CHANNEL ESTIMATION OF LONG-CODE CDMA SYSTEMS UTILIZING TRANSMISSION INDUCED CYCLOSTATIONARITY

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ABSTRACT

For long code DS-CDMA systems, where the spreading codes are aperiodic and extending over a large number of data symbols, chip-rate sampled signals and MUIs (multiuser interferences) are generally modeled as time-varying vector processes. This complicates the application of traditional blind multiuser detectors, since consistent estimation of the needed signal statistics can not be obtained by time-averaging over received data record. In this paper, we propose an equivalent time-invariant system model for long code CDMA, in which the received signals and MUIs are modeled as cyclostationary processes with modulation introduced cyclostationarity. Based on knowledge of the desired user's code sequences, channel estimation is carried out using a frequency domain subspace method.

1. INTRODUCTION

In DS-CDMA systems, each user is assigned a special signature sequence or spreading code. If the spreading codes are periodic and repeat every information symbol, the system is called *short code CDMA system*. On the other hand, if the spreading codes are aperiodic and extending over a large segment of the symbol sequence, the system is known as *long code CDMA system* in literature. More recently, systems using periodic spreading sequence which may span multiple symbols were proposed as *"semi-long-code" CDMA systems*.

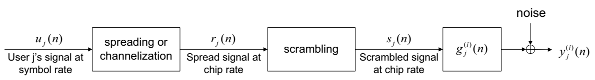

Fig. 1. Block Diagram of a long code DS-CDMA System

In the current commercial WCDMA systems, each user's signal is first spread using a code sequence spanning over just one symbol or multiple symbols. The spread signal is then further scrambled using a long-periodicity pseudo-random sequence (as shown in Fig.1). In Fig.1, after scrambling at chip level, the chip-rate sampled signal and MUIs are generally modeled as time-varying vector processes. This is equivalent to the use of an aperiodic (long) coding sequence as in *long code CDMA system*. The time-varying nature of the received signal model in the long code case severely complicates the equalizer development approaches, since consistent estimation of the needed signal statistics can not be achieved by time-averaging over the received data record. In contrast, for *short code CDMA system*, where the spreading codes repeat every information symbol, time-invariant MIMO models allow consistent estimation of the needed signal statistics. For this reason, the study for multiuser detectors has largely been limited to short code DS-CDMA systems. More recently, it has been expanded to systems with spreading codes whose period may span multiple symbols [6]. Systems with these "semi-long" codes can still be modeled as cyclostationary processes with period equals to multiple symbol periods. But this approach does not work for long code CDMA systems where the spreading code is aperiodic.

In this paper, we consider blind channel estimation for long-code CDMA systems. Under the assumption that channels remain stationary over each time slot, a time-variant model can be obtained from scrambling. The underlying channel of a long code CDMA system can actually be modeled as a time-invariant MIMO system. Motivated by this observation, we first describe an equivalent time-invariant model for long code CDMA systems. Secondly, this effort allows us to model the received signals and MUIs as cyclostationary processes with modulation induced cyclostationarity, thus providing a platform for development of blind channel estimation and equalization approaches for long code CDMA signal detection and separation. Finally, by applying periodic non-constant modulus precoding techniques (with the spreading code sequences serve as the precoding sequences in this case), multiuser detection is made possible without excess bandwidth requirement.

2. SYSTEM MODEL

Consider a DS-CDMA system with M users (M transmit antennas) and K receive antennas, as shown in Fig. 2. Assume the processing gain is N, that is, there are N chips per symbol. Let $u_j(k)$ ($j = 1, \cdots, M$) denote User j's kth symbol. Assume that the code sequence extends over L_c symbols, in this paper, without loss of generality, we choose $L_c = 2$, the results can be extended directly to the cases where $L_c > 2$. Let $\mathbf{c}_j = [c_j(0), c_j(1), \cdots, c_j(N-1), c_j(N), \cdots, c_j(2N-1)]$ denotes User j's spreading code sequence. For notations used for each individual user, please refer to Fig. 1. When k is an even integer, the spread signal (at chip rate) with respect to $u_j(k)$ and $u_j(k+1)$ is

$$\begin{aligned} & [r_j(kN), \cdots, r_j((k+1)N+N-1)] \\ = \; & [u_j(k)c_j(0), \cdots, u_j(k)c_j(N-1), \\ & u_j(k+1)c_j(N), \cdots, u_j(k+1)c_j(2N-1)]. \end{aligned}$$

