



# AN IMPROVED FEEDBACK SCHEME FOR DUAL CHANNEL IDENTIFICATION IN WIRELESS COMMUNICATION SYSTEMS

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## ABSTRACT

In an earlier paper we had presented a novel dual channel identification approach for mobile wireless communication systems. Unlike traditional channel estimation methods that rely on training symbols, this approach used a bent-pipe feedback mechanism requiring the mobile station (MS) to send portions of its received signal back to the Base Station (BS) for wireless channel identification. Using a filter-bank decomposition concept, we introduced an effective algorithm for identifying both the forward and the reverse channels based only on this feedback information. This new method permits transfer of computational burden from the MS to the resource rich BS and leads to significant savings in bandwidth consuming training signals. This paper proposes a more informative feedback method leading to significant performance improvement over our earlier scheme.

## 1. INTRODUCTION

Two important tasks in mobile wireless communications systems are channel estimation, and compensation aided by frequent transmission of training signals. In most future cellular systems the forward link, carrying data from Base Stations (BS) to a Mobile (MS), will support higher data rates than the reverse link. Consequently, the estimation and compensation of the Forward Link Channel (FLC) requires more resources and longer training sequences than that of the Reverse Link Channel (RLC). Equally, the current practice is to assign the compensation and estimation of the FLC entirely to the MS, which generally has less computational reserves than the BS.

To permit the resource rich BS to share in the compensation of the FLC, and to reduce the bandwidth consuming training of the FLC, in [2] we proposed a new approach to the estimation and compensation of the FLC in mobile wireless communication systems using a novel *bent pipe feedback* mechanism. In principle, this feedback mechanism enables the BS to estimate and compensate both the FLC the RLC, without any training signals on either link or resort to blind estimation techniques. While practical realities temper these expectations, as we demonstrated in [2], this idea has significant advantages.

Specifically, the approach of [2] requires that the MS feed back to the BS a portion of the received signal, over the time slot conventionally reserved for RLC training. Clearly, this permits the BS

to estimate the Roundtrip Channel (RTC). *However, the key novelty of our approach lies in the following discovery: By feeding back only a portion, rather than the entire received signal, one empowers the BS to identify both the FLC and the RLC from the roundtrip feedback signal alone.* This novel channel feedback does not require high speed reverse links and naturally accommodates asymmetric data link structures, and structures where the RLC and FLC have different carrier frequencies. Furthermore, no additional training signals are necessary for estimating the RLC at the BS, though some training for synchronization will still be needed.

As the BS will miss changes in the FLC that occur within feedback latency, the MS must estimate and compensate the residual ISI in the channel dynamics the BS cannot compensate. Over reasonable distances and mobile speeds these changes are modest enough to make the partially precompensated FLC dynamics relatively mild. Thus, a 5km roundtrip causes a feedback delay of  $16.67 \mu s$ , a time span over which the FLC undergoes little change. This is underscored by the fact that in GSM each data frame has a duration of  $557 \mu s$ , and training occurs only once per data frame. Thus the channel variation within the resolution of this delay occurs mainly because of Doppler effect. Yet a vehicle traveling at 100km/hr suffers a maximum Doppler shift of 55 Hz in the cellular band; a shift not large enough to cause drastic changes in the FLC characteristics over latencies of tens of microseconds. Thus the residual ISI that must be equalized at the receiver will be significantly milder leading to the need for much shorter training sequences on the FLC. Given that no training for estimation is needed on the RLC, and that feedback data occupies the RLC training slot used in conventional communication, this implies substantial savings in the bandwidth devoted to the overall training, and a significant transfer of the FLC compensation/estimation burden to the BS. Simulations presented in [2] support this contention. This scheme also permits the use of such adaptive coding at the BS as has been advocated by several authors [3]- [6].

