PERFORMANCE OF A LOW-COMPLEXITY DOWNLINK CHANNEL ESTIMATOR FOR LONG-CODE CDMA SYSTEMS

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ABSTRACT

Recently, a blind downlink channel estimator is proposed for long-code CDMA systems based on a covariance matching idea. By modeling long spreading codes as random codes and pre-computing some code-dependent quantities, the algorithm shows extremely low complexity. Due to inaccuracy of estimated data covariance from finite number of received data samples, its asymptotic performance depends on the sample size. In this paper, it is further analyzed in terms of the covariance and mean-square-error of estimated channel vector.

1. INTRODUCTION

Wireless communication has evolved into a new era with rapidly growing number of subscribers each year. Direct sequence (DS) code division multiple access (CDMA) technology has become an appealing solution to support emerging multirate multiuser communications due to its unique capabilities of simultaneous spectrum sharing, mitigation of jamming, interception and multipath fading [12].

A DS/CDMA system is interference limited mainly due to multiuser interference (MUI). Significant efforts have focused on developing multiuser detection techniques to suppress MUI [19]. Most contributions assume short code spreading, resulting in simplified methods, adaptability to a changing environment, and very importantly, tractable analysis of the system performance. However, the spreading sequence to be adopted for the new generation wireless systems is aperiodic with period much longer than the bit duration [12]. Employment of long codes bestows a CDMA system several attractive benefits such as immunity to MUI and channel fading on the average, a secure communication link in a hostile environment. However, long codes inevitably destroy cyclostationarity of CDMA signals and cause the system time-varying, making many of the existing channel estimation and detection approaches for short code CDMA systems not directly applicable. Therefore, design of advanced detection techniques for long code CDMA systems poses new challenges but remains largely under-explored.

Due to different code assignment strategies, channel characterization and synchronization, downlink and uplink are usually separately discussed in the literature. Klein *et al.* applied a channel equalization idea to CDMA uplink [8] and then modified it for CDMA downlink to restore orthogonality of the user spreading sequence [7]. Zoltowski and his team members have been investigating various ways to detect signals in a long code CDMA system by employing a space-time 2D RAKE receiver structure [2, 13, 14]. An MMSE equalizer can be obtained by the principal component method [11]. Their recent paper [9] provides an excellent overview of various downlink linear equalization techniques. Frank *et al.* proposed both symbol-level and chip-level adaptive MMSE interference suppression schemes [3]. Downlink channel equalization was also contributed by Lilleberg *et al.* [6]. By either or not employing spatial/temporal array processing, Slock *et al.* developed ZF and MMSE equalizers [4] and blind maximum SINR receivers [5]. Based on a subspace concept, blind or pilot symbol assisted channel estimation methods were proposed by Weiss & Friedlander [20]. Tong *et al.* investigated semi-blind channel estimation solutions via subspace based data projection for downlink [17] and a deterministic least-squares fitting channel estimation and sequence detection approach [18]. A novel parallel factor analysis technique was applied to multiuser detection by Sidiropoulos *et al.* [15, 16]. Poor *et al.* presented least-squares based approaches by explicitly using training symbols [1]. We have also proposed blind uplink channel estimation methods using a correlation matching technique [25]. Both centralized and decentralized estimation procedures are given. Algorithms for downlink channel estimation have been reported [24]. Low complexity methods for channel estimation are derived [22]. Signal detection techniques with unknown interference are proposed in [21].

In this paper, we further analyze the channel estimator for downlink CDMA proposed in [22]. The estimated

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channel vector is perturbed because the data covariance estimated from finite number of received data samples is perturbed. After deriving the second-order statistics of the sample covariance, the covariance and mean-square-error (MSE) of the channel estimate are derived in closed forms, and verified by computer simulations.

2. DATA MODEL AND PRIOR RESULTS

Consider a downlink CDMA system employing long spreading codes. Base station communicates with J mobile stations (or users) through a common multipath channel $g(n)$. Bit stream $w_j(n)$ for user j is spread by long codes $c_{j,n}(k)$ ($k = 0, \cdots, P-1$). For convenience, those P codes for the nth bit are collected in a code vector $\boldsymbol{c}_{j,n}$. The communication channel is assumed to be slowly fading with length q and static within a data frame. After collecting $\nu = P-q+1$ chip rate samples around the n-th bit interval in a vector $\boldsymbol{y}_n = [y(nP), \cdots, y(nP+\nu-1)]^T$, the received vector can be shown to be [22]

$$\boldsymbol{y}_n = \sum_{j=1}^{J} \boldsymbol{C}_{j,n} \boldsymbol{g} w_j(n) + \boldsymbol{v}_n \tag{1}$$

