MULTI-USER SPACE-TIME CODING IN COOPERATIVE NETWORKS

Paul A. Anghel[1], *Geert Leus*[2] and *Mostafa Kaveh*[1]

[1] Dept. of ECE, Univ. of Minnesota, 200 Union St. SE, Minneapolis, MN 55455, USA
[2] Dept. of Elec. Engr., Katholieke Universiteit Leuven, Kasteelpark Arenberg 10, 3001 Leuven, Belgium
Email: {panghel,kaveh}@ece.umn.edu, geert.leus@esat.kuleuven.ac.be

ABSTRACT

Multiple antennas at the receiver and transmitter are often used to combat the effects of fading in wireless communication systems. However, implementing multiple antennas at the mobile stations is impractical for most wireless applications due to the limited size of the mobile unit. In this paper we emulate spatial diversity using mobile relay stations, which cooperate by retransmitting the information received from a mobile station to a destination station. We propose an Alamouti based cooperative system with two relay stations and we provide an approximate formula for the average symbol error probability of this system in a Rayleigh fading environment.

1. INTRODUCTION

There is an increasing trend in the development of communication systems that allow their users to communicate "anywhere and anytime" at high data rates. Wireless networks have the potential to offer this ubiquitous high-rate communication among mobile users. A wireless network is a collection of mobile terminals that are capable of transmitting and receiving information using wireless multiple access protocols. Because the terminals in the network are mobile, communication among the terminals suffers from time-varying fading, which frequently reduces the signal level making it difficult or sometimes impossible to recover the transmitted information. In order to combat fading in wireless networks we allow cooperation among the terminals in the network [1, 2, 3].

The idea of increasing the throughput of a system using cooperation among users has been first introduced in [1, 4] for a cellular environment. The main idea is that after selecting a partner from the in-cell mobile users, each user detects a faded and noisy version of the partner's transmitted signal and combines this information with its own information data to construct its transmitted signal. It has been shown that in a flat fading environment the code division multiple access (CDMA) cooperative system of [4] achieves a higher throughput than the regular CDMA system. Instead of detecting and regenerating the cooperative signal, a simple amplification of partner's cooperative signal results in a similar performance improvement as it is illustrated in [5]. Shadowing effects have been considered in [3], where an approximate formula has been provided for the outage probability of a cooperative system in Rayleigh fading environment with lognormal shadowing.

In this paper we extend the cooperative system of [5, 6] by implementing a distributed Alamouti space-time coding system based on multi-user cooperation. We establish approximate formulas for the average symbol error probability of this system in a Rayleigh fading environment, which help us illustrate the performance improvement over the system in [5].

This work was supported in part by the National Science Foundation under the Wireless Initiative Program, Grant #9979443

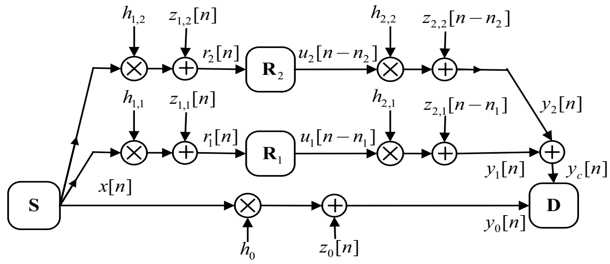

Fig. 1. Multi-relay discrete-time baseband equivalent channel

Distributed diversity systems based on user cooperation achieve diversity order equal with the overall number of transmit antennas in the system (i.e., full transmit diversity) [6]. The advantage over the point-to-point multi-antenna systems is that distributed systems with *one antenna* at each mobile could be used to facilitate transmit diversity in cases when the number of antennas per user is restricted.

2. SYSTEM MODEL

The cooperative network analyzed here uses K possibly idled mobile users R_k, $k \in [1, K]$, to relay the information transmitted by a terminal S, to the destination terminal D. The relays $\{R_k\}_{k=1}^{K}$ can decode and regenerate the received signal (regenerative system), or they can just amplify the received signal with a gain (non-regenerative system) [5, 6]. Furthermore, we assume the relays can perform simple operations on the resulting signals, which do not necessarily require regeneration of the information symbols, like delaying and conjugating. In order to reduce the size of the terminals, it is preferable to limit future cooperative systems, regenerative or non-regenerative, to only *one antenna* per terminal. In order to allow the relays to receive and transmit in the same time on a single antenna, we assume two orthogonal signal subspaces for the received and the transmitted signals, e.g., two different frequency bands. Unlike the relays that can only receive one of the signal subspaces, we assume the destination terminal D can receive both signal subspaces.