The scheme of [2] is preliminary in nature. One of its disadvantages is that it fails to make as efficient a use of the feedback slot as is desirable. In particular a large fraction of the feedback slot carries zero samples that do not contain useful information about the FLC. The key contribution of this paper is to formulate a *more informative feedback scheme that carries more information about the FLC leading to improved FLC estimation.*

## 2. THE FEEDBACK SCHEME

For the most part in this paper we assume that the ratio of the data rates supported on the FLC and RLC is  $M/L$ , with  $M > L$ . Later

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we will comment on how to accomodate the case of  $M \leq L$ . In fig. 1  $H(z)$  and  $G(z)$  are the discrete time baseband models of the FLC and RLC respectively,  $w_i(n)$  are the noise sequence at their output,  $x(n)$  and  $y(n)$  are respectively the data sequence transmitted and received by the BS. The samples  $s_1(n)$ , received at the MS are rate converted by the  $N$ -branch rate convertor, [1] that generates  $L$  samples for every  $M$  samples at its input, i.e. effects a rate conversion by a factor of  $L/M$ . The sequence  $s_2(n)$  is retransmitted over the RLC over the slots usually reserved for RLC training:  $u(k)$  models the interference caused by the normal RLC data because of imperfect synchronization.

In this arrangement  $N \leq L < M$ . Effectively, over sample lengths of  $L$ ,  $s_2(n)$  contains  $N$  out of every  $M$  samples of  $s_1(n)$ , and has in addition  $L - N$  zeros. The scheme in [2] uses *only the top branch of this arrangement*, i.e. has  $N = 1$ . Consequently in [2] out of every  $L$ -symbol feedback slot only one sample contains the data received at the MS, with the remaining  $L - 1$  symbols being zero samples. Thus in [2] the available feedback slot is under utilized as far as information exchange is considered. This causes important information to be unnecessarily discarded, reducing the ability to track time variations, resulting in larger residual ISI in the FLC compensated on the basis of the estimate at the BS. As we will demonstrate in Section 4, the more sophisticated rate convertor with  $N = L$ , leads to improved performance.

Consider the  $M$  and  $L$  fold type I and II polyphase decompositions of  $H(z)$  and  $G(z)$  respectively, i.e.  $H(z) = \sum_{i=0}^{M-1} E_i(z^M)z^{-i}$  and  $G(z) = \sum_{i=0}^{L-1} R_i(z^L)z^{-(L-i-1)}$ . Then, [1], absent noise and interference, Fig. 1 can be transformed into Fig. 2, where  $\mathbf{R}(z)$  and  $\mathbf{E}(z)$  are respectively, left and right pseudocirculant matrices given by

$$\mathbf{R}(z) = \begin{pmatrix} R_0(z) & R_1(z) & \cdots & R_{N-1}(z) \\ R_1(z) & R_2(z) & \cdots & R_N(z) \\ \vdots & \vdots & \vdots & \vdots \\ R_{L-1}(z) & z^{-1}R_0(z) & \cdots & z^{-1}R_{N-2}(z) \end{pmatrix} \quad (1)$$

and  $\mathbf{E}(z) =$

$$\begin{pmatrix} E_0(z) & E_1(z) & \cdots & E_{M-1}(z) \\ z^{-1}E_{M-1}(z) & E_0(z) & \cdots & E_{M-2}(z) \\ \vdots & \vdots & \vdots & \vdots \\ z^{-1}E_{M-N+1}(z) & z^{-1}E_{M-N+2} & \cdots & E_{M-N} \end{pmatrix}. \quad (2)$$

Define

$$E(z) = [E_0(z), \dots, E_{M-1}(z)] \quad (3)$$

$$R(z) = [R_0(z), \dots, R_{L-1}(z)]^T. \quad (4)$$

Define in Fig.2  $\mathbf{Y}(\mathbf{z}) = [Y_1(z), \dots, Y_L(z)]^T$  and  $\mathbf{X}(\mathbf{z}) = [X_1(z), \dots, X_M(z)]^T$  where  $X_i(z)$  and  $Y_i(z)$  are the  $z$ -transform of  $x_i(n)$  and  $y_i(n)$ . Then we have the following relation

$$\mathbf{Y}(\mathbf{z}) = \mathbf{R}(z)\mathbf{E}(z)\mathbf{X}(\mathbf{z}). \quad (5)$$

Since  $X_i(z)$  and  $Y_i(z)$  are known to BS, the round trip channel  $\mathbf{R}(z)\mathbf{E}(z)$  can be estimated. The question is, under what conditions can one extract  $H(z)$  and  $G(z)$  from  $\mathbf{R}(z)\mathbf{E}(z)$ . Clearly the best one can hope for is to estimate  $\mathbf{R}(z)$  and  $\mathbf{E}(z)$  to within a scaling constant and common delays among the  $E_i(z)$  and  $R_i(z)$ . In the case of [2], such an extraction is possible if either (not necessarily both) of the following conditions apply.

