ADVANCED RECEIVERS FOR MC-CDMA WITH MODIFIED DIGITAL PROLATE FUNCTIONS

I. Raos; S. Zazo

High Technical School of Telecommunications, UPM, Madrid, Spain
e-mail:ivana@gaps.ssr.upm.es

ABSTRACT

Modified Digital Prolate Functions (DPF) are adressed in this paper, applied as waveforms for downlink MC-CDMA system and providing higher transmission rates. Those waveforms, thanks to their double orthogonality and energy concentration in restricted interval properties, enable specific processing at receiver. Design and analysis of advanced receivers exploiting mentioned DPF's properties and commputer simmulations that demostrate their performance improvement compared to MC-CDMA system with Hadamard codes are presented.

1. INTRODUCTION

Wireless comunications are atractive nowdays as number of data-transmiting users is growing. Possibility of non-wired access enables their mobility and eliminates the need for annoying cabling, among other advantages. However, along with the growth of number of users, data rates to be transmitted also are increasing. These facts centred wireless channels investigation and design of systems that could meet this growth in focus of communication research community. Multi Carrier Code Division Multiple Access (MC-CDMA) is proposed by many researchers for transmission over fading channels because of its robustness to frequency selective channels and frequency diversity used for each user. [5]

Increase of required data rates stimulated authors to investigate the possibility of time-domain efficiency increase of MC-CDMA waveforms. Avoidance of time redundancy (Cyclic Prefix, Zero Padding are the most common) is accomplished when modified DPF are used as spreading codes [1]. Their double orthogonality property enables specific processing that will be analysed in more depth in this contribution. Downlink MC-CDMA is considered, as major data rates are expected in this link.

Some properties and definitions of modified DPF will be exposed in Section 2, while more detailed analysis, including discrete DPF signals used as codes, can be found in [1], [2] and [3]. System model followed by proposed enhanced receivers are presented in Sections 3 and 4. Receivers that are analysed here differ in filter integration period, due to what different parameters should be taken into account in their performance analysis. First receiver, whose integration period is equal to signal's period, suffers from Inter and possible intra Symbol Interference (ISI and iSI). The second one, with reduced integration period, requires noise analysis, as it becomes colored. In this section their performances are analysed and compared via computer simulations. Finally, conclusions and comments are presented in Section 5.

WORK PARTLY SUPPORTED BY PROJECTS $07T/0032/2000$, $IST-2001-32620$ AND $TIC2000-1395-C02-02$

2. DPF: ORTHOGONAL WAVEFORM FOR MC-CDMA

If waveform's energy is to be concentrated at the beginning of the symbol, the ratio to be maximized is:

$$\alpha = \frac{\frac{1}{T}\int_0^\tau |f(t)|^2\,dt}{\frac{1}{T}\int_0^T |f(t)|^2\,dt} = \frac{1}{E_y}\int_0^\tau |f(t)|^2\,dt \qquad (1)$$

where $f(t) = \sum_{n=0}^{N-1} y_n e^{jn\omega_0 t}$ is the signal with period T that is to be concentrated in interval $[0,\tau]$, E_y its energy, $\omega_0 = \frac{2\pi}{T}$ and N is the number of modulated exponentials.

Eq. (1) can be written in matrix form as:

$$\alpha = \frac{\mathbf{y}^H \mathbf{W}_d \mathbf{y}}{\mathbf{y}^H \mathbf{y}} \qquad (2)$$

where the entries of matrix $\mathbf{W}_d$ are:

$$\mathbf{W}_d(k,n) = \frac{\tau}{T} sinc\left(\frac{\tau}{T}(n-k)\right) e^{j\pi\frac{\tau}{T}(n-k)} \qquad (3)$$

and $\mathbf{y} = [y_0, ...y_{N-1}]^T$ is vector collecting exponentials' coefficients. Eq. (2) is known as Rayleigh Quotient and is maximized by choosing $\mathbf{y}$ to be the eigenvector corresponding to largest eigenvalue of matrix $\mathbf{W}_d$. If spectral analysis of this matrix is performed, N obtained eigenvectors are orthogonal, with their associated eigenvalues denoting their energy inside the restricted interval. Obtained waveforms are orthogonal in interval $[0,\tau]$ and orthonormal in $[0,T]$ [4].

3. SYSTEM MODEL

If modified DPF, described in previous Section, are used as orthogonal codes for MC-CDMA, $\mathbf{y}^{(k)}$ is k^{th} user's code, and $f^{(k)}(t)$ is its waveform. N eigenvectors obtained by decomposition of $\mathbf{W}_d$ define N users' codes and its N eigenvalues denominated λ_n specify the portion of signal energy inside the desired interval.

