BLIND MMSE MULTIPATH COMBINING FOR LONG-CODE CDMA SYSTEMS

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ABSTRACT

A blind MMSE multipath combining method for RAKE receiver is proposed. The method make use of the data before and after the onset of the desired user's transmission. By performing generalized eigendecomposition of the autocorrelation matrices of the post-RAKE signals in the two interval, an MMSE combiner for the desired user signal can be obtained without the need of the channel state information. The method does not require precise model nor assume whiteness of the interference. An adaptive algorithm is also developed.

1. INTRODUCTION

Multiuser detection has been identified as a key technology to improve the capacity of code division multiple access (CDMA) systems (see [1] and references therein). Past studies has focused on short-code systems, in which the period of the spreading sequence is equal to the symbol duration, though most practical systems employs long-code schemes in which the code period is typically several orders of magnitude larger than the symbol duration. The difficulty of long-code systems lies in that the user signature is time-varying and thus lack of stationarity. For this reason, it is customary in long code systems to pre-process the received signal with matched filters matched at the spreading sequences. The processed data are afterwards combined in some ways. This is what so-called RAKE receiver. The optimal combining method requires the channel of the desired user and the correlation of the interference. Suboptimal combining methods include equal power combining which simply adds up the output of all matched filters and maximal ratio combining where the combining coefficients is proportional to the power of the corresponding RAKE finger. To implement optimal combining, training symbols are needed. Blind methods have also been proposed [2, 3, 4], mostly focused on channel estimation.

In [5], the authors introduced a novel idea to construct minimum mean-square error (MMSE) receiver by exploiting the data before and after a new user enters the system. It is noted that the autocorrelation matrices of the signals at these two intervals differ by a rank-one matrix contributed by the new user's signal. The fact yields a useful property that the generalized eigenvectors corresponding to the maximum eigenvalue coincides with the MMSE receiver corresponding to the new user signal. The proposal is based on short-code schemes. Its applicability in long-code systems is also limited by the time-varying nature of the user signature. However, it is possible to convert the time-varying signature to time-invariant one through a RAKE process. Then the proposed method of [5] can be applied to obtain an MMSE combiner for the RAKE output.

2. SIGNAL MODEL

Let us consider a CDMA system in which a new user starts its transmission at time t_0. After passing through a frequency-selective fading channel, the continuous-time baseband signal received before and after t_0 can be written as

$$r_1(t) = v(t)\ , \quad t < t_0, \tag{1}$$

$$r_2(t) = \sum_{i=0}^{\infty} b[i]h_i(t - iT_b - t_0) + v(t)\ , \quad t \geq t_0, \tag{2}$$

where $h_i(t)$, $b[i]$ are the signature waveform and the ith information symbol of the said user. $v(t)$ models the background interference including other users' signals and the noise. T_b is the symbol period. The signature $h_i(t)$ can be expressed as

$$h_i(t) = s_i(t) * \phi(t) \tag{3}$$

where * denotes convolution, $\phi(t)$ is the equivalent channel impulse response and $s_i(t)$ is the signature waveform of the ith symbol, which, for long-code CDMA users can be written as

$$s_i(t) = \sum_{j=0}^{P-1} c_i[j]\psi(t - jT_c) \tag{4}$$

where $\{c_j[0], c_j[1], ...c_j[P-1]\}$ is the code sequence of the said user at the ith symbol. P is the processing gain and $T_c = T_b/P$ is the chip period. $\psi(t)$ is the chip pulse supported on $[0, T_c]$. Note that the code is a function of symbol index i, a characterization of long-code CDMA.

Combining (3) and (4), we have

$$h_i(t) = \sum_{i=0}^{P-1} c_i[j]g(t - jT_c) \tag{5}$$

where $g(t) = \psi(t) * \phi(t)$.