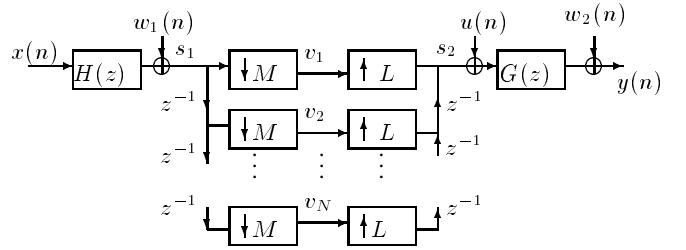


Fig. 1. System model of improved scheme: Rate changer with  $N$  branches.

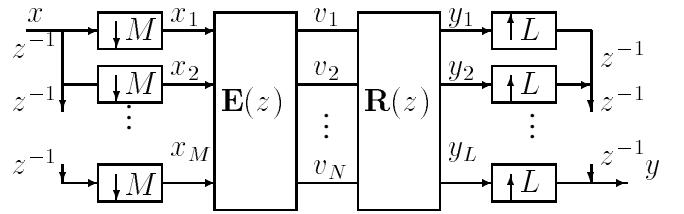


Fig. 2. Polyphase Representation.

**Assumption 1** *The greatest common divisor (gcd) of the set of polynomials  $R_i(z)$  is a pure delay  $z^{-d}$  ( $d$  integer). Further their maximum order  $l_R$  is known.*

**Assumption 2** *The gcd of set of the set of polynomials  $E_i(z)$  is a pure delay  $z^{-d}$  ( $d$  integer). Further their maximum order  $l_E$  is known.*

To see why, observe that in the setting of [2], i.e.  $N = 1$ , (5) is replaced by  $\mathbf{Y}(\mathbf{z}) = R(z)E(z)\mathbf{X}(\mathbf{z})$ . Thus the rank-1 matrix  $R(z)E(z)$  can be estimated. Observe the  $k$ -th row of  $R(z)E(z)$  is simply,  $R_k(z)[E_0(z), \dots, E_{M-1}(z)]$ . Under Assumption 2, the gcd of the elements of this row provides to within a delay and scaling,  $R_k(z)$  and hence also  $E_i(z)$  and  $H(z)$ . Similar unraveling is possible should Assumption 1 hold.

Observe in the setting of this paper the rank-1 matrix  $R(z)E(z)$  is not directly available. Yet in the next two sections, we show that under either Assumption 1 or 2,  $H(z)$  and  $G(z)$  can be obtained to within a scaling and delay from the the roundtrip dynamics captured by  $\mathbf{R}(z)\mathbf{E}(z)$ .

### 3. PROOF OF IDENTIFIABILITY

In this section we show that  $E(z)$  is identifiable to within a scaling constant, from  $\mathbf{R}(z)\mathbf{E}(z)$ , when  $M > L \geq N$ , and assumption 2 holds with the common delay  $d$  among the  $E_i$  equalling zero. In section 3.3, we discuss the case where this common delay is nonzero. The knowledge of  $E(z)$  provides  $H(z)$ . A similar result can be formulated when assumption 1 holds or when  $L \geq M > N$  or when  $L = M > N$ . *Thus even the case of  $L = M$  can be captured.* In each case the selection of  $N$  ensures that some received signal is discarded and  $s_2(n) \neq s_1(n - k)$ .

#### 3.1. Definitions and notations

For an  $M \times N$  polynomial matrix  $A(z) = \sum_{i=0}^l A(i)z^{-i}$ , where  $l$  is the degree of  $A(z)$ , define the  $mM \times (l+m)N$  generalized

Sylvester matrix of  $A(z)$  as

$$\mathcal{T}_m(A) = \begin{pmatrix} A(0) & \cdots & A(l) \\ & \ddots & \ddots & \ddots \\ & & A(0) & \cdots & A(l) \end{pmatrix}. \quad (6)$$

For the  $r \times Mm$  matrix  $B = [B(0), \dots, B(m-1)]$ , define

$$\mathcal{P}_{m,M}(B) = \sum_{i=0}^{m-1} B(i)z^{-i}. \quad (7)$$

where  $B(i)$  and  $\mathcal{P}_{m,M}(B)$  have dimension  $r \times M$ . Note that  $\mathcal{P}_{m,M}(B)$  is a function of  $z$ .

