Code-Timing Estimation for Long-Code CDMA Systems with Bandlimited Chip Waveforms

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***Abstract*— In this paper, we present a code-timing estimation scheme for asynchronous direct-sequence (DS) code-division multiple-access (CDMA) systems with *aperiodic (long)* spreading codes and *bandlimited* chip waveforms. The proposed scheme first converts the observed signal to the frequency domain by fast Fourier transform (FFT). Then, a nonlinear least-squares (NLS) criterion is invoked to fit the unknown parameters to the frequency-domain data. While the exact minimizer of the NLS criterion requires computationally prohibitive searches over a multi-dimensional parameter space, we propose an efficient approach that iteratively estimates one user/path at a time via simple linear processing and, furthermore, combines successive interference suppression (SIC) with parameter re-estimation for improved code acquisition performance. Simulation results show that the proposed scheme achieves a significantly larger user capacity and faster acquisition time than the SIC and a standard matched-filter based code acquisition techniques in time-varying fading channels.**

I. Introduction

Multiuser code-timing estimation, which parallels the well acknowledged research on multiuser detection (e.g., [1] and references therein) for direct-sequence (DS) code-division multiple-access (CDMA) systems, has been receiving increasing interest recently. Classical code-timing estimation techniques are based on correlation or matched filter processing [2], which treat the multiple-access interference (MAI) as noise and, thus, are sensitive to the so-called near-far problem [1]. Some recent studies on code-timing estimation tried to exploit the structure of the MAI to improve the user capacity and code acquisition performance. A multitude of near-far resistant code-timing estimation techniques have been proposed in the last few years (see [3], [4] and references therein). However, most of these techniques assume *short or periodic* spreading codes, whereby the code sequence periodically repeats itself for every symbol duration. In addition, most existing schemes assume *rectangular* chip waveforms that are not bandlimited. This is in contrast to *bandlimited* chip waveforms, such as the square-root raised cosine pulse, that are used in real CDMA systems.

We present herein a novel code-timing estimator for asynchronous DS-CDMA systems utilizing *aperiodic or long* spreading codes and *bandlimited* chip waveforms. We first convert the observed signal to the frequency domain by fast Fourier transform (FFT). Following that, we invoke a nonlinear least-squares (NLS) criterion to fit the unknown parameters to the frequency-domain data. Since the exact minimizer of the NLS cost function requires computationally prohibitive searches over a multi-dimensional parameter space, we propose an efficient approach that iteratively estimates one user/path at a time via simple linear processing and, furthermore, combines successive interference suppression (SIC) with parameter re-estimation for improved code acquisition performance. Simulation results show that the proposed scheme achieves a significantly larger user capacity and faster acquisition time than the SIC and a standard matched-filter based code acquisition techniques in time-varying fading channels.

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Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; $\mathrm{diag}\{\mathbf{x}\}$ is a diagonal matrix with the elements of the vector $\mathbf{x}$ placed on the diagonal; superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote transpose, conjugate and conjugate transpose, respectively; $\|\cdot\|$ denotes the matrix/vector Frobenius norm; $\lceil\cdot\rceil$ denotes the smallest integer no less than the argument; and finally, $\star$ denotes linear convolution.

II. Data Model and Problem Formulation

Consider a baseband asynchronous K-user DS-CDMA system with *long (aperiodic)* spreading codes. Let $p(t)$ denote the *bandlimited* chip waveform which is assumed to be identical for all users. The transmitted signal for user k is given by

$$x'_k(t) = \sum_{m=0}^{M-1} d_k(m) s'_{k,m}(t - mT_s), \tag{1}$$

where M is the number of symbols used for code acquisition, T_s is the symbol period, $d_k(m)$ denotes the mth symbol of user k, and $s'_{k,m}(t)$ denotes the time-varying spreading waveform that modulates the mth symbol of user k:

$$s'_{k,m}(t) = \sum_{n=0}^{N-1} c_{k,m}(n) p(t - nT_c).$$

Here and henceforth, N denotes the number of chips per symbol (i.e., *processing gain*), $T_c = T_s/N$ denotes the chip interval, and $c_{k,m}(n)$ denotes the spreading code of the mth symbol of user k.