For simplicity we only allow a one hop (i.e., the direct path from S to D) and K two-hop transmissions from S to D in our network, and we analyze the case when $K = 2$. We show in Fig. 1 the discrete-time baseband equivalent model of this multi-relay channel consisting of the three subchannels between the source and the destination. We assume that the transmissions suffer from the effects of slowly time-varying flat fading in order to write the two orthogonal signals received at the destination as

$$\begin{aligned} y_0[n] &= h_0\sqrt{\varepsilon_0}x[n] + z_0[n], \\ y_c[n] &= y_1[n] + y_2[n], \end{aligned} \tag{1}$$

where for $k \in \{1, 2\}$

$$y_k[n] := h_{2,k} u_k[n - n_k] + z_{2,k}[n], \qquad (2)$$

and where ε_0 is the transmitted symbol energy at terminal S, since we assume that the information bearing symbols $x[n]$'s are drawn from a constellation with unit energy, and $z_0[n]$, $z_{1,k}[n]$, $z_{2,k}[n]$ are additive noises. The processing and propagation delay at the kth relay is captured by $n_k \geq 1$. Let us assume that $n_1 = 1$. In order to avoid inter-symbol interference at the destination terminal we also assume that the time delay between the the two propagation paths containing a relay is negligible (i.e., $n_1 = n_2$). Nevertheless, the time delay between the direct path and the relayed paths would induce an inter-symbol interference channel (with length at least two) between the source and the destination if not for our choice of orthogonal transmissions. For non-regenerative systems, $u_k[n]$ is either $\alpha_k r_k[n] = \alpha_k (h_{1,k}\sqrt{\varepsilon_0}x[n] + z_{1,k}[n])$, or plus/minus its complex conjugate. For regenerative systems, $u_k[n]$ is either an estimate of $x[n]$, or plus/minus its complex conjugate. The effect of the slowly time-varying flat fading is captured by h_0, $h_{1,k}$, and $h_{2,k}$, which we assume to be:

a1) mutually independent complex Gaussian distributed variables with zero mean and variances Ω_0, $\Omega_{1,k}$, and $\Omega_{2,k}$, respectively.

We further assume that:

a2) the additive noises $z_0[n]$, $z_{1,k}[n]$, and $z_{2,k}[n]$ are mutually independent complex Gaussian distributed sequences with zero mean and variances N_0, $N_{1,k}$, and $N_{2,k}$, respectively.

With the fading realizations h_0, $h_{1,k}$, and $h_{2,k}$ as in a1), we find the signal to noise ratios (SNRs) per hop $\gamma_0 := \varepsilon_0|h_0|^2/N_0$, $\gamma_{1,k} := \varepsilon_0|h_{1,k}|^2/N_{1,k}$, and $\gamma_{2,k} := \varepsilon_k|h_{2,k}|^2/N_{2,k}$ to be independent and exponentially distributed with means

$$\bar{\gamma}_0 = \frac{\varepsilon_0\Omega_0}{N_0}, \quad \bar{\gamma}_{1,k} = \frac{\varepsilon_0\Omega_{1,k}}{N_{1,k}}, \quad \text{and} \quad \bar{\gamma}_{2,k} = \frac{\varepsilon_k\Omega_{2,k}}{N_{2,k}}, \qquad (3)$$

where ε_k is the average radiated energy per symbol at R_k.

