R-CONJUGATE CODES FOR MULTI-CODE CDMA

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ABSTRACT

The multi-code CDMA problem, and the corresponding data and noise model is discussed. The problem of "cross-talk", which occurs when using an MMSE receiver in colored noise, is addressed. A method to avoid "cross-talk", by choosing the eigenvectors of the interference plus noise matrix as codes, is introduced. This approach is shown to produce codes that are orthogonal not only in the conventional sense, but also with respect to the interference plus noise matrix. The gain on the codes is shown to be inversely proportional to the corresponding eigenvalue. Simulation results demonstrate that the codes corresponding to the smallest eigenvalues dramatically out-perform the MMSE receiver for Walsh-Hadamard codes in terms of mean-square error (MSE).

1. INTRODUCTION

In multi-code CDMA, the user effects serial to parallel conversion on a bit stream. Multiple information symbols are transmitted simultaneously, with each symbol stream carried by a different code. By using multiple codes in a synchronous fashion, the user increases the data transfer rate.

The data model for multi-code CDMA is:

$$\mathbf{x}[n] = \sum_{l=1}^{K} s_l[n]\mathbf{c}_l + \mathbf{v}[n] \tag{1}$$

where c_l is the N×1 code vector carrying the l^{th} symbol stream, $s_l[n]$ is the n^{th} symbol of the l^{th} information stream, x[n] is the N×1 block of received data, and v[n] is the N×1 block of interference plus noise. The resultant N×N interference plus noise correlation matrix will be represented by:

$$\mathbf{Q} = \varepsilon\left\{\mathbf{v}[n]\mathbf{v}^H[n]\right\} \tag{2}$$

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Assuming the parallel symbol streams to be independent, i.e. $\varepsilon\left\{s_k[n]s_l^*[n]\right\} = \sigma_s^2\delta_k$, the N×N signal, plus interference, plus noise correlation matrix is:

$$\mathbf{R} = \varepsilon\left\{\mathbf{x}[n]\mathbf{x}^H[n]\right\} = \sum_{i=1}^{K}\sigma_s^2\mathbf{c}_i\mathbf{c}_i^H + \mathbf{Q} \tag{3}$$

The optimal MMSE weight vector employed at receiver to extract the k^{th} information symbol at time n takes the form:

$$\mathbf{w}_k = \alpha_k\mathbf{R}^{-1}\mathbf{c}_k \tag{4}$$

Where α_k is chosen to be $1/\mathbf{c}_k\mathbf{R}^{-1}\mathbf{c}_k$ so that $\mathbf{w}_k^H\mathbf{c}_k = 1$. However, this presents an issue for conventional orthogonal codes, since applying the MMSE weight for the k^{th} code to the n^{th} data block yields:

$$\mathbf{w}_k^H\mathbf{x}[n] = \sum_{l=1}^{K} s_l[n]\mathbf{w}_k^H\mathbf{c}_l + \mathbf{w}_k^H\mathbf{v}[n] \tag{5}$$

$$= \sum_{l=1}^{K}\alpha_l s_l[n]\mathbf{c}_k^H\mathbf{R}^{-1}\mathbf{c}_l + \mathbf{c}_k^H\mathbf{R}^{-1}\mathbf{v}[n] \tag{6}$$

When the code words are standard orthogonal codes, such as Walsh-Hadamard codes, and when **R**≠**I**, $\mathbf{c}_k^H\mathbf{R}^{-1}\mathbf{c}_l \neq \delta_{kl}$. The MMSE receiver destroys the orthogonality and allows for cross-talk between the codes.

We assume that the interference plus noise matrix **Q** that is fed back from the receiver to the transmitter is relatively constant for some small period of time. Note that the interference at the receiver may be other spread spectrum users, other wireless users, jamming, etc. In previous work, including [1], [2], [3], and [4], second-order statistics have been fed back from the receiver to the transmitter in order to select codes that attempt to reduce multi-user access interference. However, all of these approaches have restricted the codes to vectors consisting only of the values 1 or -1. They also did not address the problem of multi-code transmission.

The following approach shows that by allowing the codes to take on unconstrained values, we can completely eliminate crosstalk even in the colored noise environment. In contrast to prior work, we are not trying to adjust the codes of different users; rather we are adjusting the codes or a single multi-code user.

2. DESIGN OF R-CONJUGATE CODES: EIGENVECTOR CODES

Remarkably, designing the codes such that:

$$\mathbf{c}_k^H \mathbf{R}^{-1} \mathbf{c}_l \propto \delta_{kl} \tag{7}$$

allows for an MMSE receiver while still eliminating "cross-talk" amongst the codes. Conventional Walsh-Hadamard codes do not satisfy this constraint, because they suffer from "cross-talk" amongst codes as well as from interference.