The spread spectrum signal $x'_k(t)$ passes through a frequency-selective channel with L_k distinct propagation paths. The baseband signal received at the base station, after chip-matched filtering, is given by

$$y(t) = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \alpha_{k,l} x_k(t - \tau_{k,l}) + e(t), \quad (2)$$

where $\alpha_{k,l}$ and $\tau_{k,l}$ denote, respectively, the channel attenuation and the *code timing* associated with the lth path of user k, $e(t)$ denotes the channel noise, and $x_k(t)$ denotes the output of the chip-matched filter with impulse response $p(T_c - t)$ when the input is $x'_k(t)$:

$$x_k(t) \triangleq x'_k(t) \star p(T_c - t) = \sum_{m=0}^{M-1} d_k(m) s_{k,m}(t - mT_s),$$

where $s_{k,m}(t) \triangleq s'_{k,m}(t) \star p(T_c - t)$. It is noted that $x_k(t)$ lumps together the time-varying spreading waveform as well as data symbols for user k, which are assumed known at the base station (via training). For the brevity of presentation, we assume that the maximum path delay is within one symbol period; the extension to otherwise is conceptually simple. With this assumption, the inter-symbol interference (ISI) affects only the next data symbol.

For efficient processing, we break $x_k(t)$ into non-overlapping blocks of $M_0 T_s$, where $M_0 \leq M$. Similarly and to account for ISI, we break $y(t)$ into overlapping blocks of $(M_0 + 1)T_s$, and two adjacent blocks overlap by T_s. That is, $x_k(t) = \sum_{\mu=0}^{J-1} x_{k,\mu}(t - \mu M_0 T_s)$ and $y(t) = \sum_{\mu=0}^{J-1} y_\mu(t - \mu M_0 T_s)$, where $x_{k,\mu}(t)$ and $y_\mu(t)$ have a duration of $M_0 T_s$ and $(M_0 + 1)T_s$, respectively, and $J \triangleq \lceil M/M_0 \rceil$ denotes the number of blocks. Similar to (2), we can write

$$y_\mu(t) = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \alpha_{k,l} x_{k,\mu}(t - \tau_{k,l}) + e_\mu(t), \quad \mu = 0, \ldots, J-1, \quad (3)$$

where it is noted that $e_\mu(t)$ now contains inter-block interference (IBI) due to multipath propagation. We shall choose M_0 large enough to make the IBI small (when $M_0 = M$, then IBI vanishes). Meanwhile, the complexity of the proposed code-timing estimator increases as M_0 increases. Numerical simulation suggests that a choice of around 10 for M_0 provides a good tradeoff between performance and complexity.

The problem of interest is to estimate the code-timing $\{\tau_{k,l}\}$ for all users from the chip-matched filter output $\{y_\mu(t)\}$.

III. Proposed Scheme

We first sample the output of the chip-matched filter $y_\mu(t)$ with a sampling interval $T_i = T_c/Q$: $y_\mu(i) \triangleq y_\mu(t)\big|_{t=iT_i}$, where Q is an integer referred to as the over-sampling factor; usually, $Q = 2$ is sufficient. We then form $(M_0 + 1)NQ \times 1$ vectors using samples of $y_\mu(t)$: [1] $\bar{\mathbf{y}}_\mu \triangleq [y_\mu(0),\ y_\mu(1),\ \ldots,\ y_\mu((M_0+1)NQ-1)]^T$. Likewise, let $\bar{\mathbf{x}}_{k,\mu}(l)$ and $\bar{\mathbf{e}}_\mu$ be $(M_0+1)NQ \times 1$ vectors formed by samples of $x_{k,\mu}(t - \tau_{k,l})$ [2] and $e_\mu(t)$, respectively. Then, (3) can be rewritten in vector form:

$$\bar{\mathbf{y}}_\mu = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \alpha_{k,l} \bar{\mathbf{x}}_{k,\mu}(l) + \bar{\mathbf{e}}_\mu, \quad \mu = 0, \ldots, J-1.$$

