Correlation Properties of Time-Hopping Sequences for Impulse Radio

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***Abstract*—**Time-hopping (TH) techniques have received increasing attention, as ultra wide band (UWB) impulse radio (IR) is proposed. The correlation properties of TH sequences have an important effect on the performance of IR communication system. This paper analyses the correlation properties of TH sequences. The definition of the correlation function of TH sequences is given. The upper and lower limits of correlation value of time-hopping sequences are obtained. At last, the way to improve the correlation property of TH sequences is obtained. By this means, the maximum correlation value of TH sequences will be deceased to the half of original value.

I. INTRODUCTION

TH sequences are used to implement spread spectrum multiple access (SSMA) in UWB IR communication systems, which employ the pulse techniques to generate ultra wide band communication signals that consist of trains of time shifted subnanosecond impulses[1,2,3]. Data is transmitted by using pulse-position-modulation (PPM)[1] or pulse-amplitude-modulation (PAM)[4] at a rate of many pulses per data symbol. Both modulation formats require the good correlation properties of TH sequences, namely good self and cross correlation properties. One of the important modulation scheme in IR is TH-PPM. The transmitted TH-PPM signal for user *i* can be expressed as

$$S^{(i)}(t)=\sum_{k=-\infty}^{+\infty} w(t-kT_f-y_i(k)T_c-\delta d^{(i)}_{[k/N_S]}),\qquad (1)$$

Here $w(\cdot)$ represents the transmitted monocycle waveform, and $\{y_i(k)\}$ is a TH sequence assigned to user *i*. It is periodic with period of *L* and each sequence element is an integer in the range $0\le y_i(k)\le N_h$. T_f is frame time, and T_c is TH slot time, $T_f=NT_c$, usually $N=N_h+1$. The data sequences $\{d_j^{(i)}\}$ of user *i* is a binary stream. One symbol may be conveyed by *Ns* monocycles. The notation [x] denotes the integer part of x and δ is the data shift time. According to the equation (1), we can see that the properties of TH sequences are a critical point in IR. They constitute the source of diversity that protects the desired signal from multipath interferences and MAI (multiple access interferences). They also represent a reliable source for synchronization and channel estimation. Therefore, the TH sequences should have good correlation properties.

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II. CORRELATION PROPERTIES OF TH SEQUENCES

The meaning of correlation properties of TH sequences is similar to that of frequency hopping (FH) sequences, and both describe the number of coincidences. However, they are different in operation. The correlation properties of FH sequences describe the number of coincidences in terms of frequency shift, while those of TH sequences are in terms of time shift. The number of circular shift comparison of FH sequences is *L*, while the number of circular shift comparison of TH sequences is (*N*-1) times more than L, namely *NL*. Where *N* represents the number of TH slot time. As a result, the requirement of correlation properties of TH sequences is much higher than one of FH sequences. According to the above, the correlation function of TH sequences can be defined as follows.

***Definition* 1:** Let $y_i(k)$ and $y_j(k)$ denote two TH sequences with period *L*, then the TH correlation function with time delay τ can be expressed as

$$C_{ij}(\tau)=|A\cap B|,\qquad (2)$$

where the notation "|x|" denotes the number of the elements in set x. The set $A=\{x|x=KN+y_i(K+a),\ 0\le k\le L-1,\ 0\le a\le L-1\}$ and $B=\{x|x=KN+y_j(K)+b,\ 0\le k\le L-1,\ 0\le b\le N-1\}$, here $K+a$ operates modulo *L*, while $KN+y_i(K+a)$ and $KN+y_j(K)+b$ operate modulo the period *NL*. $A\cap B$ indicates the intersection of set *A* and set *B*. *L* indicates the period of TH sequences and *N* represents the number of time slot. The symbol τ denotes time delay, $\tau=aN+b,\ 0\le\tau\le NL-1$.