Sketch of system model is depicted in Fig. 1. At trans-

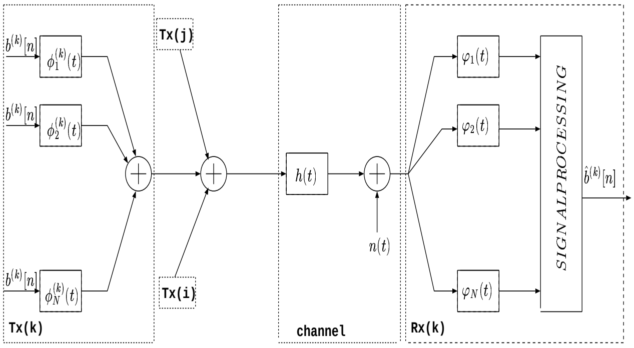

Fig. 1. System model

mitter symbol's modulation to N subcarriers is performed with user-specific waveform:

$$\phi^{(k)}(t) = \frac{1}{\sqrt{T}} \sum_m y_m^{(k)} e^{jm\omega_0 t} \Delta_t(0,T) = \sum_m \phi_m^{(k)}(t) \tag{4}$$

where $\Delta_t(x,y) = u(t-x) - u(t-y)$ is used for abbreviation, $u(t)$ beeing the step function.

Considering k^{th} user transmitting n^{th} symbol, received signal is obtained as convolution of channel impulse response and transmitted signal. In order to determine dependence of received signal and observation moment at receiver, it is divided in 3 components:

$$\begin{aligned}
& t \in nT + [0, T_c] \\
& s_1^{(k)}(t) = b^{(k)}[n] \tfrac{1}{\sqrt{T}} \textstyle\sum_m y_m^{(k)} e^{jm\omega_0 t} H(m\omega_0, 0, t) \\
& t \in nT + [T_c, T] \\
& s_2^{(k)}(t) = b^{(k)}[n] \tfrac{1}{\sqrt{T}} \textstyle\sum_m y_m^{(k)} e^{jm\omega_0 t} H(m\omega_0) \\
& t \in nT + [0, T_c] \\
& s_3^{(k)}(t) = b^{(k)}[n-1] \textstyle\sum_m y_m^{(k)} \tfrac{e^{jm\omega_0 t}}{\sqrt{T}} H(m\omega_0, t, T_c)
\end{aligned} \tag{5}$$

Channel's influence on signal depends of the different time interval considered, and the abbreviated notation is:

$$H(m\omega_0, a, b) = \int_a^b h(\tau) e^{-jm\omega_0 \tau} d\tau \tag{6}$$

where $h(t)$ stands for channel impulse response that is limited to the interval $[0, T_c]$, and a and b depend on the observation instant t.

Signal spreading due to dispersive channel is observed in these equations: symbol with original duration T, is received with duration $T + T_c$. Three different channel effects are separated: $s_1(t)$ and $s_3(t)$ are starting and tail transient periods, while $s_2(t)$ represents the steady-state response.

4. PROPOSED RECEIVERS AND THEIR PERFORMANCES

Receiver's digital front end is a filter bank, as indicated in Fig 1. Two types of filter banks will be proposed and analysed in this Section.

4.1. Filter bank design - integration period T

Considering the receiver of one arbitrary user kk and the integration period of filter bank at receiver that is equal to the symbol duration, filter bank waveforms can be represented as:

$$\varphi(t) = \frac{1}{\sqrt{T}} \sum_m e^{-jm\omega_0 t} \Delta_t(0,T) = \sum_m \varphi_m(t) \tag{7}$$

For a convenient analysis of filter bank output, the received signal should be decomposed into 3 parts, modifying Eq. (5). With this formulation, it can be interpreted as having at the beginning of period transient regimes, causing iSI and ISI, and steady-state (SS) part during whole integration period:

$$\begin{aligned}
& t \in nT + [0, T_c] \\
& s_1^{(k)}(t) = -b^{(k)}[n] \tfrac{1}{\sqrt{T}} \textstyle\sum_m y_m^{(k)} e^{jm\omega_0 t} H(m\omega_0, t, T_c) \\
& + b^{(k)}[n-1] \tfrac{1}{\sqrt{T}} \textstyle\sum_m y_m^{(k)} e^{jm\omega_0 t} H(m\omega_0, t, T_c) \\
& t \in nT + [0, T] \\
& s_2^{(k)}(t) = b^{(k)}[n] \tfrac{1}{\sqrt{T}} \textstyle\sum_m y_m^{(k)} e^{jm\omega_0 t} H(m\omega_0)
\end{aligned} \tag{8}$$