Next, we take the Fourier transform, or DFT (discrete Fourier transform) via FFT (fast Fourier transform) to take advantage of its efficiency, of $\bar{\mathbf{y}}_\mu$:

$$\mathbf{y}_\mu \triangleq \mathcal{F}\{\bar{\mathbf{y}}_\mu\} = \sum_{k=1}^{K} \sum_{l=1}^{L_k} \alpha_{k,l} \mathcal{F}\{\bar{\mathbf{x}}_{k,\mu}(l)\} + \mathbf{e}_\mu, \quad (4)$$

where $\mathcal{F}\{\cdot\}$ denotes the DFT operation and $\mathbf{e}_\mu \triangleq \mathcal{F}\{\bar{\mathbf{e}}_\mu\}$. Using the time-shifting property of Fourier transform, we have

$$\mathcal{F}\{\bar{\mathbf{x}}_{k,\mu}(l)\} = \mathrm{diag}(\mathbf{x}_{k,\mu})\mathbf{a}(\tau_{k,l}) \triangleq \mathbf{X}_{k,\mu}\mathbf{a}(\tau_{k,l}), \quad (5)$$

where $\mathbf{x}_{k,\mu} \triangleq \mathcal{F}\{\bar{\mathbf{x}}_{k,\mu}\}$ with $\bar{\mathbf{x}}_{k,\mu}$ denoting the $M_0 NQ \times 1$ vector formed from samples of $x_{k,\mu}(t)$ with no delay, and

$$\mathbf{a}(\tau_{k,l}) \triangleq \left[1, \quad e^{-j\frac{2\pi\tau_{k,l}}{(M_0+1)NQ}}, \quad \ldots, \quad e^{-j\frac{2\pi\tau_{k,l}[(M_0+1)NQ-1]}{(M_0+1)NQ}}\right]^T,$$

Note that $\mathbf{x}_{k,\mu}$ is obtained by an $(M_0+1)NQ$-point DFT with zero padding.

Let $\mathbf{A}(\boldsymbol{\tau}_k) \triangleq [\mathbf{a}(\tau_{k,1}),\ \cdots,\ \mathbf{a}(\tau_{k,L_k})]$ and $\boldsymbol{\alpha}_k \triangleq [\alpha_{k,1},\ \cdots,\ \alpha_{k,L_k}]^T$. Substituting (5) into (4) yields

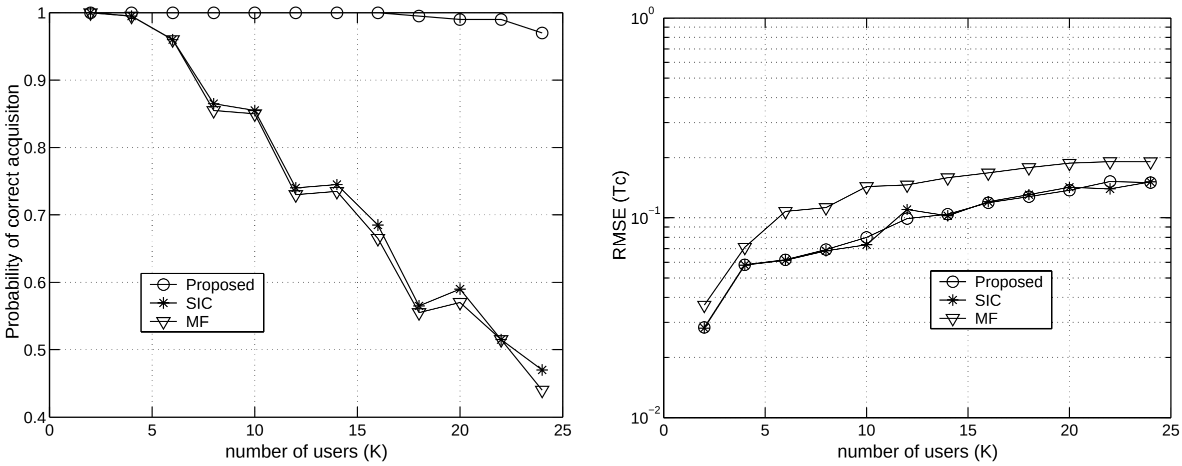
$$\mathbf{y}_\mu = \sum_{k=1}^{K} \mathbf{X}_{k,\mu}\mathbf{A}(\boldsymbol{\tau}_k)\boldsymbol{\alpha}_k + \mathbf{e}_\mu, \quad \mu = 0, \cdots, J-1; \quad (6)$$

where $\boldsymbol{\tau}_k \triangleq [\tau_{k,1},\ \cdots,\ \tau_{k,L_k}]^T$. The above equation reveals that $\mathbf{y}_\mu$ consists of K groups of *weighted* complex sinusoids contaminated by noise (and residual IBI) $\mathbf{e}_\mu$. The frequencies are determined by the code-timing parameters $\boldsymbol{\tau}_k$, while the complex amplitudes are determined by the attenuation parameters $\boldsymbol{\alpha}_k$. The weighting matrix for the kth group of sinusoids is the diagonal matrix $\mathbf{X}_{k,\mu} \triangleq \mathrm{diag}(\mathbf{x}_{k,\mu})$.