According to the equation (2), we can see that the TH sequence correlation function $C_{ij}(\tau)$ refers to the number of coincidences of the sequence $y_i(k)$ and $y_j(k)$ in a period of *NL* when time delay $\tau=aN+b$. The smaller the value of $C_{ij}(\tau)$ gets, the less the number of coincidences of two TH sequences gets. Then, MAI will be reduced.

As to the equation (2), the definition of correlation function of TH sequences is straightforward. However it is not easy to calculate, we need to transform the formulation. We note that in the set B in the equation (2) , $0 \leq y_j(K)+b \leq 2N-1$. When $y_j(K)+b \geq N$, $(KN+y_j(K)+b)_{NL}$ = $((K+1)N+(y_j(K)+b)_N)_{NL}$, here the notation $(\cdot)_p$ indicates an operation modulo p. Then, it is possible that the value of $(KN+y_j(K)+b)_{NL}$ is equal to the value of $((K+1)N+y_i(K+a))_{NL}$ in the set A . As a result, an alternative definition of correlation function is required.

***Definition* 2:** Let $y_i(k)$ and $y_j(k)$ denote two TH sequences with period L, then the correlation function with time delay τ can be given by

$$C_{ij}(\tau) = \sum_{K=0}^{L-1} h[KN + y_i(K+a), KN + y_j(K) + b] + \sum_{K=0}^{L-1} h[(K+1)N + y_i(K+1+a), KN + y_j(K) + b] , \quad (3)$$

where, $h[n_1, n_2] = \begin{cases} 1, & n_1 = n_2, (\bmod NL) \\ 0, & n_1 \neq n_2, (\bmod NL) \end{cases}$, the other notations are the same as that in definition 1.

In the definition 2, it is obvious that the value of $C_{ij}(\tau)$ contains two parts. The first part,

$$\sum_{K=0}^{L-1} h[KN + y_i(K+a), KN + y_j(K) + b]$$

gives the number of $(KN+y_i(K+a))_{NL}=(KN+y_j(K)+b)_{NL}$ *when* $y_j(K)+b<N$. On the condition that $y_j(K)+b<N$ and two TH sequences have a corresponding comparison in terms of K, the value of $((K+1)N+y_i(K+1+a))_{NL}$ and the value of $(KN+y_j(K)+ b)_{NL}$ in the second part of $C_{ij}(\tau)$ are not equal. The second part of $C_{ij}(\tau)$,

$$\sum_{K=0}^{L-1} h[(K+1)N + y_i(K+1+a), KN + y_j(K) + b] ,$$

gives the number of $((K+1)N+y_i(K+1+a))_{NL}$ = $(KN+y_j(K)+b)_{NL}$ when $y_j(K)+b \geq N$. Like the former, on the condition that $y_j(K)+b \geq N$ and two TH sequences have a corresponding comparison in terms of K, the value of $(KN+y_i(K+a))_{NL}$ and the value of $(KN+y_j(K)+b)_{NL}$ in the first part of $C_{ij}(\tau)$ are not equal.

According to the above analysis, we can conclude that the TH sequences correlation function defined in definition 2 is divided into two parts which do not include each other, and the sum of them is equal to the quantities of circular shift coincidences in the period NL.

III. THE LIMIT OF CORRELATION VALUE OF TIME-HOPPING SEQUENCES

In practice, for the correlation properties of TH sequence we are interested in the maximum values of correlation function, i.e., the maximum self correlation value, saying S_{max}, and the maximum cross correlation value, saying C_{max}. In this section, we will give the lower and upper limits of the correlation values of TH sequences so that we study TH sequences. Firstly, the lower limit is given. Before we evaluate the lower limit, we give the definition of correlation function of FH sequences, namely Hamming correlation.