The chip-rate discrete-time form of (2) is obtained by sampling it at time $t_0 + nT_c$

$$\begin{aligned} r_2[n] &= r_2(t)|_{t=t_0+nT_c} \\ &= \sum_{i=0}^{\infty} b[i]h_i[n - iP] + v[n_0 + n] \end{aligned} \tag{6}$$

where

$$h_i[n] = h_i(t)|_{t=nT_c} = \sum_{j=0}^{P-1} c_i[j]g[n - j] \tag{7}$$

is the discrete-time signature and $g[n]$ is the equivalent discrete-time channel impulse response. We assume the channel impulse response has finite support, so $g[n]$ is an FIR, i.e., $g[0], g[1] ... g[Q-1]$. Thus we have $P+Q-1$ samples for one whole symbol. We further assume that $Q \ll P$, therefore we can discard the first and last $Q-1$ samples interfered by neighboring symbols of the same user and only collect $L = P - Q + 1$ samples that are free of *intersymbol interference*(ISI).

We now group the discrete samples into vector forms by defining $\mathbf{r}_2[i] = [r[iP+Q-1], r[iP+Q], ..., r[iP+P]]^H$, $\mathbf{v}[i] = [v[iP], ..v[iP+P-Q]]^H$ with $i \geq 0$ and $\mathbf{g} = [g[0], ..., g[Q-1]]^H$ (the superscript H represents conjugate transpose), we end up with a concise vector form

$$\mathbf{r}_2[i] = \mathbf{C}[i]\mathbf{g}b[i] + \mathbf{v}[i] \tag{8}$$

where

$$\mathbf{C}[i] = \begin{bmatrix} c_i[Q] & & c_i[0] \\ \vdots & \ddots & \vdots \\ c_i[P] & & c_i[P-Q] \end{bmatrix}_{L \times Q} \tag{9}$$

The discrete-time form of (1) is straight-forward: using similar definition of $\mathbf{r}_1[i]$ as $\mathbf{r}_2[i]$, we have

$$\mathbf{r}_1[i] = \mathbf{v}[i] \tag{10}$$

Note that we have not used specific models for the interferences either before or after time t_0. This is because we assume that the receiver has no knowledge of the interfering signals, so that $v(t)$ is just modeled as a stationary process[6]. Actually, as we will see later, our algorithm does not rely on how the interference is modeled. For example, the other users are not necessarily to be synchronized to the new user, and the noise is not required to be white. On the other hand, the desired user's information bits are uncorrelated to the interference.

3. BLIND MMSE RAKE COMBINING

If short-code system is considered, the autocorrelation matrices of (8) and (10) are different by only a rank-one matrix. The generalized eigendecomposition of them gives an MMSE solution to the new user. The procedure is well established in [5]. In long-code system, however, the autocorrelation matrices are functions of symbol index i. In order to obviate the time-varying part introduced by the codes, we project the received signals onto a lower-dimensional subspace and deal with the outputs instead of the original data (8) and (10). In our case, we use a branch of matched filter outputs matched to the code of the said user directly. This is equivalent to premultiply the received signals by $\mathbf{C}^H[i]$, i.e.

$$\begin{aligned} \mathbf{y}_1[i] &= \mathbf{C}^H[i]\mathbf{r}_1[i] \\ &= \mathbf{C}^H[i]\mathbf{v}[i] && (11) \\ \mathbf{y}_2[i] &= \mathbf{C}^H[i]\mathbf{r}_2[i] \\ &= \mathbf{C}^H[i]\mathbf{C}[i]\mathbf{g}b[i] + \mathbf{C}[i]^H\mathbf{v}[i] && (12) \end{aligned}$$

Note that the data before t_0 are still premultiplied by the code matrices as if the new user were transmitting information at this time. (12) is exactly what the conventional RAKE receiver does. After that, the information bit is estimated by combining the outputs. The optimal combiner can be shown to be[7]

$$\tilde{b}_{rake}[i] = \mathbf{g}^H\mathbf{y}_2[i] \tag{13}$$

Another approach is to obtain an MMSE solution at the same level as RAKE receiver, which is:

$$\begin{aligned} \tilde{b}[i] &= ({\mathbf{R}_{\mathbf{y2}}}^{-1}[i]\mathbf{C}^H[i]\mathbf{C}[i]\mathbf{g})^H\mathbf{y}_2[i] \\ &\overset{P\to\infty}{=} ({\mathbf{R}_{\mathbf{y2}}}^{-1}[i]\mathbf{g})^H\mathbf{y}_2[i] \\ &= ({\mathbf{R}_{\mathbf{y1}}}^{-1}[i]\mathbf{g})^H\mathbf{y}_2[i] && (14) \end{aligned}$$