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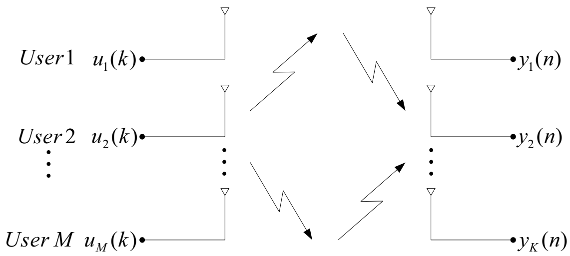

Fig. 2. A wireless communications system with multiple transmit and receive antennas

The successive scrambling process is achieved by

$$\begin{aligned} & [s_j(kN), \cdots, s_j((k+1)N+N-1)] \\ = \; & [r_j(kN), \cdots, r_j((k+1)N+N-1)] .* [d_j(kN), \cdots, \\ & d_j(kN+N-1), d_j((k+1)N), \cdots, d_j((k+2)N-1)] \quad (1) \end{aligned}$$

where ".*" stands for point-wise multiplication, and $[d_j(kN), d_j(kN+1), \cdots, d_j(kN+N-1)]$ denotes the chip rate scrambling sequence with respect to symbol $u_j(k)$. Define

$$\begin{aligned} & [v_j(kN), \cdots, v_j((k+1)N+N-1)] \\ \triangleq \; & [u_j(k)d_j(kN), \cdots, u_j(k)d_j(kN+N-1), \\ & u_j(k+1)d_j((k+1)N), \cdots, u_j(k+1)d_j((k+1)N+N-1)], \end{aligned}$$

we get

$$\begin{aligned} & [s_j(kN), s_j(kN+1), \cdots, s_j((k+1)N+N-1)] \\ = \; & [v_j(kN), v_j(kN+1), \cdots, v_j((k+1)N+N-1)] \\ & .* [c_j(0), c_j(1), \cdots, c_j(2N-1)]. \qquad (2) \end{aligned}$$

If we regard the chip rate $v_j(n)$ as the input signal of User j, then $s_j(n)$ is the precoded transmit signal corresponding to the jth user and

$$s_j(n) = v_j(n)c_j(n), \quad n \in Z, \; j = 1, 2, \cdots M. \qquad (3)$$

Obviously, $c_j(n) = c_j(n+2N)$ is a periodic precoding sequence with period $2N$. We note that this form of periodic precoding has been suggested by Serpedin and Giannakis in [1] to introduce cyclostationarity in the transmit signal, thereby making blind channel identification based on second-order statistics in symbol-rate sampled single carrier system possible. More general idea of transmitter-induced cyclostationarity has been suggested previously in [2, 3]. In [4], non-constant precoding technique has been applied to blind channel identification and equalization in OFDM-based multiantenna systems.

Based on Fig. 1 and Fig. 2, the received chip-rate signal at the pth antenna ($p = 1, 2, \cdots, K$) can be expressed as

$$y_p(n) = \sum_{j=1}^{M} \sum_{l=0}^{L-1} g_j^{(p)}(l) s_j(n-l) + w_p(n). \qquad (4)$$

where $w_p(n)$ is the pth antenna additive white noise. Let $\mathbf{s}(n) = [s_1(n), s_2(n), \cdots, s_M(n)]^T$ be the precoded signal vector. Collect the samples at each receiver antenna and stack them into a $K \times 1$ vector, we get the following *time-invariant* MIMO system model

$$\mathbf{y}(n) = [y_1(n), y_2(n), \cdots, y_K(n)]^T = \sum_{l=0}^{L-1} \mathbf{H}(l)\mathbf{s}(n-l) + \mathbf{w}(n), \qquad (5)$$

where

$$\mathbf{H}(l) = \begin{bmatrix} g_1^{(1)}(l) & g_2^{(1)}(l) & \cdots & g_M^{(1)}(l) \\ g_1^{(2)}(l) & g_2^{(2)}(l) & \cdots & g_M^{(2)}(l) \\ \vdots & \vdots & \ddots & \vdots \\ g_1^{(K)}(l) & g_2^{(K)}(l) & \cdots & g_M^{(K)}(l) \end{bmatrix}_{K \times M} \qquad (6)$$

and $\mathbf{w}(n) = [w_1(n), w_2(n), \cdots w_K(n)]^T$.