Signal at filter bank output is obtained after integration of received signal and waveforms defined in eq. (7):

$$r_m[n] = \sum_k \int_0^T s^{(k)}(t) \varphi_m(t) dt \tag{9}$$

with $s^{(k)}(t) = s_1^{(k)}(t) + s_2^{(k)}(t)$, considering signals of all users all users. Eq. (9) is decomposed using matrix notation in the following two parts, as explained before, and at slicer SS part and transient regimes result in:

$$r_{SS}^{(kk)}[n] = \sum_k \mathbf{x}^{(kk)^T} \mathbf{H} \mathbf{y}^{(k)} b^{(k)}[n] \tag{10}$$

$$\begin{aligned}
r_{iSI+ISI}^{(kk)}[n] = & - \textstyle\sum_k \mathbf{x}^{(kk)^T} \mathbf{A} \mathbf{y}^{(k)} b^{(k)}[n] \\
& + \textstyle\sum_k \mathbf{x}^{(kk)^T} \mathbf{A} \mathbf{y}^{k} b^{(k)}[n-1]
\end{aligned} \tag{11}$$

where, for abbreviation, the following notation was introduced:

$$a(n,m) = \int_0^{T_c} e^{j(m-n)\omega_0 t} H(n\omega_0, t, T_c) dt \tag{12}$$

$a(n,m)$ are hermitian Toeplitz matrix $\mathbf{A}$ entries and $\mathbf{H} = diag(H(0), H(\omega_0), ...H((N-1)\omega_0)$ is channel's frequency response, supposing N subcarriers, and $\mathbf{x}^{(k)}$ is receiver's code that should be chosen.

Different types of signal processing are possible from this point:

$a)$ Receiver similar to standard MC receivers is welcome because of its computational simplicity, by equalizing SS part of received signal. This approach results in receivers' codes of the form:

$$\mathbf{x}_1^{(kk)^T} = \mathbf{y}^{(kk)^H} \mathbf{H}^{-1} \quad (13)$$

if Zero-Forcing (ZF) equalization is used. This receiver equalizes just SS channel's part leading to error floor because of non-equalized transient period. However, because of its simplicity, it will be analyzed further.

$b)$ Codes can be designed to equalize the symbol, including starting transient period. In this case, codes should satisfy:

$$\mathbf{x}_2^{(kk)^T} = \mathbf{y}^{(kk)^H} \left(-\mathbf{A} + \mathbf{H}\right)^{-1} \quad (14)$$

This main drawback of this receiver is inversion of the non-diagonal matrix that requires more complex computation. Furthermore, matrix $\mathbf{A}$ is channel dependent. This computational complexity force us to simplify signal processing at receiver. It can be performed with already proposed receiver (Eq. 14), or with

$c)$ diagonal approximation of matrix $\mathbf{A}$:

$$\mathbf{x}_3^{(kk)^T} = \mathbf{y}^{(kk)^H} \left(\mathbf{H} - diag(\mathbf{A})\right)^{-1} \quad (15)$$

Former processing is justified in case of channel resulting in mostly diagonal matrix $\mathbf{A}$.

In order to verify the system performances, computer simulations are performed, for all 3 proposed receivers, and results are shown in Fig. 2. Simulations are performed for 2-ray channel with fixed rays' amplitudes of 1 and 0.8. ISI power is shown on y-axe, and it is calculated for all 64 users. Loss due to receiver simplification is not drastical, but difference of ISI power of Hadamard and DPF waveforms for a certain number of users is obvious.

In Fig. 3 is shown ISI-power comparison of MC-CDMA with Hadamard and DPF codes. Scenario is 2-ray channel, with different distance between rays. Power of ISI is shown on y-axis depending on the rays' distance, for each one of 16 possible users. Signal period is $10ms$ ans maximum distance between rays is $1.2ms$. This figure demostrates that ISI power of users' signals with DPF codes is gradually increasing, while signals with Hadamard codes all have comparable ISI. In standard MC-CDMA system, this has the consequence that all users will suffer from comparable ISI if no time redundancy is added. However, in case of DPF codes, amount of ISI power will be negligible for most of users, and a few last ($\sim 12\%$ in this concrete scenario) will suffer from ISI comparable to the system with Hadamard codes.