where the last step uses matrix inversion lemma[8] and the covariance matrices appeared above are

$$\begin{aligned} \mathbf{R}_{\mathbf{y1}}[i] &= \mathbf{E}\{\mathbf{y}_1[i]\mathbf{y}_1^H[i]\} \\ &= \mathbf{C}^H[i]\mathbf{E}\{\mathbf{v}[i]\mathbf{v}^H[i]\}\mathbf{C}[i] \\ &= \mathbf{C}^H[i]\mathbf{R}_\mathbf{v}\mathbf{C}[i] && (15) \\ \mathbf{R}_{\mathbf{y2}}[i] &= \mathbf{E}\{\mathbf{y}_2[i]\mathbf{y}_2^H[i]\} \\ &= \mathbf{C}^H[i]\mathbf{C}[i]\mathbf{g}\mathbf{g}^H\mathbf{C}^H[i]\mathbf{C}[i] \\ &\quad +\mathbf{C}^H[i]\mathbf{R}_\mathbf{v}\mathbf{C}[i] && (16) \end{aligned}$$

the expectation above is conditioned on the code matrix $\mathbf{C}[i]$ and with respect to the information symbols and the interference.

However, these auto correlation matrices are not well-defined in practise. Now we do the time-averaging of the instant samples $\mathbf{y}_1[i]\mathbf{y}_1^H[i]$ and $\mathbf{y}_2[i]\mathbf{y}_2^H[i]$ respectively.

$$\begin{aligned} \bar{\mathbf{R}}_1 &= \frac{1}{N_1}\sum_{i=0}^{N_1-1}\mathbf{y}_1[i]\mathbf{y}_1^H[i] \\ &= \frac{1}{N_1}\sum_{i=0}^{N_1-1}\mathbf{C}^H[i]\mathbf{v}[i]\mathbf{v}[i]\mathbf{C}^H[i] && (17) \\ \bar{\mathbf{R}}_2 &= \frac{1}{N_2}\sum_{i=0}^{N_2-1}\mathbf{y}_2[i]\mathbf{y}_2^H[i] \\ &= \frac{1}{N_2}\sum_{i=0}^{N_2-1}\{\mathbf{C}^H[i]\mathbf{C}[i]\mathbf{g}\mathbf{g}^H\mathbf{C}^H[i]\mathbf{C}[i] \\ &\quad +\mathbf{C}^H[i]\mathbf{C}[i]\mathbf{g}\mathbf{v}^H[i]\mathbf{C}[i]b[i] \\ &\quad +\mathbf{C}^H[i]\mathbf{v}[i]\mathbf{g}^H\mathbf{C}^H[i]\mathbf{C}[i]b[i] \\ &\quad +\mathbf{C}^H[i]\mathbf{v}[i]\mathbf{v}^H[i]\mathbf{C}[i]\} && (18) \end{aligned}$$

where N_1 and N_2 are the number of samples employed and the information bits are normalized without loss of generality.

With little performance loss, the time-varying covariance matrix $\mathbf{R}_{\mathbf{y1}}[i]$ in (14) can be replaced by the time-averaged $\bar{\mathbf{R}}_1$

$$\tilde{b}[i] = (\bar{\mathbf{R}}_1^{-1}\mathbf{g})^H\mathbf{y}_2[i] \tag{19}$$

When N_1 and N_2 are large enough, the second and third terms in (18) are negligible because the information bits and the interference are assumed to be uncorrelated zero-mean random variables. Since $\mathbf{C}^H[i]\mathbf{C}[i] = \frac{1}{P-Q+1}\mathbf{I} + \mathbf{R}[n]$ where $\mathbf{I}$ is identity matrix(the code sequence are normalized during one symbol) and diagonal elements of **R**[i] are zero while the rest are zero-mean random variables.

