sim \frac{1}{2}\mathcal{L}(w, \sigma, \sigma_a) + \frac{1}{2}\mathcal{L}(-w, \sigma, \sigma_a)$ where $\mathcal{L}$ distribution has the pdf $f(x)$ as

$$\begin{aligned} Y_1 &= \frac{\sigma}{(2\pi)^{M/2}\sigma_s} \\ Y_2(x) &= \exp(-\frac{x^T x}{2\sigma^2}) \\ Y_3(x) &= \exp(\frac{x^T w x^T w \sigma_a^2}{2\sigma^2\sigma_s^2}) \frac{x^T w \sigma_a}{\sigma\sigma_s} \mathrm{erfc}(-\frac{x^T w \sigma_a}{\sqrt{2}\sigma\sigma_s}) \\ f(x) &= Y_1 Y_2(x) \left[Y_3(x)\sqrt{\frac{\pi}{2}} + 1 \right] \end{aligned} \tag{12}$$

where $\mathrm{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_y^\infty \exp(-t^2) dt$ is the complementary error function and $\sigma_s^2 = \sigma^2 + \sigma_a^2 w^T w$.

V. Numerical Examples

We use the same system configuration for simulation as in [5] with processing gain $N = 31$, $K = 10$ active users so that $\mathbf{S} = [s_1, s_2, ..., s_{10}]$, the remaining $M = 21$ signature sequences form $\mathbf{Z} = [z_1, z_2, ..., z_{21}]$, equal amplitudes $A_1 = A_2 = ... = A_{10} = 3$, noise variance $\sigma^2 = 1$, new user amplitude $A_{11} = 3$. We assume that only one user enters the system at a time with equal a priori probability $p_j = p/M$. The SSPRT starts with initial probabilities $\pi_0 = 0.99, \pi_1 = \pi_2 = ... = \pi_M = 0.01/M$.

Since the Matrix CUSUM algorithm does not take into account the a priori probability of the change time, we can not use the same simulation setup in [5] to compare it with SSPRT. Furthermore, the problem of detection/isolation here is not simply a multiple hypotheses testing problem but more complicated due to the change in number of hypotheses and dependence of that number on the correctness of the previous detection/isolation. Therefore we design our simulation scheme as follows

Step1: generate T change points $\alpha_0 = 0$, $\alpha_1, \alpha_2, ..., \alpha_T$ with $\mathbf{E}[\alpha_{t+1} - \alpha_t] = 1/p$. Initialize $NumFalse = NumMiss = NumDetect = TimeDelay = t = 0$

Step2: generate statistic x in (5) for the period from α_t to $\alpha_{t+1} - 1$ under H_0 and from α_{t+1} to α_{t+2} under $\mathcal{H}_j$

Step3: there are 3 possibilities

1. 1. if there is a false alarm or isolation, then $NumFalse = NumFalse + 1$
2. 2. if there is a true detection with delay d, then $NumDetect = NumDetect + 1, TimeDelay = TimeDelay + d$
3. 3. if there is no alarm, then $NumMiss = NumMiss + 1$

Step4: $t = t + 1$ and if $t \le T - 2$, then go to Step2.

Step5: The mean detection delay is defined as $\overline{T_d} = \frac{TimeDelay}{NumDetect}$ Because a false alarm or a false isolation in the past decision will cause the miss in the future decisions (the set of actual hypotheses will be different from the set of testing hypotheses), it makes sense to take the sum of $NumFalse + NumMiss$ and use it and the mean detection delay $\overline{T_d}$ as the performance indexes in comparison between the two algorithms.

We use $T = 10000$, $p = 0.01$ in all simulations below.

The second simulation compare MMSE-Based CUSUM and SSPRT in the case where amplitude of new user has Rayleigh pdf and SSPRT with known mean amplitude only for a set of different thresholds. The Rayleigh distribution is specified by $\sigma_a^2 = 2\mathbf{E}[a]^2/\pi$ where mean amplitude $\mathbf{E}[a] = 3$. The performance comparison is shown in Fig. 1 where SSPRTs (even SSPRT with known mean amplitude only)*give much lower number of false and misses at a considerably shorter mean detection delay* (16 versus 120 and 6 versus 12 respectively). It can also be noted that SSPRT with known distribution achieves better performance than SSPRT with known mean amplitude only.

VI. Conclusion

In this paper, we have shown by simulation that an algorithm (SSPRT) taking into account available knowledge about the system parameters can outperform those ignore that information. The venue for further work is to prove the performances of SSPRT analytically.

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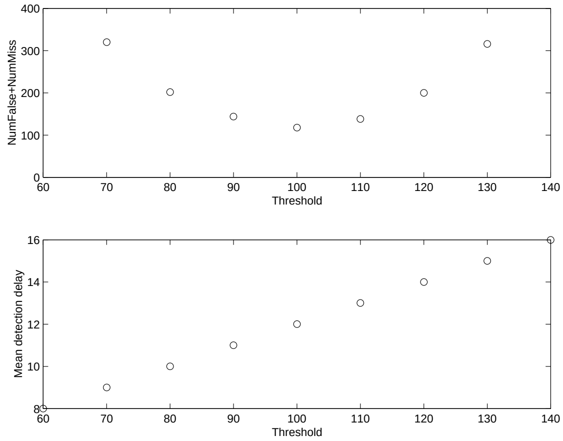
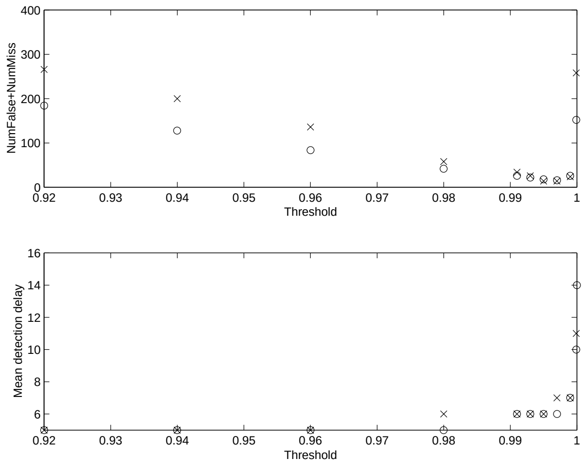


(a) MMSE CUSUM



(b) SSPRT, 'o' - known distribution, 'x' - known mean amplitude

Figure 1: Random new user amplitude with Rayleigh distribution case