***Definition* 3[5]:** Let $y_i(k)$ and $y_j(k)$ denote two frequency hopping sequences with period L, then the hamming correlation function with time delay τ is given by

$$H_{ij}(\tau) = \sum_{k=0}^{L-1} h[y_i(k), y_j(k+\tau)], \ 0 \leq \tau \leq L-1 , \quad (4)$$

where $h[y_i(k), y_j(k+\tau)] = \begin{cases} 1, & y_i(k) = y_j(k+\tau) \\ 0, & y_i(k) \neq y_j(k+\tau) \end{cases}$, and $k+\tau$ operates modulo L.

Let F_S_{max} and F_C_{max} represent the maximum self correlation and cross correlation of FH sequences, respectively. In the following, we will commence the analysis of the lower limit.

***Theorem* 1:** For given sequences, the maximum correlation values S_{max} and C_{max} of the TH sequences are larger than those of the FH sequences, i.e., $S_{max} \geq F\_S_{max}$, $C_{max} \geq F\_C_{max}$.

Proof: As to the equation (3), specially, when $b=0$, it can be simplified as follows.

$$C_{ij}(\tau) = \sum_{K=0}^{L-1} h[y_i(K+a), y_j(K)] + \sum_{K=0}^{L-1} h[N + y_i(K+1+a), y_j(K)].$$

Because $0 \leq y_i(K+1+a) \leq N-1$, we have $N \leq N+y_i(K+1+a) \leq 2N-1$. Meanwhile $0 \leq y_j(K) \leq N-1$, thus

$$\sum_{K=0}^{L-1} h[N + y_i(K+1+a), y_j(K)] = 0,$$

Therefore, $C_{ij}(\tau) = \sum_{K=0}^{L-1} h[y_i(K+a), y_j(K)]$.

The above equation is the same as the equation (4). For b=0, $C_{ij}(\tau)$ describes the hamming correlation function $H_{ij}(\tau)$ of two TH sequences $y_i(k)$ and $y_j(k)$ when the time delay $\tau=a$. Then, $S_{max} \geq F\_S_{max}$, $C_{max} \geq F\_C_{max}$.

Q.E.D.

From the above operation, it is can be seen that if the correlation property of sequences regarded as FH sequences is not good, the correlation property of the sequences regarded as TH sequences will be also not good surely. This implies that we may construct TH sequences using the knowledge of FH sequences. In the following, we will consider the upper limit.

***Theorem* 2:** For TH sequences with period L, if $N=L$, then the upper limit is given by

$$S_{\max}, C_{\max} \leq 2MAX\left(\sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L]\right),$$ where

$MAX\left(\sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L]\right)$ denotes the maximum value of $\sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L]$.

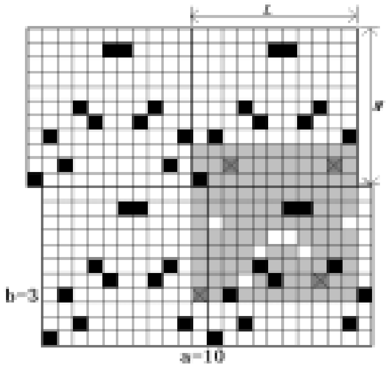

Fig.1 the place of the maximum correlation value of user 3 and user 5 (here C_{max}=4) when $N=N_h+1=L=11$ and $\tau = aN+b = 10 \times 11+3 = 113$. In this figure, coincidences are denoted by a cross ×.

Proof: According to the equation (3), we have

$$C_{ij}(\tau)=\sum_{K=0}^{L-1} h[y_i(K+a), y_j(K)] + \sum_{K=0}^{L-1} h[N+y_i(K+1+a), y_j(K)] \quad ,$$

the first part of $C_{ij}(\tau)$ is $\sum_{K=0}^{L-1} h[y_i(K+a), y_j(K)]$. Note that it operates modulo NL. When it operates modulo L, the possibility of coincidences of $(y_i(k+a))_L$ and $(y_j(k)+b)_L$ is larger than that of $(y_i(k+a))_{NL}$ and $(y_j(k)+b)_{NL}$. Then, $\sum_{K=0}^{L-1} h[y_i(K+a), y_j(K)+b] \le \sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L]$.