$$\begin{aligned} \mathbf{R}_1 &= \lim_{N_1\to\infty}\bar{\mathbf{R}}_1 && (20) \\ \mathbf{R}_2 &= \lim_{N_2\to\infty}\bar{\mathbf{R}}_2 \end{aligned}$$

$$= \lim_{N_2\to\infty}\left(\frac{1}{P-Q+1}\mathbf{I}+\mathbf{R}[i]\right)\mathbf{g}\mathbf{g}^H \cdot \left(\frac{1}{P-Q+1}\mathbf{I}+\mathbf{R}[i]\right)+\mathbf{R}_1$$

$$= \alpha\mathbf{g}\mathbf{g}^H+\mathbf{R}_1 \quad (21)$$

where $\alpha = \frac{1}{(P-Q+1)^2}$ is a scalar.

So far we have shown that $\mathbf{R}_2$ is different from $\mathbf{R}_1$ by only a rank-one matrix $\alpha\mathbf{g}\mathbf{g}^H$, From Theorem 4 of [5], we know that among the generalized eigenvectors of $\mathbf{R}_2$ with respect to $\mathbf{R}_1$, there is one, denoted by $\mathbf{q} = \beta\mathbf{R}_1^{-1}\mathbf{g}$ which corresponds to the largest generalized eigenvalue. Hence we can directly apply this eigenvector as the combining coefficients without estimating the channel coefficients first.

4. ADAPTIVE APPROACH

Now let's look at the computation complexity of the proposed method. Notice that the dimensions of the matrices $\bar{\mathbf{R}}_1$ and $\bar{\mathbf{R}}_2$ are just Q which is much less than the processing gain. This means that the generalized eigendecomposition can be easily implemented. Furthermore it can be shown that an adaptive solution can be applied to solve the generalized eigendecomposition part, which again reduces the complexity.

The key part of the algorithm is to find the maximum generalized eigenvector of $\bar{\mathbf{R}}_2$ with respect to $\bar{\mathbf{R}}_1$. It is well known[9] that this is equivalent to finding a vector $\mathbf{f}$ that maximize the Rayleigh quotient

$$\mathbf{f} = arg\ \max_{\mathbf{f}} \frac{\mathbf{f}^H\bar{\mathbf{R}}_2\mathbf{f}}{\mathbf{f}^H\bar{\mathbf{R}}_1\mathbf{f}} \quad (22)$$

This can be achieved by maximizing the numerator while keeping the denominator constant(this constant is set to 1 in the following without any loss of generality), which is

$$\mathbf{f} = arg\ \max_{\mathbf{f}}\ \mathbf{f}^H\bar{\mathbf{R}}_2\mathbf{f},\ \ subject\ to\ \mathbf{f}^H\bar{\mathbf{R}}_1\mathbf{f}=1, \quad (23)$$

Since the constraint above is in a quadratic form, which leads to some inconvenience. Notice that $\bar{\mathbf{R}}_1$ is a symmetric positive definite matrix, it can be expressed in the form: $\bar{\mathbf{R}}_1 = \mathbf{F}\times\mathbf{F}^H$ where $\mathbf{F}$ is a lower triangular matrix[10]. Hence,(23) is equivalent to

$$\mathbf{f} = arg\ \max_{\mathbf{f}}\ \mathbf{f}^H\bar{\mathbf{R}}_2\mathbf{f},\ \ s.t.\ \mathbf{F}^H\mathbf{f}=\mathbf{q}\ and\ ||\mathbf{q}||=1, \quad (24)$$

The adaptive solutions are well studied in [11]. Here we only introduce one of its LMS methods.