In the following section, channels are estimated based on the desired user's code sequence and the following assumptions:

- **(A1)** The multiuser sequences $\{u_j(k)\}_{j=1}^M$ are zero mean, mutually independent and i.i.d. Take $E\{|u_j(k)|^2\} = 1$ by absorbing any non-identity variance of $u_j(k)$ into the channel.
- **(A2)** The scrambling sequences $\{d_j(k)\}_{j=1}^M$ are mutually independent i.i.d. BPSK sequences, independent of the information sequences.
- **(A3)** The noise is zero mean Gaussian, independent of the information sequences, with $E\{\mathbf{w}(k+l)\mathbf{w}^H(k)\} = \sigma_w^2 \mathbf{I}_K \delta(l)$ where $\mathbf{I}_K$ is the $K \times K$ identity matrix.

3. BLIND CHANNEL IDENTIFICATION BASED ON MODULATION-INDUCED CYCLOSTATIONARITY

From the previous section, it can be seen that the scrambled sequence $s_j(n)$ is obtained by multiplying the individual stream $v_j(n)$ by a periodic precoding sequence $c_j(n)$ (see (3)), where $c_j(n)$ is User j's channelization code. In this section, it will be seen that by applying periodic non-constant modulus precoding in the transmitter, the cyclostationarity induced by precoding enables channel identification of each individual channel $g_j^{(p)}(l)$ from jth user to pth antenna, $p = 1, 2, \cdots, K$ and $j = 1, 2, \cdots, M$. Precoding sequences need to be distinct for each transmit antenna, and they are chosen in such a way that for a given cycle, all but one antennas are nulled out. It is therefore possible to identify each individual channel based on the cyclostationary statistics.

Consider the $K \times K$ auto-correlation matrix

$$\begin{aligned} & \mathbf{R}_y(n,k) \triangleq E\{\mathbf{y}(n)\mathbf{y}^H(n-k)\} \\ = \; & E\{\sum_{l=0}^{L-1}\sum_{m=0}^{L-1} \mathbf{H}(l)[\mathbf{s}(n-l)\mathbf{s}^H(n-k-m)]^H \mathbf{H}^H(m)\} \\ & + \mathbf{R}_w(k) \\ \triangleq \; & \mathbf{R}_{y_s}(n,k) + \sigma_n^2 \mathbf{I}_K \delta(k). \qquad (7) \end{aligned}$$

Consider

$$\begin{aligned} \mathbf{R}_s(n,k) & \triangleq E\{\mathbf{s}(n)\mathbf{s}^H(n-k)\} \\ & = \text{diag}[|c_1(n)|^2\delta(k), \cdots, |c_M(n)|^2\delta(k)], \quad (8) \end{aligned}$$

it follows that $\mathbf{R}_s(n,k)$ is periodic with respect to n

$$\mathbf{R}_s(n,k) = \mathbf{R}_s(n+2N,k)$$

(where N is the processing gain) since $c_j(n) = c_j(n+2N)$ for $j = 1, 2, \cdots, M$. Note that $\mathbf{R}_s(n,k) = 0$ for any $k \neq 0$, define $\mathbf{R}_s(n) \triangleq \mathbf{R}_s(n,0)$, then

$$\mathbf{R}_s(n) = \mathbf{R}_s(n+2N), \qquad (9)$$

and its Fourier series coeffient matrice are therefore given by

$$\mathbf{C}_s(k) = \sum_{n=0}^{2N-1} \mathbf{R}_s(n) e^{-i\frac{\pi}{N}kn}, \quad k = 0, 1, \cdots, N-1. \quad (10)$$

From (7)-(9), it follows that

$$\begin{aligned} \mathbf{R}_y(n,k) &= \sum_{l=0}^{L-1} \mathbf{H}(l)\mathbf{R}_s(n-l)\mathbf{H}^H(l-k) + \mathbf{R}_w(k) \\ &\triangleq \mathbf{R}_{y_s}(n,k) + \mathbf{R}_w(k) \quad (11) \end{aligned}$$

and $\mathbf{R}_y(n,k)$ is also periodic in n

$$\mathbf{R}_y(n,k) = \mathbf{R}_y(n+2N,k). \quad (12)$$

The Fourier series coefficient matrices of $\{\mathbf{R}_y(n,k)\}$ for a fixed k are then given by

$$\mathbf{C}_y(m,k) = \sum_{n=0}^{2N-1} \mathbf{R}_{y_s}(n,k) e^{-i\frac{\pi mn}{N}} + \mathbf{R}_w(k) \cdot \delta(m),$$