Define an  $M \times M$  polynomial matrix  $Q_M(z)$  as

$$Q_M(z) = \begin{pmatrix} \mathbf{0}_{(M-1) \times 1} & \mathbf{I}_{M-1} \\ z^{-1} & \mathbf{0}_{1 \times (M-1)} \end{pmatrix} \quad (8)$$

where  $\mathbf{I}_{M-1}$  denotes  $(M-1) \times (M-1)$  identity matrix;  $\mathbf{0}_{m \times n}$  denotes the  $m \times n$  zero matrix.

### 3.2. Identifiability

With  $\mathbf{C}(z) = \mathbf{R}(z)\mathbf{E}(z)$ ,

$$\mathcal{T}_m(\mathbf{C}^T) = \mathcal{T}_m(\mathbf{E}^T)\mathcal{T}_{l_E+1+m}(\mathbf{R}^T) \quad (9)$$

Whenever  $G(z)H(z) \neq 0$ ,  $\mathcal{T}_m(\mathbf{R}^T)$  and  $\mathcal{T}_m(\mathbf{E})$  are full rank for all integers  $m > 0$ . Hence  $\mathcal{T}_m(\mathbf{C}^T)$  and  $\mathcal{T}_m(\mathbf{E}^T)$  have identical left nullspaces. Thus the knowledge of  $\mathbf{C}(z)$  provides the left nullspace of  $\mathcal{T}_m(\mathbf{E}^T)$ . The following theorem shows that the left null space of  $\mathcal{T}_m(\mathbf{E}^T)$  under assumption 2 provides  $H(z)$  to within a scaling.

**Theorem 1** Suppose  $E(z)$  and  $\mathbf{E}(z)$  are defined in (3) and (2) respectively and assumption 2 is in force, with  $d = 0$  and  $M > L \geq N$ . Then for any integer  $m > Nl_E + N - 1$ ,  $\mathcal{T}_m(\mathbf{E}^T)$  has a nontrivial left null space. Suppose  $B$  is a matrix whose rows span the left nullspace of  $\mathcal{T}_m(\mathbf{E}^T)$ . Let

$$\mathbf{B}(z) = [(\mathcal{P}_{m,M}(B))^T, \dots, Q_M^{N-1}(\mathcal{P}_{m,M}(B))^T]. \quad (10)$$

Consider an  $M$ -dimensional nonzero polynomial row vector  $\hat{E}(z)$  with degree  $l_E$ . Then

$$\mathcal{T}_1(\hat{E})\mathcal{T}_{l_E}(\mathbf{B}) = 0 \text{ iff } \hat{E}(z) = cE(z) \quad (11)$$

where  $c$  is a nonzero constant.

Thus indeed the left null space of  $\mathcal{T}_{l_E}(\mathbf{B})$ , constructed from the left nullspace of  $\mathcal{T}_m(\mathbf{C}^T)$ , provides  $E(z)$  to a scaling.

### 3.3. A Subspace Algorithm

Assume assumption 2 holds with  $d = 0$ . Suppose the signals  $x(n)$ ,  $u(n)$ ,  $w_1(n)$  and  $w_2(n)$  are zero mean, white and mutually uncorrelated. In view of the noise free model of (5), and the knowledge of  $x(n)$  and  $y(n)$  at the BS, a standard least squares scheme provides an estimate of  $\mathcal{T}_m(\mathbf{C}^T)$  and hence its SVD:

$$\mathcal{T}_m(\mathbf{C}^T) + \mathcal{N} = (V_s \ V_n) \begin{pmatrix} \Sigma_s & 0 \\ 0 & \Sigma_n \end{pmatrix} \begin{pmatrix} W_s^H \\ W_n^H \end{pmatrix} \quad (12)$$