For a non-regenerative system, an automatic gain control (AGC) front-end is required at the relay in order to prevent $r_k[n]$ from saturating the relay amplifier. Besides adjusting the input power to the amplifier, the AGC facilitates control on the relay's output power. Specifically, if we want to constrain the average radiated energy per symbol at the kth relay to be ε_k, a good choice is to adopt an AGC that employs

$$\alpha_k = \frac{h^*_{1,k}}{|h_{1,k}|}\sqrt{\frac{\varepsilon_k}{\varepsilon_0|h_{1,k}|^2 + N_{1,k})}}, \quad k \in \{1, 2\}. \qquad (4)$$

3. AN ALAMOUTI-BASED MULTI-USER SPACE-TIME DIVERSITY SYSTEM

In this section, we consider a specific implementation of the more general system described above. More specifically, we implement a system based on the Alamouti's space-time coding scheme. We then compare this Alamouti-based scheme with the distributed diversity system of [5].

3.1. The proposed system

In the non-regenerative multi-user space-time diversity (MSTD) system, terminal S broadcasts a block of two symbols $\mathbf{x}[i] := [x[2i], x[2i-1]]^{\mathrm{T}}$, which is received by the two relays $\{R_k\}_{k=1}^2$ (e.g., two nearby idled mobile users) and by terminal D. In the same time slot i, relay R_1 transmits the block $\mathbf{u}_1[i-1] := [\alpha_1 r_1[2i-2], -\alpha_1^* r_1^*[2i-3]]^{\mathrm{T}}$, and relay R_2 transmits the block $\mathbf{u}_2[i-1] := [\alpha_2 r_2[2i-3], \alpha_2^* r_2^*[2i-2]]^{\mathrm{T}}$, which corresponds to the Alamouti space-time coding scheme (see [7]) with the two transmit antennas distributed over two different terminals. The Alamouti receiver is used at the destination terminal D to processed the signals received from R_1 and R_2. In order to recover $\mathbf{x}[i-1]$ the output of the Alamouti receiver is combined with $[y_0[2i-2], y_0[2i-3]]^{\mathrm{T}}$ using a Maximum Ratio Combiner (MRC). Of course, processing of $\mathbf{x}[i]$ at terminal D has to be delayed by 2 symbols until the relays have transmitted $\mathbf{u}_1[i]$ and $\mathbf{u}_2[i]$. The average symbol error probability (ASEP) of the non-regenerative system depends on the distribution of the post-detection SNR at the destination terminal D. If we denote with $N_1 := |h_{2,1}\alpha_1|^2 N_{1,1} + N_{2,1}$ the power of the noise in $y_1[n]$, and with $N_2 := |h_{2,2}\alpha_2|^2 N_{1,2} + N_{2,2}$ the power of the noise in $y_2[n]$, the post-detection SNR at D is
$\gamma_t := \gamma_0 + \varepsilon_0 \left(\sum_{k=1}^2 |h_{2,k}\alpha_k h_{1,k}|^2\right)^2 / \sum_{k=1}^2 N_k|h_{2,k}\alpha_k h_{1,k}|^2$.
We find that

$$\gamma_t \leq \gamma_T := \gamma_0 + \gamma_1 + \gamma_2, \qquad (5)$$

where $\gamma_k := \varepsilon_0|h_{2,k}\alpha_k h_{1,k}|^2/N_k$, $k \in \{1, 2\}$, is the SNR in (2), and where equality holds for $N_1 = N_2$. Because the distribution of γ_t is not easily tractable, we prefer to find and use the distribution of γ_T to lower bound the ASEP of the non-regenerative system. As we will see from simulations this lower bound is extremely tight if we assume the same distribution for N_1 and N_2 i.e., $\Omega_{j,1} = \Omega_{j,2}$, $N_{j,1} = N_{j,2}$, $j \in \{1, 2\}$, and $\varepsilon_1 = \varepsilon_2$.

With the relay gain as in (4), it turns out $\gamma_k = \gamma_{1,k}\gamma_{2,k}/(\gamma_{1,k} + \gamma_{2,k} + 1)$, $k \in \{1, 2\}$. In order to find a lower bound for the performance of the space-time coding system, we first derive the moment generating functions (MGFs) of $\{\gamma_k\}_{k=1}^2$, which are