for $m = 0, 1, \cdots, 2N-1$. Let $H(z) \triangleq \sum_{l=0}^{L-1} \mathbf{H}(l) z^{-l}$ be the Z-transform of the channel coefficient matrice $\{\mathbf{H}(l)\}_{l=1}^{L-1}$, the Z-transform of $\mathbf{C}_y(m,k)$ w.r.t. k is

$$\begin{aligned} & \mathbf{S}_y(m,z) = \sum_{k=-\infty}^{\infty} \mathbf{C}_y(m,k) z^{-k} \\ =& \sum_{k=-\infty}^{\infty} \sum_{n=0}^{2N-1} \sum_{l=0}^{L-1} \mathbf{H}(l)\mathbf{R}_s(n-l)\mathbf{H}^H(l-k) e^{-i\frac{\pi mn}{N}} z^{-k} \\ & + \mathbf{S}_w(z)\delta(m) \\ =& H(ze^{i\frac{\pi m}{N}})\mathbf{C}_s(m)[H(\frac{1}{z^*})]^H + \mathbf{S}_w(z)\delta(m) \quad (13) \end{aligned}$$

where

$$\begin{aligned} \mathbf{C}_s(m) &= \text{diag}\left\{ \sum_{n=0}^{2N-1} |c_1(n)|^2 e^{-i\frac{\pi mn}{N}}, \cdots, \right. \\ & \left. \sum_{n=0}^{2N-1} |c_M(n)|^2 e^{-i\frac{\pi mn}{N}} \right\} \\ &\triangleq \text{diag}\{c_{s_1}(m), c_{s_2}(m), \cdots, c_{s_M}(m)\}. \end{aligned}$$

Clearly, when $m \neq 0$, the noise effect disappears and

$$\mathbf{S}_y(m,z) = H(ze^{i\frac{\pi m}{N}})\mathbf{C}_s(m)[H(\frac{1}{z^*})]^H. \quad (14)$$

Similar to [1], the basic idea of this channel estimation algorithm is to design precoding code sequences $\{c_j(n)\}_{n=0}^{2N-1}$ ($j = 1, 2, \cdots, M$) such that for a given cycle $m = m_j$, $c_{s_j}(m_j) \neq 0$ and $c_{s_k}(m_j) = 0$ for all $k \neq j$. That is, all but one entry in $\mathbf{C}_s(m)$ are zero. Choose a different cycle m_j for each user(obviously, we need $2N > M$), blind identification of each individual channel can then be achieved by applying the subspace method in frequency domain[1].

In fact, suppose that for a given cycle $m = m_j, m_j \neq 0$, the codes are designed such that $c_{s_j}(m_j) \neq 0$ and $c_{s_k}(m_j) = 0$ for all $k \neq j$, then

$$\mathbf{S}_y(m_j,z) = H(ze^{i\frac{\pi m_j}{N}}) \,\text{diag}\{0, \cdots, c_{s_j}(m_j), \cdots, 0\}[H(\frac{1}{z^*})]^H \quad (15)$$

Define $\alpha_j \triangleq \frac{\pi m_j}{N}$ and

$$G_j^{(p)}(z) \triangleq \sum_{l=0}^{L-1} g_j^{(p)}(l) z^{-l}, \quad (16)$$

it follows that

$$\mathbf{S}_y(m_j,z) = c_{s_j}(m_j) \begin{bmatrix} G_j^{(1)}(ze^{i\alpha_j})G_j^{(1)}(\frac{1}{z^*})^* & \cdots & G_j^{(1)}(ze^{i\alpha_j})G_j^{(K)}(\frac{1}{z^*})^* \\ \vdots & & \vdots \\ G_j^{(K)}(ze^{i\alpha_j})G_j^{(1)}(\frac{1}{z^*})^* & \cdots & G_j^{(K)}(ze^{i\alpha_j})G_j^{(K)}(\frac{1}{z^*})^* \end{bmatrix} \quad (17)$$

where $G_j^{(p)}(\frac{1}{z^*})^* = [G_j^{(p)}(\frac{1}{z^*})]^*$.