where $w_j(n)$ has zero-mean and same variance σ_w^2 in downlink, $\boldsymbol{C}_{j,n}$ is a Toeplitz matrix constructed from $\boldsymbol{c}_{j,n}$ with properly added zeros, $\boldsymbol{g}$ is the unknown channel vector of length q. Noise $\boldsymbol{v}_n$ is modeled as a zero-mean process with covariance matrix $\sigma_v^2 \boldsymbol{I}_\nu$ where $\boldsymbol{I}_\nu$ is an identity matrix of degree ν. For the k-th mobile user, its matched filter correlates $\boldsymbol{y}(n)$ with code matrices $\boldsymbol{C}_{k,n}^H$, yielding output

$$\boldsymbol{y}_{k,n} \triangleq \boldsymbol{C}_{k,n}^H \boldsymbol{y}_n = \boldsymbol{C}_{k,n}^H \sum_{j=1}^{J} \boldsymbol{C}_{j,n} \boldsymbol{g} w_j(n) + \boldsymbol{C}_{k,n}^H \boldsymbol{v}_n. \tag{2}$$

The autocorrelation matrix of $\boldsymbol{y}_{k,n}$ and output power of $\boldsymbol{y}_n$ become

$$\boldsymbol{R}_k = \sum_{j=1}^{J} E\{\boldsymbol{C}_{k,n}^H \boldsymbol{C}_{j,n} \boldsymbol{G} \boldsymbol{C}_{j,n}^H \boldsymbol{C}_{k,n}\} + \nu\sigma_c^2\sigma_v^2 \boldsymbol{I}_q \tag{3}$$

$$\eta = J\nu\sigma_c^2 \boldsymbol{a}^T \boldsymbol{d} + \nu\sigma_v^2, \quad \boldsymbol{a} = vec(\boldsymbol{I}_q) \tag{4}$$

where $\boldsymbol{G} = \sigma_w^2 \boldsymbol{g}\boldsymbol{g}^H$, $\boldsymbol{d} = vec(\boldsymbol{G})$, long codes have been approximated by random codes with variance σ_c^2, and an identity is used: $trace(\boldsymbol{X}_1\boldsymbol{X}_2) = vec^T(\boldsymbol{X}_1^T)vec(\boldsymbol{X}_2)$. After taking a vec operation on $\boldsymbol{R}_k$ to stack all its columns in a big vector $\boldsymbol{r}_k$ and eliminating σ_v^2 based on (3) and (4), it can be found that $\boldsymbol{S}\boldsymbol{d} = \boldsymbol{z}$ where $\boldsymbol{S}$ depends on the autocorrelation and cross-correlation of code matrices

$$\begin{aligned}\boldsymbol{S} &= \sum_{j=1}^{J} E\{\boldsymbol{Q}_{k,n}^H \boldsymbol{Q}_{j,n}\} - J\nu\sigma_c^4 \boldsymbol{a}\boldsymbol{a}^T \\ &= \boldsymbol{S}_a + (J-1)S_c - J\nu\sigma_c^4 \boldsymbol{a}\boldsymbol{a}^T, \end{aligned} \tag{5}$$

$$\boldsymbol{S}_a = E\{\boldsymbol{Q}_{k,n}^H \boldsymbol{Q}_{k,n}\}, \quad \boldsymbol{Q}_{k,n} = \boldsymbol{C}_{k,n}^* \otimes \boldsymbol{C}_{k,n},$$
$$\boldsymbol{S}_c = E\{\boldsymbol{Q}_{k,n}^H \boldsymbol{Q}_{j,n}\} \ \text{for} k \neq j,$$
$$\boldsymbol{z} = \boldsymbol{r}_k - \sigma_c^2 \eta \boldsymbol{a}, \tag{6}$$

and $\otimes$ is the Kronecker product. A channel estimation method has been developed for such a system [22]. $\boldsymbol{R}_k$ is estimated from N data vectors by $\widehat{\boldsymbol{R}}_k = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{y}_{k,n}\boldsymbol{y}_{k,n}^H$, and η by the trace of $\boldsymbol{R}$ which is estimated by $\widehat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{y}_n \boldsymbol{y}_n^H$. By matching $\boldsymbol{z}$ with its estimate and minimizing the matching error, $\boldsymbol{d}$ can be estimated as

$$\widehat{\boldsymbol{d}} = \boldsymbol{S}^{-1}\widehat{\boldsymbol{z}}, \quad \widehat{\boldsymbol{z}} = \widehat{\boldsymbol{r}}_k - \sigma_c^2 \boldsymbol{a}\boldsymbol{b}^T \widehat{\boldsymbol{r}}, \quad \boldsymbol{b} = vec(\boldsymbol{I}_\nu) \tag{7}$$