$$\begin{aligned}\mathcal{M}_{\gamma_k}(s) = &\ \frac{\overline{\gamma}_{s,k}\overline{\gamma}_{\delta,k} - 4\overline{\gamma}_{p,k}}{\overline{\gamma}_{\rho,k}^2} \\ &- \frac{s\overline{\gamma}_{p,k}(\overline{\gamma}_{\rho,k} + 2\overline{\gamma}_{p,k})}{\overline{\gamma}_{\rho,k}^3} e^{\frac{\overline{\gamma}_{\delta,k} - \overline{\gamma}_{\rho,k}}{2\overline{\gamma}_{p,k}}} E_1\left(\frac{\overline{\gamma}_{\delta,k} - \overline{\gamma}_{\rho,k}}{2\overline{\gamma}_{p,k}}\right) \\ &- \frac{s\overline{\gamma}_{p,k}(\overline{\gamma}_{\rho,k} - 2\overline{\gamma}_{p,k})}{\overline{\gamma}_{\rho,k}^3} e^{\frac{\overline{\gamma}_{\delta,k} + \overline{\gamma}_{\rho,k}}{2\overline{\gamma}_{p,k}}} E_1\left(\frac{\overline{\gamma}_{\delta,k} + \overline{\gamma}_{\rho,k}}{2\overline{\gamma}_{p,k}}\right),\end{aligned} \qquad (6)$$

for $\mathcal{R}e\{s\} < \overline{\gamma}_{s,k}/\overline{\gamma}_{p,k} + 2/\sqrt{\overline{\gamma}_{p,k}}$, where $\overline{\gamma}_{\rho,k} := \sqrt{\overline{\gamma}_{\delta,k}^2 - 4\overline{\gamma}_{p,k}}$, $\overline{\gamma}_{\delta,k} := \overline{\gamma}_{s,k} - \overline{\gamma}_{p,k}s$, $\overline{\gamma}_{s,k} := \overline{\gamma}_{1,k} + \overline{\gamma}_{2,k}$, and $\overline{\gamma}_{p,k} := \overline{\gamma}_{1,k}\overline{\gamma}_{2,k}$, $k \in \{1, 2\}$, and where $E_1(\cdot)$ is the exponential integral function defined as in [8, Eq. 5.1.1]. The proof of (6) is given in Appendix A.

Second, we use the MGF-based approach of [9, Eq. 9.21] along the independence of $\{\gamma_k\}_{k=0}^2$ in (5) to obtain the following closed-form lower bound on the ASEP of the non-regenerative MSTD system employing M-QAM modulation:

$$\begin{aligned}P_e^{(nr)} \geq &\ \frac{4}{\pi}\frac{\sqrt{M}-1}{\sqrt{M}}\left[\int_0^{\pi/2} \frac{\prod_{k=1}^2 \mathcal{M}_{\gamma_k}\left(\frac{-g_{\mathrm{QAM}}}{\sin^2\phi}\right)}{1 + \frac{g_{\mathrm{QAM}}\overline{\gamma}_0}{\sin^2\phi}} d\phi \right. \\ &\left. - \frac{\sqrt{M}-1}{\sqrt{M}} \int_0^{\pi/4} \frac{\prod_{k=1}^2 \mathcal{M}_{\gamma_k}\left(\frac{-g_{\mathrm{QAM}}}{\sin^2\phi}\right)}{1 + \frac{g_{\mathrm{QAM}}\overline{\gamma}_0}{\sin^2\phi}} d\phi\right],\end{aligned} \qquad (7)$$

where $g_{\mathrm{QAM}} := 3/[2(M-1)]$.

For the regenerative MSTD system, the communication scheme remains the same except for the relays, which now detect the information symbols received from terminal S. After the relays have regenerated the information symbols at time slot $i-1$, R_1 transmits $[\hat{x}[2i-2], -\hat{x}^*[2i-3]]^{\mathrm{T}}$, and R_2 transmits $[\hat{\hat{x}}[2i-3], \hat{\hat{x}}^*[2i-2]]^{\mathrm{T}}$, where $\hat{x}[n]$ and $\hat{\hat{x}}[n]$ are ML estimates of $x[n]$. The receiver at terminal D assumes perfect recovery of $x[n]$ at the relays. If $\{h_{1,k}\}_{k=1}^2$ and $\{N_{1,k}\}_{k=1}^2$ are communicated to D, an overall ML receiver can be used to recover $x[n]$ at D (see [4, 5]), but this case is not considered here.