One approach for choosing codes that satisfy **(7)** is to use eigenvectors of the interference plus noise correlation matrix **Q** as code words:

$$\mathbf{Q}\mathbf{c}_k = \lambda_k \mathbf{c}_k \tag{8}$$

Since **Q** is Hermitian symmetric, it follows that c_k is also an eigenvector of $\mathbf{Q}^{-1}$:

$$\mathbf{Q}^{-1}\mathbf{c}_k = \frac{1}{\lambda_k}\mathbf{c}_k \tag{9}$$

Further, since **Q** and $\mathbf{Q}^{-1}$ are both Hermitian symmetric, their eigenvectors are orthonormal (conjugate in the conventional sense):

$$\mathbf{c}_k^H \mathbf{c}_l = \delta_{kl} \tag{10}$$

It follows from (9) and (10) that:

$$\mathbf{c}_k^H \mathbf{Q}^{-1} \mathbf{c}_l = \frac{1}{\lambda_k}\mathbf{c}_k^H \mathbf{c}_l \tag{11}$$

Thus, the eigenvectors of $\mathbf{Q}^{-1}$ (which are the same as the eigenvectors of **Q**) are conjugate in the conventional sense **and** are $\mathbf{Q}^{-1}$-conjugate as well:

$$\mathbf{c}_k^H \mathbf{Q}^{-1} \mathbf{c}_l \propto \delta_{kl} \tag{12}$$

Furthermore, selecting eigenvectors of **Q** as codes implies they are **R**-conjugate **and** $\mathbf{R}^{-1}$-conjugate, as well as **Q**-conjugate, $\mathbf{Q}^{-1}$-conjugate, and **I**-conjugate (the last one meaning orthogonal in the conventional sense). To prove this, recall the structure of **R** in (3):

$$\mathbf{R}\mathbf{c}_k = \left\{\sum_{l=1}^{K} \sigma_s^2 \mathbf{c}_l \mathbf{c}_l^H + \mathbf{Q}\right\}\mathbf{c}_k \tag{13}$$

$$= \sum_{l=1}^{K} \sigma_s^2 \mathbf{c}_l \left(\mathbf{c}_l^H \mathbf{c}_k\right) + \mathbf{Q}\mathbf{c}_k \tag{14}$$

$$= \sigma_s^2 \mathbf{c}_k + \lambda_k \mathbf{c}_k \tag{15}$$

$$= \gamma_k \mathbf{c}_k \tag{16}$$

This implies that $\mathbf{c}_k$ is an eigenvector of **R** with an eigenvalue of $\gamma_k = \sigma_s^2 + \lambda_k$. Furthermore, since **R** is Hermitian symmetric, c_k is also an eigenvector of $\mathbf{R}^{-1}$:

$$\mathbf{R}^{-1}\mathbf{c}_k = \frac{1}{\gamma_k}\mathbf{c}_k \tag{17}$$

This is crucial, since it implies if we select an eigenvector of **Q** as the k^{th} code, the MMSE weight vector for extracting the k^{th} symbol is equal to the code itself:

$$\mathbf{w}_k = \frac{\mathbf{R}^{-1}\mathbf{c}_k}{\mathbf{c}_k \mathbf{R}^{-1}\mathbf{c}_k} = \frac{\frac{1}{\gamma}\mathbf{c}_k}{\frac{1}{\gamma}\mathbf{c}_k^H \mathbf{c}_k} = \mathbf{c}_k \tag{18}$$

The denominator reduces to one, since $\mathbf{c}_k$ is an eigenvector of $\mathbf{R}^{-1}$ with unit length. This result is directly analogous to using Walsh-Hadamard codes in white noise: the MMSE receiver for the k^{th} Walsh-Hadamard code is the code itself.

Applying the $\mathbf{c}_k$ receiver weight to the n^{th} data block gives, recalling (10):

$$\mathbf{w}_k^H \mathbf{x}[n] = \mathbf{c}_k^H \mathbf{x}[n] \tag{19}$$

$$= \sum_{l=1}^{K} s_l[n] \mathbf{c}_k^H \mathbf{c}_l + \mathbf{c}_k^H \mathbf{v}[n] \tag{20}$$

$$= s_k[n] + \mathbf{c}_k^H \mathbf{v}[n] \tag{21}$$

In summary, if we select eigenvectors of **Q**, the interference plus noise correlation matrix, as codes, we find that $\mathbf{c}_k$, k = 1..N, are orthogonal in all of the following ways:

I-conjugate $\Rightarrow$ $\mathbf{c}_k^H \mathbf{c}_l = \delta_{kl}$

Q-conjugate $\Rightarrow$ $\mathbf{c}_k^H \mathbf{Q} \mathbf{c}_l \propto \delta_{kl}$