Set up the Langrangian cost function as following

$$\mathbf{J} = \mathbf{f}^H\bar{\mathbf{R}}_2\mathbf{f}+\lambda^H(\mathbf{F}^H\mathbf{f}-\mathbf{q})+\lambda(\mathbf{f}^H\mathbf{F}-\mathbf{q}^H) \quad (25)$$

where λ is the Lagrange multiplier. The update forms of $\mathbf{f}$ and $\mathbf{g}$ are

$$\mathbf{f}_{n+1} = \mathbf{f}_n+\mu_f\nabla_{\mathbf{f}}\mathbf{J} = \mathbf{f}_n+\mu_f(\bar{\mathbf{R}}_2\mathbf{f}_n+\mathbf{F}\lambda_n) \quad (26)$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n-\mu_q(\mathbf{I}-\frac{\mathbf{q}_n\mathbf{q}_n^H}{\mathbf{q}_n^H\mathbf{q}_n})\nabla_{\mathbf{q}}\mathbf{J} = \mathbf{q}_n-\mu_q(\mathbf{I}-\frac{\mathbf{q}_n\mathbf{q}_n^H}{\mathbf{q}_n^H\mathbf{q}_n})(\rho\mathbf{q}_n-\lambda_n) \quad (27)$$

(28)

Step 1:	Obtain $\mathbf{R}_1$ and do the Cholesky Factorization: $\mathbf{R}_1 = \mathbf{F}\mathbf{F}^H$
Step 2:	Define μ_f, μ_q and initialize $\mathbf{q}_0 = [1,\underbrace{0,0...0}_{\mathbf{Q}-1}]^T$, $\mathbf{f}_0 = (\mathbf{F}^H)^{-1}\mathbf{q_0}$
Step 3:	Pre-compute $\prod = \mathbf{I}-\mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H$
Step 4:	For n = 0,1,2,... • Update $\mathbf{f}_{n+1} = \prod\ [\mathbf{q}_n+\mu_q\mathbf{y}_2[n]\mathbf{y}_2^H[n]\mathbf{q}_n]+\mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{q}_n$ • Update $\mathbf{q}_{n+1} = \mathbf{q}_n + \frac{\mu_q}{\mu_f}(\mathbf{I}-\frac{\mathbf{q}_n\mathbf{q}_n^H}{\mathbf{q}_n^H\mathbf{q}_n})(\mathbf{F}^H\mathbf{F})^{-1}[\mu_f\mathbf{F}^H\bar{\mathbf{R}}_2\mathbf{f}_n-\mathbf{q}_n+\mathbf{F}^H\mathbf{f}_n]$ and normalize $\mathbf{q}_{n+1}$ by $\mathbf{q}_{n+1} = \frac{\mathbf{g}_{n+1}}{\lVert\mathbf{g}_{n+1}\rVert}$

Table 1. Adaptive Algorithm

By using the constraint $\mathbf{F}^H\mathbf{f}_{n+1} = \mathbf{q}_n$ we can solve λ_n

$$\lambda_n = \frac{1}{\mu_f}(\mathbf{F}^H\mathbf{F})^{-1}(\mathbf{g}_n-\mathbf{F}^H\mathbf{f}_n-\mu_f\mathbf{F}^H\bar{\mathbf{R}}_2\mathbf{f}_n) \quad (29)$$

Another constraint $||\mathbf{q}|| = 1$ is guaranteed by normalizing $\mathbf{q}_n$ at each iteration. Combining (26) (27) and (29), we have the recursions

$$\mathbf{f}_{n+1} = \prod\ [\mathbf{q}_n+\mu_q\bar{\mathbf{R}}_2\mathbf{q}_n]+\mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{q}_n \quad (30)$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n+\frac{\mu_q}{\mu_f}(\mathbf{I}-\frac{\mathbf{q}_n\mathbf{q}_n^H}{\mathbf{q}_n^H\mathbf{q}_n})(\mathbf{F}^H\mathbf{F})^{-1}[\mu_f\mathbf{F}^H\bar{\mathbf{R}}_2\mathbf{f}_n - \mathbf{q}_n+\mathbf{F}^H\mathbf{f}_n] \quad (31)$$

where $\prod = \mathbf{I}-\mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H$ can be pre-computed.

In practise $\bar{\mathbf{R}}_1$ is always ready to use before the adaptive recursion starts and $\bar{\mathbf{R}}_2$ is replaced by instant samples $\mathbf{y}_2[n]\mathbf{y}_2^H[n]$. The algorithm is summarized in Table 1.