where η has been replaced by $trace(\boldsymbol{R}) = \boldsymbol{b}^T\boldsymbol{r}$. Then $\widehat{\boldsymbol{G}}$ is reconstructed from $\widehat{\boldsymbol{d}}$ by the reverse operation of vec. This operation can be well represented by a linear transformation after introducing a unitary vector $\boldsymbol{e}_i$ of length q but with 1 only in the ith position

$$\widehat{\boldsymbol{G}} = [\boldsymbol{T}_1\widehat{\boldsymbol{z}}, \cdots, \boldsymbol{T}_q\widehat{\boldsymbol{z}}], \quad \boldsymbol{T}_i = (\boldsymbol{e}_i^T \otimes \boldsymbol{I}_q)\boldsymbol{S}^{-1}. \tag{8}$$

Singular value decomposition (SVD) on $\widehat{\boldsymbol{G}}$ is performed to obtain an estimate for $\boldsymbol{g}$. Due to an estimation error in $\boldsymbol{z}$: $\delta\boldsymbol{z} = \widehat{\boldsymbol{z}} - \boldsymbol{z}$, the estimated channel vector $\widehat{\boldsymbol{g}}$ will be different from $\boldsymbol{g}$. We will analyze the covariance and MSE of these channel estimators next.

3. PERFORMANCE EVALUATION

For simplicity, assume $\|\boldsymbol{g}_1\| = 1$. We also assume N is large. Therefore, $\delta\boldsymbol{r}$ is a small perturbation. Then

$$\delta\boldsymbol{G} = [\boldsymbol{T}_1\delta\boldsymbol{z}, \cdots, \boldsymbol{T}_q\delta\boldsymbol{z}]. \tag{9}$$

From our definition of $\boldsymbol{G}$, $\boldsymbol{g}$ is an eigenvector corresponding to its unique non-zero eigenvalue σ_w^2. If $\boldsymbol{G}$ is perturbed by $\delta\boldsymbol{G}$, then the first-order perturbation in this eigenvector becomes [23]

$$\delta\boldsymbol{g} \approx \sigma_w^{-2}\boldsymbol{\Pi}\delta\boldsymbol{G}\boldsymbol{g}, \quad \boldsymbol{\Pi} = \boldsymbol{U}_n\boldsymbol{U}_n^H \tag{10}$$

where $\boldsymbol{U}_n$ spans a $(q-1)$-dimensional space orthogonal to $\boldsymbol{g}$. Using (9) and (10), we obtain the covariance $\mathrm{Cov}(\delta\boldsymbol{g})$

$$\mathrm{Cov}(\delta\boldsymbol{g}) \approx \sigma_w^{-4}\boldsymbol{\Pi}\Big(\sum_{i=1}^{q} g(i)\boldsymbol{T}_i\Big)\boldsymbol{\Phi}(\widehat{\boldsymbol{z}})\Big(\sum_{i=1}^{q} g(i)\boldsymbol{T}_i\Big)^H\boldsymbol{\Pi} \tag{11}$$

where $\boldsymbol{\Phi}(\widehat{\boldsymbol{z}})$ is the covariance of $\widehat{\boldsymbol{z}}$: $\boldsymbol{\Phi}(\widehat{\boldsymbol{z}}) = E\{\delta\boldsymbol{z}\delta\boldsymbol{z}^H\}$. If we denote the instantaneous estimate of $\boldsymbol{z}$ as $\boldsymbol{z}_n$, instantaneous estimate of $\boldsymbol{r}$ as $\boldsymbol{r}_n = vec(\boldsymbol{y}_n\boldsymbol{y}_n^H)$, then according to (6), they are related by

$$\boldsymbol{z}_n = \boldsymbol{B}_n\boldsymbol{r}_n, \quad \boldsymbol{B}_n = \boldsymbol{Q}_{k,n}^H - \sigma_c^2\boldsymbol{a}\boldsymbol{b}^T.$$

Therefore,

$$\Phi(\widehat{z}) = E\{B_n \Phi(\widehat{r}) B_n^H\}$$

where $\Phi(\widehat{r})$ is a conditional covariance of $\widehat{r}$ given spreading codes of user k.