$\mathbf{Q}^{-1}$-conjugate $\Rightarrow$ $\mathbf{c}_k^H \mathbf{Q}^{-1} \mathbf{c}_l \propto \delta_{kl}$

R-conjugate $\Rightarrow$ $\mathbf{c}_k^H \mathbf{R} \mathbf{c}_l \propto \delta_{kl}$

$\mathbf{R}^{-1}$-conjugate $\Rightarrow$ $\mathbf{c}_k^H \mathbf{R}^{-1} \mathbf{c}_l \propto \delta_{kl}$

By selecting code c_k as an eigenvector of interference plus noise correlation matrix **Q**, the MMSE weight for extracting symbol carried by k^{th} code becomes equal to the code itself. Because the codes are **I**-conjugate, this guarantees no "cross-talk" amongst codes when applying the MMSE weight vector to extract the symbol carried by k^{th} code.

In contrast, the MMSE weight for extracting the k^{th} Walsh-Hadamard code in "colored noise" (interference), where $\mathbf{Q} \neq \mathbf{I}$, lacks orthogonality and therefore causes "cross-talk" between the codes.

3. SINR PERFORMANCE OF EIGENVECTOR CODES

Since the use of eigenvector codes eliminates "cross-talk" between the codes, calculating the theoretical SINR for the k^{th} received code is relatively simple. Recalling (10), (11), and (18):

$$SINR_k = \frac{\left|\mathbf{w}_k^H \mathbf{c}_k\right|^2}{\mathbf{w}_k^H \mathbf{Q} \mathbf{w}_k} = \frac{\left|\mathbf{c}_k^H \mathbf{c}_k\right|^2}{\mathbf{c}_k^H \mathbf{Q} \mathbf{c}_k} = \frac{1}{\lambda_k} \qquad (21)$$

This result reveals that the SINR of each received code is inversely proportional to that code's corresponding eigenvalue of **Q**. The preceding analysis presumes that the codes are of unit length; if we scale the codes by a factor of $\sqrt{N}$, so that they match the power of standard Walsh-Hadamard codes, then the received SINR will have a corresponding gain of $\sqrt{N}$.

Equation (21) suggests that the best codes to use for multi-code transmission are the codes corresponding to the smallest eigenvalues of **Q**. Higher-order constellations could also be used for the codes corresponding to the smallest eigenvalues, while lower order constellations are used for transmission of the codes corresponding to the larger eigenvalues. Such a scheme takes advantage of their higher SINR to further increase the data transmission rate.

4. SIMULATION RESULTS

For the simulation, the case of length 16 codes was examined, with 9 parallel symbol streams being used by the transmitter. 16-QAM symbols were used to transmit the data. Two types of codes were used: standard Walsh-Hadamard codes, and codes consisting of the eigenvectors corresponding to the 9 smallest eigenvalues of the interference plus noise matrix **Q**. In the case of the Walsh-Hadamard codes, an MMSE receiver was constructed by formulating $\mathbf{R}^{-1}\mathbf{c}_k$.

The colored noise or interference environment consisted of 16 random frequency sinusoidal interferers, each with power equal to one tenth the signal power. There was also a relatively small white noise component (power = .01 times signal power).

Figure 1 shows received constellations with Walsh Hadamard codes and an MMSE receiver. Most of the codes have errors so large that the bit error rate approaches 50%. This large spread is due to cross-talk as well as interference. Note that a couple codes (codes 2 and 9 in this case) achieve fairly accurate reconstruction. Previous research in this area has centered on finding binary spreading codes such as these, that minimize multi-user access interference on the output of the MMSE receiver.

The final plot shows the performance of the eigenvector codes. The first seven of nine codes, which correspond to the smallest eigenvalues of **Q**, have error rates dramatically lower than the MMSE Walsh-Hadamard receiver, suggesting the use of a higher-order constellation. The performance of the eigenvector codes associated with the larger eigenvalues of **Q** approaches that of the MMSE receiver with Walsh-Hadamard Codes.