In order to find a closed form expression for the ASEP of the regenerative MSTD system we approximate the S to D nonlinear subchannel that includes the regenerative relay R_k (i.e., the subchannel k) with an equivalent Rayleigh fading channel with the same individual ASEP. Hence, we end up with 3 Rayleigh fading subchannels between S and D, and we can use the approximation in [9, p.275] to obtain the overall ASEP of the system

$$P_e^{(r)} \approx \left(\frac{M}{M-1}\right)^2 \prod_{k=0}^{2} P_{e,k}, \tag{8}$$

where $P_{e,k}$ is the individual ASEP for subchannel k (subchannel 0 is the direct path from S to D).

In order to find $P_{e,k}$ we need to find the ASEP for a two-hop system that uses *only* subchannel k and undergoes two decoding processes, one at R_k and one at the destination D. For BPSK $P_{e,k} = P_{1,k} + P_{2,k} - 2P_{1,k}P_{2,k}$, where $P_{1,k}$ is the ASEP for transmissions between S and R_k, and $P_{2,k}$ is the ASEP for transmissions between R_k and D (see [10, Eq.(11.4.12)]). We can extend the result in [10, Eq.(11.4.12)]) by computing an exact expression of $P_{e,k}$ for an M-QAM modulation or we can use the following approximation:

$$P_{e,k} \approx P_{1,k} + P_{2,k} - P_{1,k}P_{2,k} - \frac{P_{1,k}P_{2,k}}{M-1}, \; k \in \{1,2\} \tag{9}$$

The last term in (9) compensates for the case when an error-free transmission is achieved between S and D even though errors occur between S and R_k and between R_k and D (i.e., two consecutive errors that cancel out).

3.2. The Distributed Diversity System of [5]

The cooperative design in [5] requires the assignment of orthogonal signal subspaces for every transmitting terminal in the network, e.g., a different frequency band is allocated to each terminal. Hence, when $K = 2$, we need to employ three frequency bands. In this case, the increase in bandwidth requirements is 1.5-fold compared with the Alamouti-based MSTD system. Therefore, $M^{1.5}$-QAM modulation has to be selected if bandwidth constraints are to be met.

The major drawback of the cooperative system of [5] is that the bandwidth increases with the number of relays. However, implementing a generalized orthogonal space-time block coding scheme at the relays only comes with a 2-fold bandwidth expansion. The price paid is an increase in delay at terminal D. This delay is at least equal to the number of relays times the symbol period.

4. PERFORMANCE COMPARISON

In order to check how well (7) and (8) approximate the performance of the non-regenerative system and the performance of the regenerative system respectively, for equal power allocation among transmitters, i.e., $\varepsilon_0 = \varepsilon_1 = \varepsilon_2$, unit power channels, i.e., $\Omega_0 = \Omega_{1,k} = \Omega_{2,k} = 1$, $k \in \{1,2\}$, and equal power noises, i.e., $N_0 = N_{1,k} = N_{2,k}$, $k \in \{1,2\}$, we have selected a 4-QAM modulation, and we have simulated both the regenerative and the non-regenerative system. We have plotted in Fig. 2 the average symbol error rate obtained by simulation along with the formulas in (7) and (8) versus E_s/N_0, where $E_s := \sum_{k=0}^{2} \varepsilon_k = 3\ \varepsilon_0$. We can see from Fig. 2 that the formula in (7) is an tight lower bound for the ASEP of the non-regenerative system. This is because with the same distribution for the noise powers N_1 and N_2 it is unlikely that $N_1 \gg N_2$ (or viceversa), and $\gamma_t \approx \gamma_T$ (i.e., the Alamouti receiver performs very close to an MRC receiver). We can also see