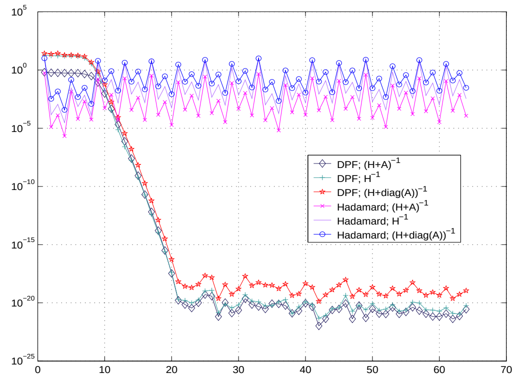

Fig. 2. ISI power depending of user's waveform

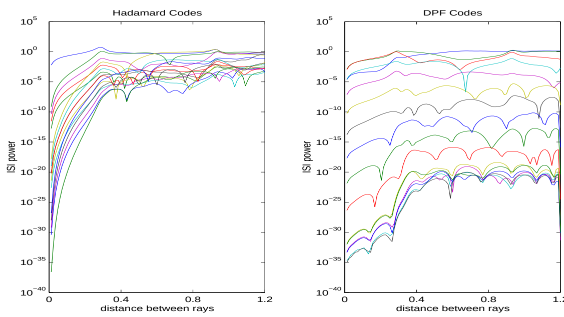

Fig. 3. ISI power depending of rays' distance

4.2. Filter bank design - Integration period τ

All of those three proposed receivers have the drawback that ISI can be avoided only for the users' codes that are concentrated in period $[0, \tau]$, and this cannot be accomplished for all users. This disadvantage can be overcome if integration period of signals is restricted to interval (T_c, T):

$$r_m[n] = \sum_k \int_{T_c}^{T} \left(s^{(k)}(t) + n(t)\right) \varphi_m(t)dt \quad (16)$$

with

$$\varphi(t) = \sum_m e^{-jm\omega_0 t} \Delta_t(T_c, T) = \sum_m \varphi_m(t) \quad (17)$$

and $n(t)$ representing additive white noise with power σ^2. Following this notation, signal's samples after filter bank can be written in matrix form as:

$$\mathbf{r}_1[n] = \sum_k \mathbf{W}_d^{(2)} \mathbf{H} \mathbf{y}^{(k)} b^{(k)}[n] + \mathbf{w}[n] \quad (18)$$

where elements of matrix $\mathbf{W}_d^{(2)}$ differ from $\mathbf{W}_d$, defined in Eq. 3, in a phase shift factor, $\mathbf{W}_d^{(2)} = \mathbf{W}_d^*$, and $w[n]$ are colored noise samples with noise covariance matrix $\mathbf{C}_w = \sigma^2 \mathbf{W}_d^{(2)}$. Noise color requires whitening filter after filter bank, providing signal's samples in the following form:

$$\mathbf{r}_2[n] = \sum_k \mathbf{W}_d^{T-1/2}\mathbf{H}\mathbf{y}^{(k)}b^{(k)}[n] + \mathbf{n}[n] \tag{19}$$

Afterwards, signal's equalization should be performed with vector $\mathbf{x}^{(kk)^T}$, to be designed:

a) If frequency flat channel is considered ($\mathbf{H}(n,n) = H,\ \forall n$), double orthogonality property of DPF can be exploited, as it remains unaltered and codes at receiver can be chosen as $\mathbf{x}^{(kk)^T} = \mathbf{y}^{(kk)H}H^{-1}$.

However, multicarrier modulations are usually applied in frequency selective channels, and receivers are designed for this case.

b) If simple ZF equalization followed by symbol's despreading is performed, noise variance becomes:

$$\sigma_{ZF}^2 = \sigma^2 \left(\mathbf{y}^{(kk)}\right)^H \mathbf{H}^{-1}\mathbf{W}_d^{-T}\mathbf{H}^{-H}\mathbf{y}^{(kk)} \tag{20}$$

Noise enhancement depends on both channel and waveform type and it can be dramatically increased, due to inversion of matrices $\mathbf{W}_d^*$ and $\mathbf{H}$, as eigenvalues of their product may take very small values, those corresponding to waveforms whose energy is primarly outside the integration interval.

Because of noise enhancement of this type of receiver, more sophisticated ones are developed. Instead of working in a basis of dimension N, advanced receiver would project received waveforms into a smaller one, using a rank reduction strategy. Not all waveforms will remain orthogonal with this processing, meaning that when number of users is greater than dimension of reduced subspace, they cause interference. However, rank reduction will cancel the need for matrix $\mathbf{W}_d^*$ inversion, decreasing noise variance at slicer.