Similarly, the second part of $C_{ij}(\tau)$

$$\sum_{K=0}^{L-1} h[N+y_i(K+1+a), y_j(K)+b] \le \sum_{K=0}^{L-1} h[(N+y_i(K+1+a))_L, (y_j(K)+b)_L]$$

$= \sum_{K=0}^{L-1} h[(y_i(K+1+a))_L, (y_j(K)+b)_L]$ (Here since $N=L$, we have $(N+y_i(k+1+a))_L = (y_i(k+1+a))_L$). It is also clear that

$$MAX(\sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L])$$

$= MAX(\sum_{K=0}^{L-1} h[(y_i(K+1+a))_L, (y_j(K)+b)_L])$. Therefore,

$$S_{\max}, C_{\max} = MAX(\sum_{K=0}^{L-1} h[y_i(K+a), y_j(K)+b]$$

$$+ \sum_{K=0}^{L-1} h[N+y_i(K+1+a), y_j(K)+b])$$

$$\le 2MAX(\sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L]) \quad .$$

Q.E.D.

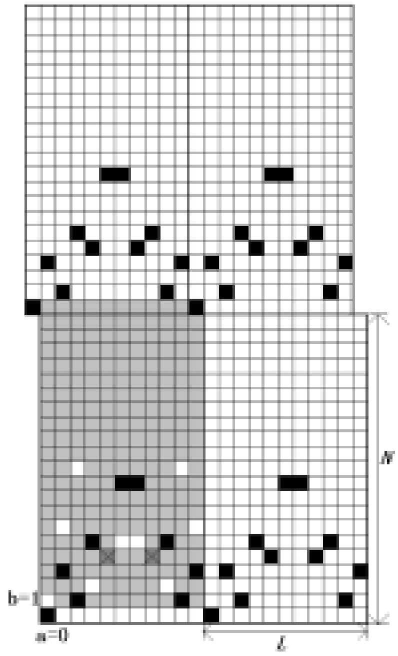

Fig.2 the place of the maximum correlation value of user 3 and user 5 (here C_{max}=2) when $N=2N_h+1=2L-1=21$ and $\tau = aN+b = 0 \times 21+1 = 1$. In this figure, coincidences are denoted by a cross ×.

IV. IMPROVEMENT OF CORRELATION PROPERTIES OF TH SEQUENCES

In this section, we will give a new theorem that improves the correlation property of TH sequences. The results are shown obviously in Fig.1 and Fig.2. The theorem is expressed as follows.

***Theorem* 3:** For the correlation function given by definition 3, let $N \ge 2N_h+1$ (here N_h denotes the maximum value in sequences set $\{y_i(k)\}$, i.e., $0 \le y_i(k) \le N_h$.), then we have

$$C_{ij}(\tau)=\begin{cases} \sum_{K=0}^{L-1} h[y_i(K+a), \quad y_j(K)+b], 0 \le b < N_h+1 \\ \sum_{K=0}^{L-1} h[N+y_i(K+1+a), \quad y_j(K)+b], N_h+1 \le b < N \end{cases} ,$$

and $S_{\max}, C_{\max} \le MAX(\sum_{K=0}^{L-1} h[(y_i(K+a))_L, (y_j(K)+b)_L])$.

where $h[n_1, n_2] = \begin{cases} 1, & n_1 = n_2, (\text{mod } NL) \\ 0, & n_1 \ne n_2, (\text{mod } NL) \end{cases}$, $0 \le a \le L-1$, $0 \le b \le N-1$, $K=0,1,2,\cdots,L-1$, and $\tau = aN+b$, $0 \le \tau \le NL-1$.