5. REFERENCES

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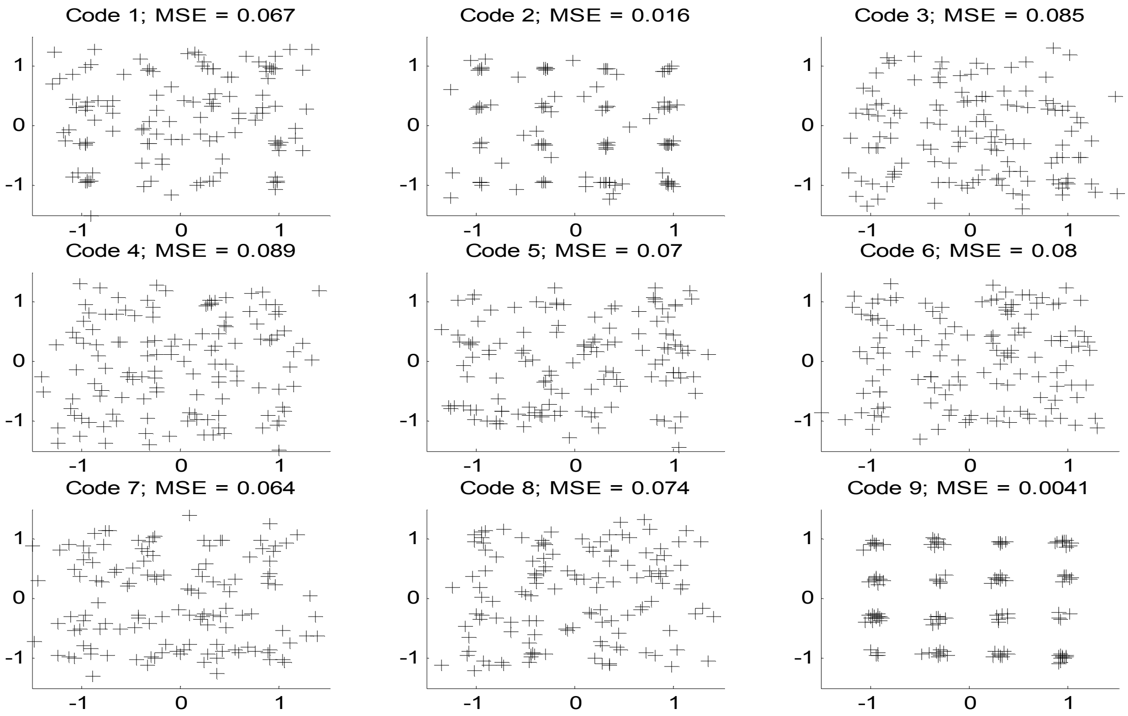

Figure 1: Mean-Square Error (MSE) of Walsh-Hadamard codes with MMSE receiver

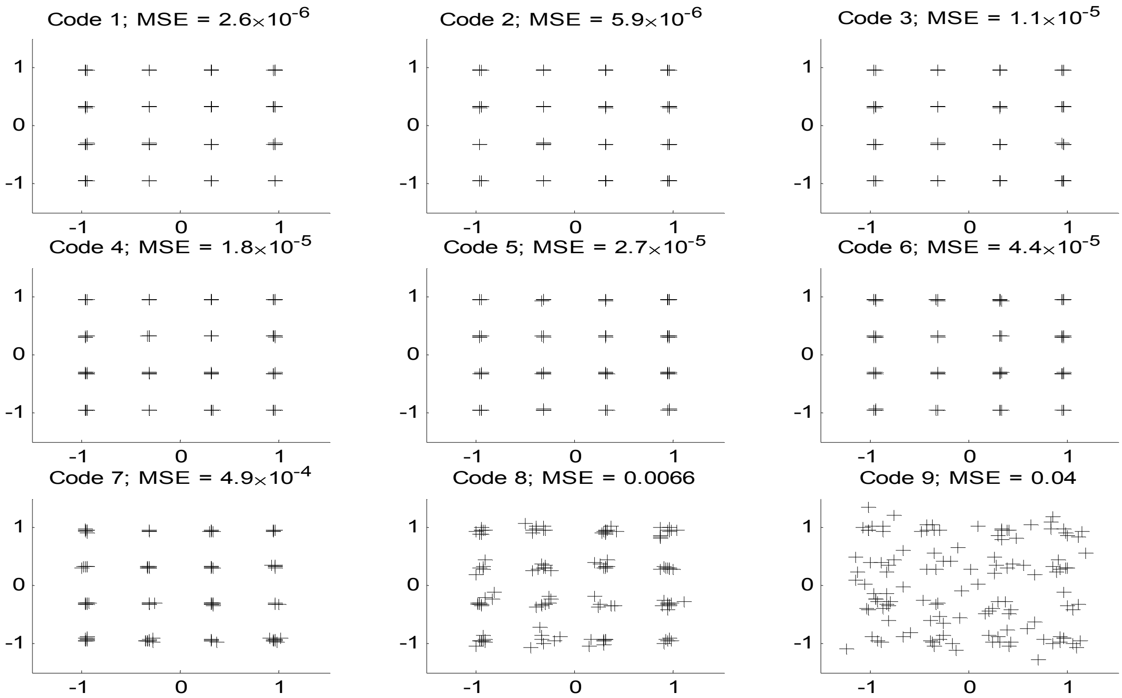

Figure 2: Mean-Square Error (MSE) of Eigenvector codes