Find  $B$  whose rows span the left nullspace of  $\mathcal{T}_m(\mathbf{C}^T)$ , where  $m > Nl_E + N - 1$ , and construct  $\mathbf{B}(z)$  defined in (10). Because of the assumption of lack of correlation between  $x(n)$  and the noise and interference,  $B$  is provided by  $V_n$ . Then solving for  $\mathcal{T}_1(\hat{E})$  as the eigenvector corresponding to the smallest eigenvalue of  $\mathcal{T}_{l_E}(\mathbf{B})\mathcal{T}_{l_E}(\mathbf{B})^H$ , where  $(\cdot)^H$  indicates transpose conjugate, provides  $E(z)$ . Since  $\mathcal{T}_{l_E+1}(\mathbf{E})$  has full row rank one finds  $G(z)$  also to a scaling constant, using

$$\mathcal{T}_1(\mathbf{R}) = \mathcal{T}_1(\mathbf{C})(\mathcal{T}_{l_E+1}(\mathbf{E}))^\dagger.$$

If  $E_i(z)$  have a common delay then this manifests in certain columns of  $\mathcal{T}_m(\mathbf{C}^T)$  being zero. Then applying the above procedure on the matrix with these zero columns removed provides  $H(z)$  and  $G(z)$  to within a scaling and a delay.

## 4. SIMULATIONS

We present two simulation examples. The first example shows the basic performance of the scheme in this paper relative to that of [2]. The second example illustrates the reduction of training levels needed on the FLC when channel parameters change with time.

### 4.1. Simulation I

In the simulation, FLC and RLC are generated from two delayed raised-cosine pulse  $C(t, \alpha)$ , where  $\alpha$  is the roll-off factor.  $C(t, \alpha)$  is limited in 8T for FLC and in 6T for RLC. The FLC and RLC have the respective analog models:  $0.3C(t, 0.25) + 0.8C(t - T/2, 0.25)$  and  $0.5C(t, 0.10) + 0.6C(t - T/3, 0.10)$ . We use downsampling factor  $M = 3$  and upsampling factor  $L = 2$  and  $N = L$ . Noises  $w_1(n)$  and  $w_2(n)$  are zero mean and have the same variance. The input signal  $x(n)$  is i.i.d BPSK. To obtain a performance measure of channel estimation, we define the normalized root-mean-square error (NRMSE) as

$$\text{NRMSE} = \frac{1}{\|h\|_2} \sqrt{\frac{1}{M_t} \sum_{i=1}^{M_t} \|\hat{h}_{(i)} - h\|_2^2} \quad (13)$$

where  $M_t$  is the number of Monte Carlo runs;  $h$  is the actual channel and  $\hat{h}_{(i)}$  is the  $i$  estimation. In our simulation  $M_t = 100$ . In each run 600 symbols are used. We call the scheme in [2] old scheme and the scheme in this paper new scheme. Fig. 3 shows NRMSE versus input SNR, where input SNR is defined as

$$\text{Input SNR} = 10 \log 10 \text{E}(x^2(n)) / \text{E}(w_1^2(n)) \quad (14)$$

A zero-forcing preequalizer is constructed for FLC and a post equalizer for the RLC, on the basis of the FLC and RLC estimates. Both are housed at the BS. Equalizer SNR for the FLC, is computed using the signal power at the FLC input. For RLC, it uses the signal power at the equalizer output. BER versus equalizer SNR is displayed in Fig. 4, where the input symbol is BPSK signal. Both figures show the performance improvement due to the more informative feedback of this paper.

### 4.2. Simulation II: Reduced training for a time varying channel

We use COST-207 Typical Urban(TU) [7] model with 100 echo paths, BPSK data and maximum Doppler frequency 55Hz. We assume the channels to be quasistatic, i.e., time-invariant in one frame and time-variant from frame to frame. The receive filters