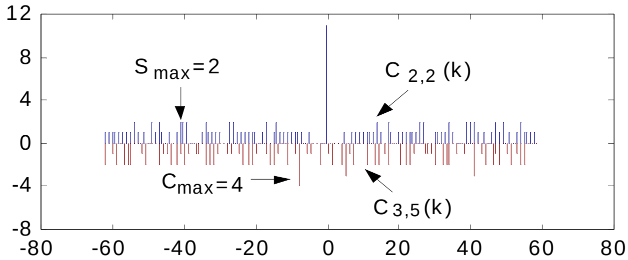

Fig. 3 the distributing situation of self-correlation value of user 2 and cross-correlation value of user 3 and user 5 when the number of time slot $N=N_h+1=L=11$

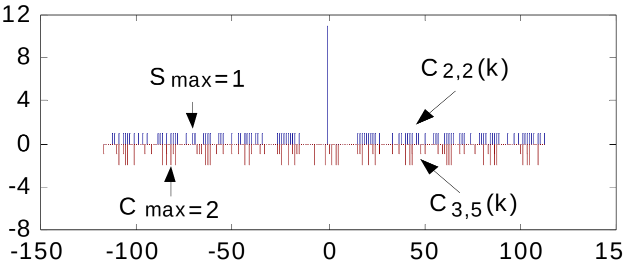

Fig. 4 the distributing situation of self-correlation value of user 2 and cross-correlation value of user 3 and user 5 when the number of time slot $N=2N_h+1=2L-1=21$

Proof: Consider

$$C_{ij}(\tau)=\sum_{K=0}^{L-1}h[y_i(K+a),y_j(K)]+\sum_{K=0}^{L-1}h[N+y_i(K+1+a),y_j(K)],$$

On the one hand, for $0\leq b<N_h+1$, $0\leq y_j(K)+b<2N_h+1$. But, $N\geq 2N_h+1$, thus $N+y_i(K+1+a)\geq 2N_h+1$, thus,

$$\sum_{K=0}^{L-1}h[N+y_i(K+1+a),y_j(K)+b]=0,\text{ then}$$

$$C_{ij}(\tau)=\sum_{K=0}^{L-1}h[y_i(K+a),y_j(K)+b],\quad \text{if } 0\leq b<N_h+1.$$

On the other hand, for $N_h+1\leq b<N$, $0\leq y_i(K+a)<N_h+1$. Thus, $y_j(K)+b\geq N_h+1$.

Therefore, $\sum_{K=0}^{L-1}h[y_i(K+a),y_j(K)+b]=0$, then

$$C_{ij}(\tau)=\sum_{K=0}^{L-1}h[N+y_i(K+1+a),y_j(K)+b]\quad \text{if } N_h+1\leq b<N.$$

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According to the theorem 2, we get

$$S_{\max},C_{\max}\leq MAX(\sum_{K=0}^{L-1}h[(y_i(K+a))_L,(y_j(K)+b)_L])\quad .$$

Q.E.D.

According to theorem 3, when the number of TH time slot $N\geq 2N_h+1$, the correlation properties of TH sequences will be improved. In fact, the possibility of coincidences decreases when the number of time slot N increases. Thus, we can improve the correlation properties by increasing the number of time slot.

To show how theorem 3 works we now give an example. In this example we use the existing quadratic congruence sequences[6]. For quadratic congruence sequences, $y_i(k)=(k^2i)_p$, where p is a prime, $0\leq k\leq p-1$, $i=1,2,\cdots,p-1$, and the notation $(\cdot)p$ denotes a modulo p operation. Let $p=11$, then $L=11$, and the number of users $U_N=10$. From Fig.1 and Fig.2, we can see that the maximum correlation value of quadratic congruence sequences decrease from $S_{max}\leq 2$ and $C_{max}\leq 4$ to $S_{max}\leq 1$ and $C_{max}\leq 2$. In order to show the distributing situation of correlation value in a period NL, we also give Fig.3 and Fig.4.

V. CONCLUSIONS

In this paper, we described the concept of correlation function of TH sequences and showed the lower and upper limits of correlation value of TH sequences. Furthermore, we gave a method to improve correlation properties of TH sequences. In this way, the maximum correlation value of the TH sequences can be decreased to the half of original value.

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