One might opt to perform frequency deconvolution by dividing elementwise both sides of (6) by $\mathbf{x}_{k,\mu}$ to isolate the kth sinusoidal group from the others, so that the rich literature on sinusoidal parameter estimation (e.g., [5]) can be utilized for code acquisition. However, this is not recommended since $\mathbf{x}_{k,\mu}$, which lumps aperiodic spreading codes, are likely to have many spectral nulls. Such a division would boost drastically the interference/noise power at those spectral nulls. To overcome the problem, one might perform a partial frequency deconvolution by which the frequency samples at places where spectral nulls are likely to happen (usually at the end frequencies of the DFT grid) are discarded [6]. We do not recommend such an approach

[1] Throughout this paper, we use notation $\bar{(\cdot)}$ to denote a time-domain quantity if its frequency-domain counterpart is also used for estimation.

[2] Note that $x_{k,\mu}(t - \tau_{k,l})$ has support in $0 \leq t \leq (M_0+1)T_s$ due to the propagation delay (within one T_s).



(a) (b)

Fig. 1. Performance versus K, the user capacity, in time-varying Rayleigh fading channels when $Q = 2$, $M = 60$, $M_0 = 15$, SNR $= 15$ dB, NFR $= 10$ dB and $f_D T_s = 0.0067$. (a) Probability of correct acquisition. (b) RMSE.

either, since throwing away those frequency samples amounts to throwing away useful information; in principle, it would be better to exploit every sample of data for estimation.

To seek fast acquisition, deterministic processing in general should be preferred. We consider the following deterministic nonlinear least squares (NLS) criterion, by which the parameter estimates are chosen to minimize the Euclidean distance:

$$C_{\text{nls}}(\boldsymbol{\tau}_k, \boldsymbol{\alpha}_k) = \sum_{\mu=0}^{J-1} \left\| \mathbf{y}_\mu - \sum_{k=1}^{K} \mathbf{X}_{k,\mu} \mathbf{A}(\boldsymbol{\tau}_k) \boldsymbol{\alpha}_k \right\|^2. \quad (7)$$

The NLS cost function is highly nonlinear and cannot be easily minimized unless for small K. The linearly separable structure of (6) suggests that the successive interference canceling (SIC) with power ranking, which was originally proposed to solve multiuser detection problem (e.g., [1]), can be modified to solve the code acquisition problem. The basic idea is to estimate one user at a time, then subtract that user from $\mathbf{y}_\mu$, and use the remainder to estimate the next user, and subtract it again, and estimate the next one, and so on and so forth. Yet, We found via numerical simulations that the performance of SIC is still quite limited (see Section IV).

To find an alternative solution, we consider the so-called relaxation based optimization technique, by which we not only subtract previously estimated users to help the estimation of the current user, but also re-estimate those previously determined user paths through an iterative fashion [7]. To elaborate, let us assume that all code timing and attenuation coefficients are known except for $\tau_{k,l}$ and $\alpha_{k,l}$, i.e., the delay and attenuation of path l of user k. Let

$$\mathbf{y}_\mu(k, l) \triangleq \mathbf{y}_\mu - \sum_{k} \sum_{j \neq l} \hat{\alpha}_{k,j} \mathbf{X}_{k,\mu} \mathbf{a}(\hat{\tau}_{k,j}), \quad (8)$$

where $\hat{\alpha}_{k,j}$ and $\hat{\tau}_{k,j}$ are assumed known for $j \neq l$. It is easy to show that the LS (least squares) estimates of the unknown parameters are (e.g., [5])