According to the previous result, r depends on the second order information of spreading codes. Then $\Phi(\widehat{r})$ relies on their fourth order statistics. Together with the definition of B_n, it is expected that $\Phi(\widehat{z})$ requires up to the eight-th order statistics of spreading codes. Though it is desirable to derive a closed-form expression, the complexity is prohibitive. Instead, $\Phi(\widehat{r})$ can be easily obtained. It can be shown by lengthy but straightforward derivation that for a given system model

$$y_n = H b_n + v_n, \tag{12}$$

with i.i.d. input sequence and AWGN, $\Phi(\widehat{r})$ can always be simplified. It depends on whether the system is real or complex. If all quantities are real-valued, then

$$\begin{aligned}\Phi(\widehat{r}) &= \frac{\kappa_{4b}}{N}\sum_{l=1}^{L}(h_l h_l^T)\otimes(h_l h_l^T) \\ &+ \frac{\sigma_b^4}{N}\sum_{l=1}^{L}(H\otimes h_l)(h_l\otimes H)^T \\ &+ \frac{1}{N}R\otimes R + \frac{1}{N}\sum_{j=1}^{\nu}(T\otimes t_j)(t_j\otimes T)^T \\ &+ \frac{\sigma_b^2}{N}[H\otimes w_1,\cdots,H\otimes w_\nu](I\otimes H^T) \\ &+ \frac{\sigma_b^2}{N}(I\otimes H)[H\otimes w_1,\cdots,H\otimes w_\nu]^T\end{aligned} \tag{13}$$

while for a complex system it satisfies

$$\Phi(\widehat{r}) = \frac{\kappa_{4b}}{N}\sum_{l=1}^{L}(h_l^* h_l^T)\otimes(h_l^* h_l^T) + \frac{1}{N}R^*\otimes R \tag{14}$$

where L is the dimension of b_n whose entries have same variance σ_b^2, fourth-order absolute moment m_{4b} and fourth-order cumulant κ_{4b}, $h_l = H(:,l)$, $W = \sigma_v^2 I_\nu$, $T = W^{\frac{1}{2}}$, $w_j = W(:,j)$, $t_j = T(:,j)$. κ_{4b} is related to σ_b^2 and m_{4b} [10]. For a real system, $\kappa_{4b} = m_{4b} - 3\sigma_b^4$, while for a complex system, $\kappa_{4b} = m_{4b} - 2\sigma_b^4$.

In order to apply these statistical results, let us examine model (1). Define $H(:,j)$ to be the signature waveform $C_{j,n}g$ of symbol $w_j(n)$. Correspondingly all of these vectors are stacked into matrix H and inputs of all users at time n into a vector b_n. Then we set $L = J$. Once covariances of $\widehat{z}$ and $\widehat{r}$ are obtained either in a real system or complex system, the MSE of channel estimate can be found from the trace of the corresponding covariance matrix $\mathrm{Cov}(\delta g)$.

4. SIMULATION

We provide a numerical example to verify our analysis for downlink communications. Consider a synchronous CDMA system with 10 users. All users transmit binary sequences with equal power. Spreading sequences are randomly generated with binary digits ± 1 at equal probability. Spreading factor is set to be 16. Channel coefficients are randomly selected from Gaussian distribution with 4 chip-spacing equal power paths. The channel estimation MSE for different number of received data vectors is presented in Fig. 1. The solid line represents the average from 100 realizations, and the dashed line represents the analytical result. It is clear that these two lines agree with each other for a wide range of number of snapshots.

5. FUTURE WORK

In [22], both downlink and uplink channel estimation methods are derived. In the uplink, there are scenarios with either AWGN or colored noise. The base station may have no knowledge about interfering users. Meanwhile, different users experience different multipath channels. The method employs outputs of J code matched filters in order to estimate all channel parameters simultaneously. Those outputs are not independent due to filtering of the same received data and correlation of the codes with the data. It requires derivation of second-order statistics of covariance of J outputs. This topic will be further investigated and corresponding results be reported in the near future.

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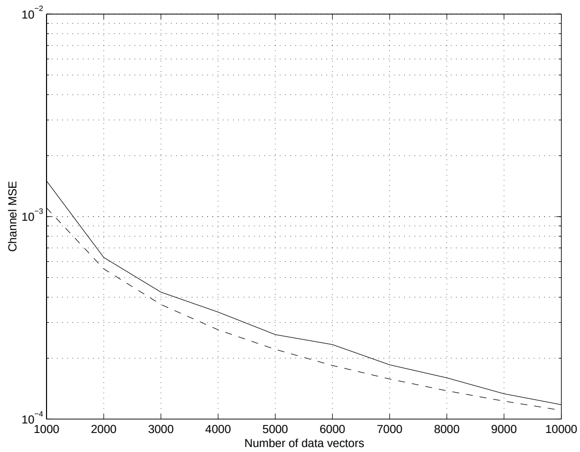

Fig. 1. Channel estimation error.

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