In (17), considering the diagonal entries of both sides, it follows that

$$\mathbf{S}_y(m_j,z)_{(p,p)} = c_{s_j}(m_j) G_j^{(p)}(ze^{i\alpha_j})[G_j^{(p)}(\frac{1}{z^*})]^*, \quad (18)$$

where $j = 1, \cdots, M, p = 1, \cdots, K$. Change variables $z \leftrightarrow e^{-i\alpha_j}z$, $z \leftrightarrow e^{-i2\alpha_j}/z^*$ in (18), we get

$$\mathbf{S}_y(m_j, e^{-i\alpha_j}z)_{(p,p)} = c_{s_j}(m_j) G_j^{(p)}(z)[G_j^{(p)}(\frac{e^{-i\alpha_j}}{z^*})]^*, \quad (19)$$

$$\mathbf{S}_y^*(m_j, \frac{e^{-i2\alpha_j}}{z^*})_{(p,p)} = c_{s_j}^*(m_j)[G_j^{(p)}(\frac{e^{-i\alpha_j}}{z^*})]^* G_j^{(p)}(e^{-i2\alpha_j}z). \quad (20)$$

It then follows that

$$\begin{aligned} & \mathbf{S}_y(m_j, e^{-i\alpha_j}z)_{(p,p)} \; c_{s_j}^*(m_j) \; G_j^{(p)}(e^{-i2\alpha_j}z) \; z^{-(L-1)} \\ =& \mathbf{S}_y^*(m_j, \frac{e^{-i2\alpha_j}}{z^*})_{(p,p)} \; c_{s_j}(m_j) \; G_j^{(p)}(z) \; z^{-(L-1)}, \quad (21) \end{aligned}$$

where $z^{-(L-1)}$ was introduced to ensure the polynomials on both sides causal.

Since the multiplication of two polynomials is essentially the convolution of two coefficient sequences, and convolution can be represented in matrix form using Toeplitz matrix. For an arbitrary polynomial $A(z) = \sum_{i=0}^{L_a-1} a(i)z^{-i}$, and an integer $\widetilde{L} > 0$, we associate with $A(z)$ the $(\widetilde{L}+L_a) \times (\widetilde{L}+1)$ Toeplitz matrix $\mathcal{T}_a(\widetilde{L})$ with the first column as $[a(0), a(1), a(L_a-1), 0, \cdots, 0]'$ and the first row as $[a(0), 0, \cdots, 0]$.

Let $\mathcal{T}_{s_1}(L-1)$ and $\mathcal{T}_{s_2}(L-1)$ denote the $(3L-2) \times L$ Toeplitz matrix associated with the $2(L-1)$th-order polynomials $c_{s_j}^*(m_j)z^{-(L-1)}\mathbf{S}_y(m_j, e^{-i\alpha_j}z)_{(p,p)}$ and $c_{s_j}(m_j)z^{-(L-1)} *$ $\mathbf{S}_y^*(m_j, \frac{e^{-i2\alpha_j}}{z^*})_{(p,p)}$ respectively. Define

$$\mathbf{D}_{\alpha_j}(\widetilde{L}) = \begin{bmatrix} 1 & & & \\ & e^{i2\alpha_j} & & \\ & & \ddots & \\ & & & e^{i2\alpha_j\widetilde{L}} \end{bmatrix},$$

and let $\mathbf{g}_j^{(p)} = [g_j^{(p)}(0), \cdots, g_j^{(p)}(L-1)]$ denote the coefficient vector of $G_j^{(p)}(z)$, actually $\mathbf{g}_j^{(p)}$ is the channel impulse response from user j to the pth receive antenna. Let $\mathbf{g}_{\alpha_j}^{(p)}$ be the coefficient vector of $G_j^{(p)}(e^{-i2\alpha_j}z)$, then

$$\mathbf{g}_{\alpha_j}^{(p)} = \mathbf{D}_{\alpha_j}(L-1)\mathbf{g}_j^{(p)}. \qquad (22)$$

From (21) it follows that

$$\mathcal{T}_{s_1}(L-1)\mathbf{g}_{\alpha_j}^{(p)} = \mathcal{T}_{s_2}(L-1)\mathbf{g}_j^{(p)}. \qquad (23)$$

Combine (22) and (23), we have

$$(\mathcal{T}_{s_1}(L-1)D_{\alpha_j}(L-1)-\mathcal{T}_{s_2}(L-1))\mathbf{g}_j^{(p)} \triangleq \mathcal{T}_{\alpha_j}(L-1)\mathbf{g}_j^{(p)} = 0. \qquad (24)$$

From (24), it can be seen that $\mathbf{g}_j^{(p)}$ **is uniquely identifiable if the null space of $\mathcal{T}_{\alpha_j}(L-1)$ is of rank one, i.e.**

$$\mathbf{dim}\{Null(\mathcal{T}_{\alpha_j}(L-1))\} = 1.$$

In [1], it was shown that $\dim\{Null(\mathcal{T}_{\alpha_j}(L-1))\} = 1$ if and only if

$$e^{i2\alpha_j l} \neq 1 \text{ for } l = 1, \cdots, L-1. \qquad (25)$$

It was also pointed out that a necessary condition for α_j to satisfy (25) is the period of the code sequence $c_j(n)$ is greater or equal to the channel length. In our case, the period is $2N$, thus $2N \geq L$ is a necessary condition for channel identifiability. Under this condition, choose $\alpha_j = \pi j/N$ with $gcd(j,N) = 1$, then condition (25) is fulfilled.