$$\begin{aligned}\hat{\tau}_{k,l} &= \arg\max_{\tau_{k,l}} \frac{\left| \sum_{\mu=0}^{J-1} \mathbf{a}^H(\tau_{k,l}) \mathbf{X}_{k,\mu}^H \mathbf{y}_\mu(k,l) \right|^2}{\sum_{\mu=0}^{J-1} \mathbf{a}^H(\tau_{k,l}) \mathbf{X}_{k,\mu}^H \mathbf{X}_{k,\mu} \mathbf{a}(\tau_{k,l})} \\ &= \arg\max_{\tau_{k,l}} \left| \mathbf{a}^H(\tau_{k,l}) \sum_{\mu=0}^{J-1} \mathbf{X}_{k,\mu}^H \mathbf{y}_\mu(k,l) \right|^2, \quad (9)\end{aligned}$$

$$\hat{\alpha}_{k,l} = \sum_{\mu=0}^{J-1} \frac{\mathbf{a}^H(\hat{\tau}_{k,l}) \mathbf{X}_{k,\mu}^H \mathbf{y}_\mu(k,l)}{\mathbf{x}_{k,\mu}^H \mathbf{x}_{k,\mu}} \quad (10)$$

where in (9) we used the fact that $\mathbf{X}_{k,\mu}$ is diagonal and the Vandermonde vector $\mathbf{a}(\tau_{k,l})$ is formed by all unit-modulus elements, so that $\mathbf{a}^H(\tau_{k,l}) \mathbf{X}_{k,\mu}^H \mathbf{X}_{k,\mu} \mathbf{a}(\tau_{k,l}) = \mathbf{x}_{k,\mu}^H \operatorname{diag}^H\{\mathbf{a}(\tau_{k,l}\} \operatorname{diag}\{\mathbf{a}(\tau_{k,l})\} \mathbf{x}_{k,\mu} = \mathbf{x}_{k,\mu}^H \mathbf{x}_{k,\mu}$ is a constant, which is independent of the code timing and can be precomputed. Note that $\mathbf{X}_{k,\mu}^H \mathbf{y}_\mu(k,l)$ is just the elementwise multiplication between $\mathbf{x}_{k,\mu}^*$ and $\mathbf{y}_\mu(k,l)$. Effectively, *(9) indicates the $\hat{\tau}_{k,l}$ is the location of the dominant of the periodogram of $\sum_{\mu=0}^{J-1} \mathbf{X}_{k,\mu} \mathbf{y}_\mu(k,l)$ [5], which can be efficiently computed by using the FFT (with zero padding)*. With the above preparation, we now summarize the proposed scheme as follows:

1) Assume $K = 1$. For $l = 1 : L_1$, use (8) to compute $\mathbf{y}_\mu(1,l)$ (assuming there are a total of l paths for user 1 during the calculation of $\mathbf{y}_\mu(1,l)$), and use $\mathbf{y}_\mu(1,l)$ to estimate one path at a time via (9)–(10).
2) Assume $K = 2$. For $l = 1 : L_2$, use (8) to compute $\mathbf{y}_\mu(2,l)$ (assuming there are a total of l paths for user 2 during the calculation of $\mathbf{y}_\mu(2,l)$), and use $\mathbf{y}_\mu(2,l)$ to estimate one path at a time via (9)–(10). Next, recompute $\mathbf{y}_\mu(1,l)$ by using the parameter estimates for user 2, and redetermine the code timing and attenuation estimates for user 1. Iterate between the above two substeps a few times till convergence (typically one iteration is sufficient).