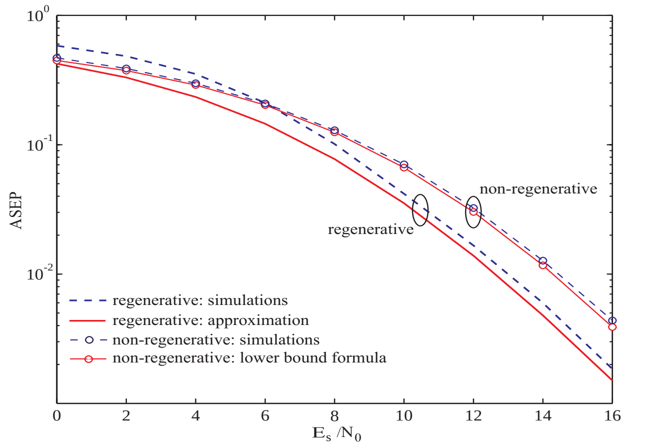

Fig. 2. Simulation results for a regenerative system with 4-QAM

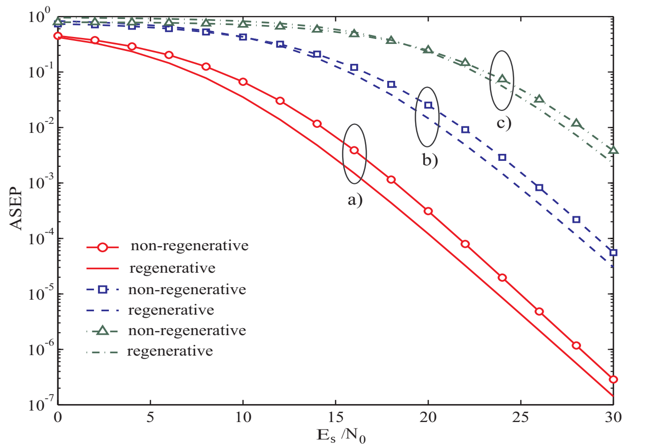

Fig. 3. a) benchmark b) Alamouti-based MSTD system c) the full FDMA cooperative system of [5].

from Fig. 2 that at high SNR, (8) approximates well the ASEP of the regenerative system. This can be explained by arguing that since fewer errors are made at the relay as the SNR available at the relay increases, the S to D subchannel k reduces to the Rayleigh fading channel between R_k and D. Consequently, at high SNR we end up with 3 independent Rayleigh fading channels between S and D.

To compare the ASEP of the MSTD system with the ASEP of the FDMA system in [5] we have selected the same parameters as above. However, in order to guarantee the same bandwidth for the two systems we have selected a 16-QAM modulation for the Alamouti-based MSTD system, and a 64-QAM modulation for the system in [5]. Using (7) and (8) we have plotted the ASEP of the two systems. As a benchmark, we also show the performance of the Alamouti-based MSTD system in case we can design two orthogonal signal subspaces without any bandwidth expansion, e.g., by using ideal polarized antennas. For this ideal benchmark case, we can use a 4-QAM modulation. We clearly notice the large improvement in performance of the proposed design.

APPENDIX A — PROOF OF (6)

In order to prove (6) we need to find the MGF of $\Gamma(X,Y) := XY/(X+Y+1)$, where X and Y are two independent and exponentially distributed random variables with mean $\overline{X}$ and $\overline{Y}$. From [11] we know the cumulative distribution function (CDF) of $\Gamma(X,YX)$ to be for $\gamma > 0$

$$F_\Gamma(\gamma) = 1 - \frac{2\sqrt{\gamma^2+\gamma}}{\sqrt{p}}\, e^{-\gamma\frac{\sigma}{p}} K_1\left(\frac{2\sqrt{\gamma^2+\gamma}}{\sqrt{p}}\right), \tag{10}$$

where $\sigma := \overline{X} + \overline{Y}$, $p := \overline{XY}$, and $K_\nu(\cdot)$ denotes the modified Bessel function of the second kind and order ν .

We take the derivative with respect to γ in (10) to obtain the probability density function (PDF) of $\Gamma(X,Y)$ and we use [8, Eq. 9.6.26] for the derivative of $K_1(\gamma)$ in order to establish the following PDF for $\Gamma(X,Y)$:

$$\begin{aligned} f_\Gamma(\gamma) = &\frac{4\gamma+2}{p}\, e^{-\gamma\frac{\sigma}{p}} K_0\left(\frac{2}{\sqrt{p}}\sqrt{\gamma^2+\gamma}\right) \\ &+ \frac{2\sigma\sqrt{\gamma^2+\gamma}}{p\sqrt{p}}\, e^{-\gamma\frac{\sigma}{p}} K_1\left(\frac{2}{\sqrt{p}}\sqrt{\gamma^2+\gamma}\right),\ \gamma > 0. \end{aligned} \tag{11}$$