Similarly, if equation (18) holds for two-cycles $m_{j_1}, m_{j_2}(j_1 \neq j_2)$, then we have

$$\frac{\mathbf{S}_y(m_{j_1},z)_{(p,p)}}{\mathbf{S}_y(m_{j_2},z)_{(p,p)}} = \frac{c_{s_{j_1}}(m_{j_1}), G_j^{(p)}(ze^{i\alpha_{j_1}})}{c_{s_{j_2}}(m_{j_2})G_j^{(p)}(ze^{i\alpha_{j_2}})} \qquad (26)$$

In this case, $\mathbf{g}_j^{(p)}$ is uniquely identifiable if and only if $e^{i(\alpha_{j_1}-\alpha_{j_2})} \neq 1$ for $l = 1, \cdots, L-1$.

After the channel estimation, equalization/desired user extraction can be carried out using a MMSE cyclic Wiener filter. Therefore, periodic non-constant-modulus precoding makes multiuser detection possible without excess bandwidth requirement.

4. SIMULATION EXAMPLE

We consider the case of two users (i.e. two transmit antennas) and two receive antennas. Each user transmit QPSK signals with a processing gain of $N = 8$, The channelization code sequences spread over two symbols, chosen to be

$$\mathbf{c}_1 = [0.6935, 0.4449, 0.6247, 0.4273, 0.5549, 0.2370, 0.47490.2021, 0.3671, 0.2021, 0.4749, 0.2370, 0.5549, 0.4273, 0.6247, 0.4449]$$
$$\mathbf{c}_2 = [0.6828, 0.4558, 0.2234, 0.4495, 0.6051, 0.4989, 0.34590.4932, 0.5159, 0.4932, 0.3459, 0.4989, 0.6051, 0.4495, 0.2234, 0.4558]$$

The multipath channels have 3 rays, the multipath amplitudes are Gaussian with zero mean and identical variance, the tranmission delays uniformly spread over 4 chip intervals, and $p(t)$ is the raised-cosine pulse with roll-off factor $\beta = 0.22$. That is,

$$g_j^{(p)}(t) = \sum_{k=0}^{2} a_j^{(p)}(k)p(t-\tau_k) \qquad (27)$$

where $j = 1, 2$ and $p = 1, 2$. Complex zero mean white Gaussian noise was added to the received signals. Assumed that User 1 is the desired user. The normalized mean-square-error (MSE)of channel estimates for the desired user is defines as

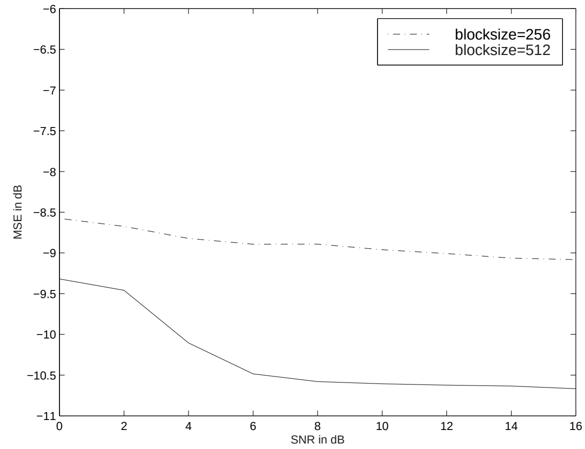

Fig. 3. Normalized MSE of channel estimation versus SNR

$$MSE = \frac{1}{2IL}\sum_{i=1}^{I}\sum_{p=1}^{2}\frac{\|\hat{\mathbf{g}}_1^{(p)} - \mathbf{g}_1^{(p)}\|^2}{\|\mathbf{g}_1^{(p)}\|^2} \qquad (28)$$

where I stands for the number of Monto Carlo runs. And SNR refers to the signal-to-noise ratio with respect to the desired user and is chosen to be the same at both receivers. The result is averaged over $I = 100$ Monto Carlo runs, the channel is randomly generated in each run and the system is simulated for a block size (i.e. the number of symbols users per run) of 256 and 512 respectively.

5. REFERENCES

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