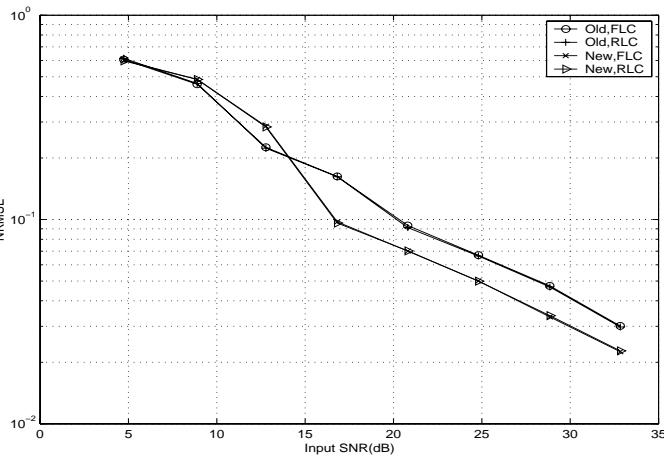


Fig. 3. NRMSE versus input SNR for both schemes

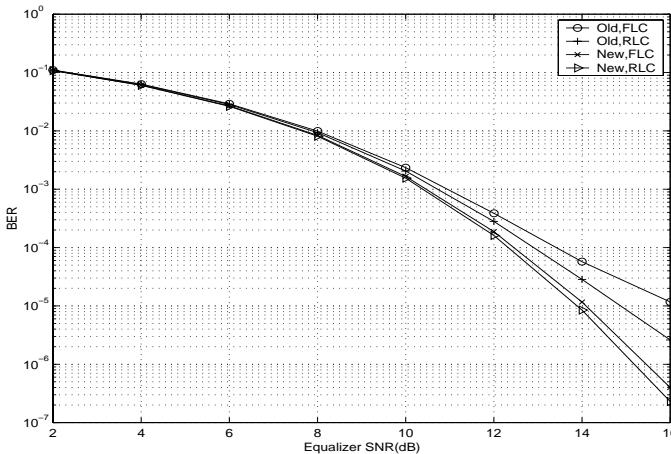


Fig. 4. BER versus equalizer SNR for both schemes.

for FLC and RLC are raised cosine functions with roll-off factors 0.2 and 0.1 respectively. The FLC sustains a data rate of 1 Mbps, and the RLC supports 0.667 Mbps. We use downsampling factor  $M = 3$  and upsampling factor  $L = 2$ . The schemes with  $N = 1$  and  $N = 2$  are called old scheme and new scheme respectively. In both cases we compare two settings:

- Training aided equalization of FLC at the MS and of the RLC at the BS, with no feedback.
- No training on the RLC, but instead sending feedback data of the same length as the RLC training data in (a). A precorrector, obtained by the new scheme or the old scheme is used on the FLC and is augmented by a post-equalizer estimated at the receiver using reduced training.

Methods in (a) and (b) use the same signal power at the FLC input. Fig. 5 shows  $n_b/n_t$  versus input SNR for methods in (a) and (b) to achieve the same BER. Here  $n_t$  is the length of the FLC training sequence used in (a) and  $n_b$  the length of training used in (b), so that the same FLC BER is obtained in both cases. As is evident in Fig. 5, in order to achieve the same BER as the conventional method, the new scheme needs less training data and

thus saves more bandwidth than the old scheme when input SNR > 6dB. By way of further comparison we note that at 18dB SNR, while the training sequence length in [2] is 30% of (a), the length for the new scheme is only 15%, i.e. half that needed by [2].

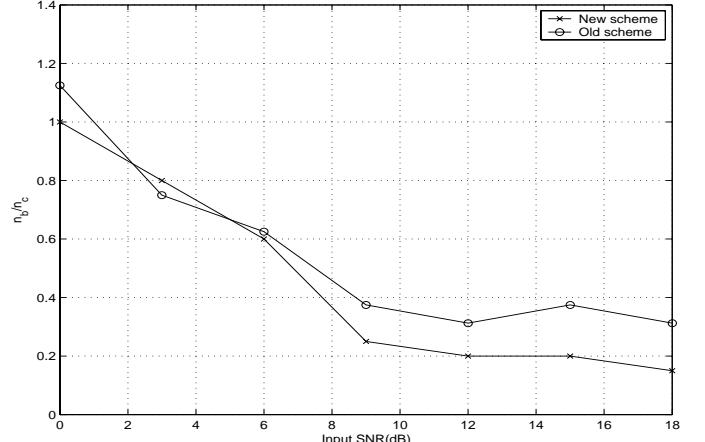


Fig. 5.  $n_b/n_t$  versus SNR at the same BER.

## 5. CONCLUSION

In this paper, a bent pipe multi-branch feedback scheme is used to estimate FLC and RLC from RTC. By exploiting the properties of the nullspace of pseudocirculant matrices, the identifiability result and unravelling method are derived for this improved scheme. Since this improved scheme uses more information from FLC, we get performance improvement compared to the scheme in [2].

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