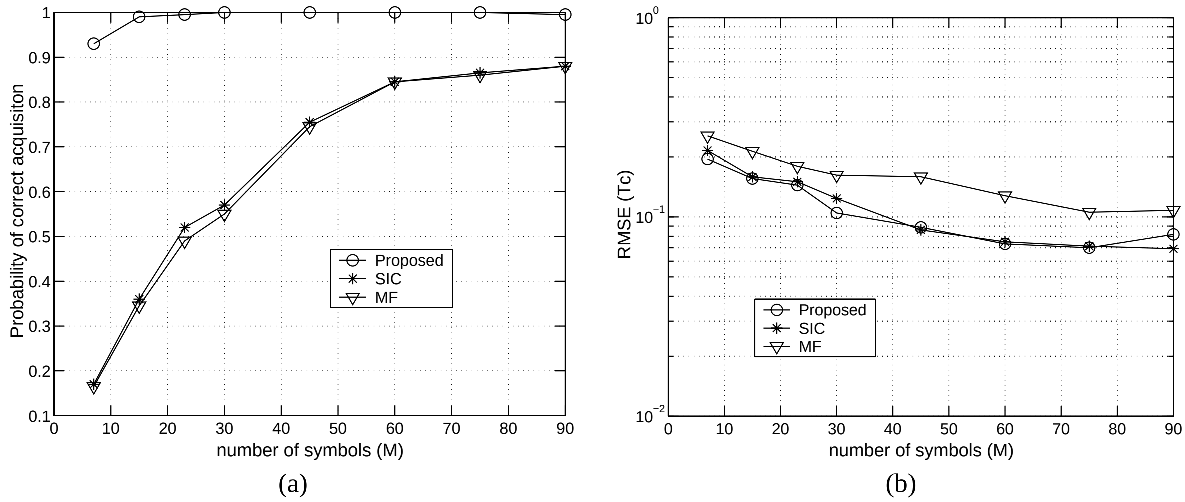


Fig. 2. Performance versus M, the acquisition time, in time-varying Rayleigh fading channels when $Q = 2$, $K = 10$, $M_0 = 15$, SNR $= 15$ dB, NFR $= 10$ dB and $f_D T_s = 0.0067$. (a) Probability of correct acquisition. (b) RMSE.

3) Assume $K = 3$. For $l = 1 : L_3$, use (8) to compute $\mathbf{y}_\mu(3, l)$ (assuming there are a total of l paths for user 3 during the calculation of $\mathbf{y}_\mu(3, l)$), and use $\mathbf{y}_\mu(3, l)$ to estimate one path at a time via (9)–(10). Next, recompute $\mathbf{y}_\mu(1, l)$ by using the parameter estimates for users 2 and 3, and redetermine the parameter estimates for user 1. Similarly, redetermine the parameter estimates for user 2. Iterate the previous three substeps a few times (typically one iteration) till convergence.
4) Continue in a similar fashion till K is equal to the real number of users.

Note that although the above approach is iterative, it is computationally quite fast due to the use of FFT during each iteration.

IV. Simulation Results

Consider a K-user asynchronous DS-CDMA system using *long spreading codes* and *bandlimited chip waveforms*. The spreading codes are randomly generated, which modulate BPSK (binary phase shift keying) symbols with a processing gain of $N = 16$. The bandlimited chip waveform is a square-root raised-cosine pulse. We consider a near-far environment whereby the transmitted power P_1 for the desired user is scaled so that $P_1 = 1$, whereas the power for all other interfering users in *all* simulations follows a log normal distribution: $P_k/P_1 = 10^{0.1P}, P \sim N(10, 100), k = 2, \cdots, K$. Note that all the interfering users transmit at a mean power level 10 dB higher than that of the desired user, i.e., the *near-far ratio (NFR)* is 10 dB for all examples. Even though our estimator was derived assuming the channel is static during code acquisition, we consider here a *time-varying* fading channel model whereby the fading $\alpha_{k,l}(t)$ is parameterized by the normalized Doppler rate $f_D T_s$, with f_D denoting the maximum Doppler rate. In the following, we use $f_D T_s = 0.0067$. The fading process is generated by the Jakes' model [8] and is updated continuously every T_i (recall $T_i = T_c/Q$). The channel is assumed frequency-nonselective, i.e., $L_k = 1$ for all k. We consider two performance measures. One is the *probability of correct acquisition*, defined as the probability of the event that the code timing estimate is within $T_c/2$ of the true code timing. The other measure is the *root mean-squared error (RMSE)* of the code-timing estimate normalized by T_c, given correct acquisition. The results shown here are averaged over hundreds independent realizations of the time-varying fading channels.

Since for long-code DS-CDMA systems, the only well recognized code acquisition techniques in the literature are correlation-based methods, we compare with the **Matched Filter (MF)** [2, Section 5-5], which achieves much faster code acquisition than serial-search based methods. We also compare with the **Successive Interference Canceling (SIC)** approach we outlined earlier. Figure 1 depicts the performance versus K, the number of users, when $M = 60$ (broken into $J = 4$ blocks of $M_0 = 15$) and SNR $= 15$ dB. It is seen that the proposed scheme can support a fully loaded system (i.e., $K = N$), with a user capacity much larger than that of the SIC and MF schemes. Figure 2 depicts the performance as a function of M, the acquisition time. It is seen that the proposed scheme has a much faster acquisition time than the other two.

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