Spectral decomposition of matrices' product $\mathbf{W}_d^{T-1/2}\mathbf{H}$ can be performed as:

$$\mathbf{W}_d^{T-1/2}\mathbf{H} = \mathbf{U}_1\Lambda_1\mathbf{V}_1^H + \mathbf{U}_2\Lambda_2\mathbf{V}_2^H \tag{21}$$

where diagonal matrix Λ_1 colects eigenvalues greater than a certain limit. Rank reduction is then performed by multiplication of received signal with $\Lambda_1^{-1}\mathbf{U}_1^H$, providing following signal's samples:

$$\mathbf{r}_3[n] = \sum_k \mathbf{V}_1^H\mathbf{y}^{(k)}b^{(k)}[n] + \mathbf{n}_1[n] \tag{22}$$

New, reduced dimension, user's codes are $\hat{\mathbf{y}}^{(k)} = \mathbf{V}_1^H\mathbf{y}^{(k)}$. If all users' samples are colected in one vector, it is:

$$\mathbf{r}_3[n] = \hat{\mathbf{Y}}\mathbf{b}[n] + \mathbf{n}_2[n] \tag{23}$$

c) Matrix $\hat{\mathbf{Y}}$ is full rank matrix, and ZF equalization can be performed with vector $\mathbf{x}_1^{(kk)^T}$ that is the row of matrix $\hat{\mathbf{Y}}^{-1}$ corresponding to reduced dimension vector of user kk.

d) Signal processing can be performed with MMSE detector as well, minimizing Eq. (24) and obtaining as solution Eq. (25):

$$E\left\{\left|\mathbf{x}_2^{(kk)^T}\left(\hat{\mathbf{Y}}\mathbf{b}[n] + \mathbf{n}_2[n]\right) - b^{(kk)}[n]\right|^2\right\} \tag{24}$$

$$\mathbf{x}_2^{(kk)^H} = \left(\left(\hat{\mathbf{Y}}\hat{\mathbf{Y}}^H + \mathbf{C}_2\right)^{-1}\hat{\mathbf{y}}^{(kk)}\right)^H \tag{25}$$

where $\mathbf{C}_2 = \sigma^2\Lambda^{-2}$ is the noise covariance matrix.

Simulations comparing last 2 receivers when spreading codes are DPF and Hadamard are performed. Results in terms of BER are shown in Fig. 4 for 2 -ray Rayleigh distributed channel with power profile $[0\ 3]dB$ and separation $T_c/2$. Averaging is performed over 5000 independent channel realizations.

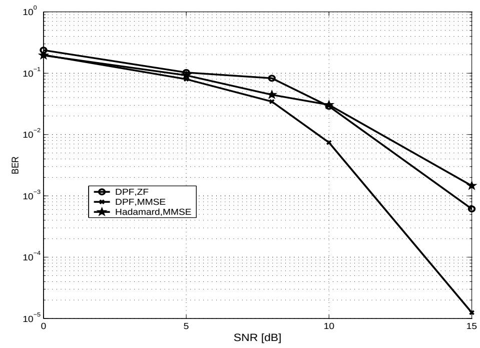

Fig. 4. Comparison of proposed receivers

5. CONCLUSIONS

Receivers for downlink MC-CDMA system employing modified DPF as user specific codes are proposed and analysed. Depending of the integration interval of filters bank at receiver's digital front end, analysis may need to concern all iSI, ISI and noise, depending on waveform and equalizer type that is used, when integration interval is equal to symbol period; if it is reduced, main concern should be noise enhancement. Those factors are analysed, confirming that energy concentration and double orthogonality property make these codes more suitable than usually employed Hadamard codes.

6. REFERENCES

[1] I.Raos, S.Zazo, F.Bader"Prolate Spheroidal Functions: A General Framework for MC-CDMA Waveforms" Proc of PIMRC 2002

[2] I.Raos, S.Zazo, J.M.Páez-Borrallo "Reduced Interference MC-CDMA System Using Discrete Prolate Codes", Proc of ICASSP 2002

[3] I.Raos, S.Zazo, J.M.Páez-Borrallo "MC-CDMA with Prolate-Spheroidal Functions-based Orthogonal Codes" Proc of EUSIPCO 2002,

[4] A.Papoulis"Signal Analysis" McGraw- Hill, Inc

[5] G.B.Giannakis, Z.Wang, A. Scaglione, S. Barbarosa "Mutually orthogonal transceivers for blind uplink CDMA irrespective of multipath channel nulls" Proc of ICASSP 99