5. NUMERICAL RESULTS

A CDMA system is simulated to test the proposed method. We compare it's bit error rate with that of the optimal conventional RAKE receiver exploiting perfect channel estimation.

Fig. 1 shows the bit error rate(BER) versus the signal to noise ratio(SNR). There are $K = 3$ equal-power asynchronous users exploiting random selected codes in the system which is corrupted by white Gaussian noise at the level σ^2. The SNR is defined as $SNR = 10log_{10}(E_b/\sigma^2)$ where E_b is the bit energy of the said user. Each of the delays are generated randomly from a uniform distribution over the whole symbol period. The processing gain P and the discrete channel length Q is set to be 30 and 3 for all users and the channel coefficients are independent Gaussian random variables. The number of samples used in the startup algorithm(N_1 and N_2) are both 500.

The convergence property is observed by changing N_1 and N_2 while fixing the other conditions. We have $N_1 = N_2 = N$ in every simulation in Fig. 2. We can see that the startup algorithm begins to converge only after around 50 symbols. The adaptive result is also shown in the graph.

The performance of adaptive method is illustrated in Fig. 3. N_1 is set to 500 in each experiment while $N_2 = N$ is the iteration.

6. CONCLUSION

In this paper, a startup receiver without estimating the channel coefficients is studied. This is obtained by exploiting the statistical information before and after the onset of the new user while the other users' information are required the least. Simulations show that the proposed method has almost the same performance as the RAKE receiver.

7. REFERENCES

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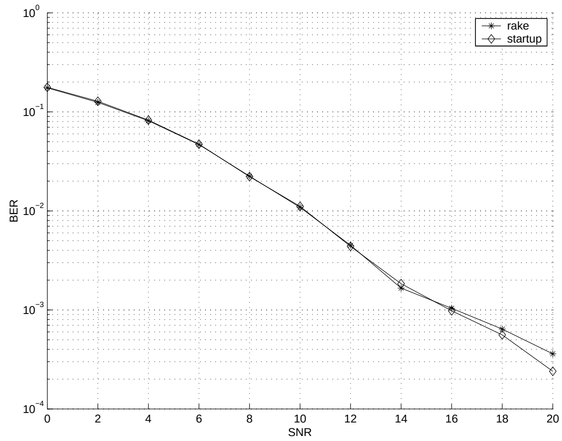

Fig. 1. BER versus SNR in asynchronous CDMA system

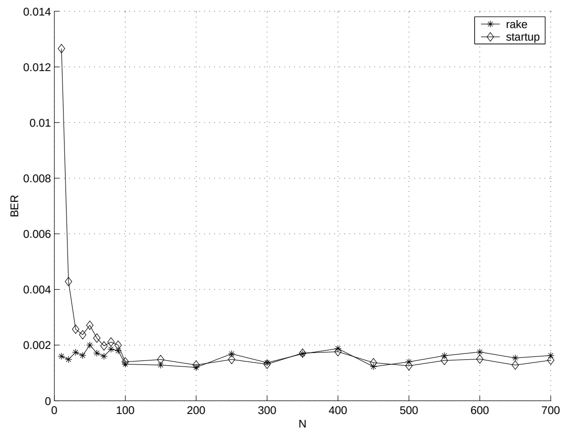

Fig. 2. BER versus N

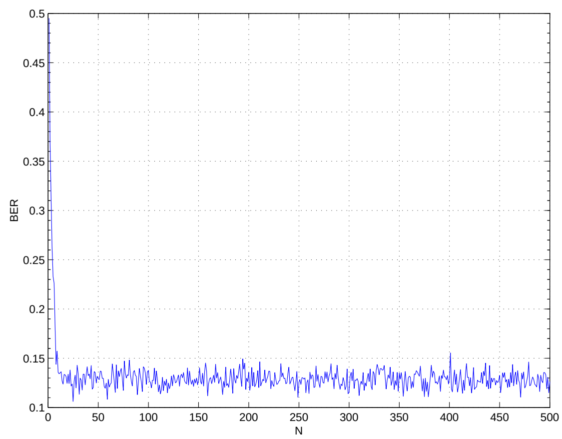

Fig. 3. convergence property