We use the definition of the moment generating function along with (11) to write:

$$\begin{aligned} \mathcal{M}_\Gamma(s) = &\frac{2}{p}\int_0^\infty \Bigg[(2\gamma+1) e^{-\gamma\frac{\sigma-ps}{p}} K_0\left(\frac{2\sqrt{\gamma^2+\gamma}}{\sqrt{p}}\right) \\ &+ \frac{\sigma}{\sqrt{p}}\sqrt{\gamma^2+\gamma}\, e^{-\gamma\frac{\sigma-ps}{p}} K_1\left(\frac{2}{\sqrt{p}}\sqrt{\gamma^2+\gamma}\right)\Bigg] d\gamma. \end{aligned} \tag{12}$$

Using the change of variable $\gamma \to \gamma - 1/2$ and after some manipulations we obtain:

$$\mathcal{M}_\Gamma(s) = \frac{2}{p} e^{\frac{\alpha}{2}} \left[2\mathcal{J}_0(s) + \frac{\sigma}{\sqrt{p}}\mathcal{J}_1(s)\right], \tag{13}$$

where

$$\mathcal{J}_0(s) := \int_{1/2}^\infty \gamma e^{-\alpha\gamma} K_0\left(\beta\sqrt{\gamma^2 - (1/2)^2}\right) d\gamma \tag{14}$$

and

$$\mathcal{J}_1(s) := \int_{1/2}^\infty \sqrt{\gamma^2 - (1/2)^2}\, e^{-\alpha\gamma} K_1\left(\beta\sqrt{\gamma^2 - (1/2)^2}\right) d\gamma, \tag{15}$$

and where $\alpha := (\sigma - ps)/p,\ \beta := 2/\sqrt{p}$.

In order to simplify the integrand in (14), we write $\mathcal{J}_0(s)$ as:

$$\mathcal{J}_0(s) = \frac{-\partial}{\partial\alpha}\left[\int_{1/2}^\infty e^{-\alpha\gamma} K_0\left(\beta\sqrt{\gamma^2 - (1/2)^2}\right) d\gamma\right]. \tag{16}$$

To solve the integral in (16) we use [12, Eq. 6.646] to obtain

$$\begin{aligned} \mathcal{J}_0(s) = \frac{\partial}{\partial\alpha}\Bigg\{ &\frac{-1}{2\sqrt{\alpha^2-\beta^2}} \Bigg[e^{-\frac{\sqrt{\alpha^2-\beta^2}}{2}} E_1\left(\frac{\alpha - \sqrt{\alpha^2-\beta^2}}{2}\right) \\ &- e^{\frac{\sqrt{\alpha^2-\beta^2}}{2}} E_1\left(\frac{\alpha + \sqrt{\alpha^2-\beta^2}}{2}\right)\Bigg]\Bigg\}, \end{aligned}$$

and after differentiating with respect to α, and then letting $\alpha = (\sigma - ps)/p$ we find

$$\begin{aligned} \mathcal{J}_0(s) = &\frac{p\, e^{-\frac{\delta}{2p}}}{4\rho^3} \left[\delta(\delta+2p)\, e^{\frac{\delta-\rho}{2p}} E_1\left(\frac{\delta-\rho}{2p}\right)\right. \\ &\left. + \delta(\delta-2p)\, e^{\frac{\delta+\rho}{2p}} E_1\left(\frac{\delta+\rho}{2p}\right) - 4\rho p\right], \end{aligned} \tag{17}$$

for $\mathcal{R}e\{s\} < \sigma/p + 2/\sqrt{p}$, where $\rho := \sqrt{\delta^2 - 4p}$ and $\delta := \sigma - ps$.

Similar to (16), we write $\mathcal{J}_1(s)$ in (15) as

$$\mathcal{J}_1(s) = \frac{-\partial}{\partial\beta}\left[\int_{1/2}^\infty e^{-\alpha\gamma} K_0\left(\beta\sqrt{\gamma^2 - (1/2)^2}\right) d\gamma\right]. \tag{18}$$

We note that the above integral is the same as the integral in (16). Consequently, we use [12, Eq. 6.646] again to find the integral in (18), and after differentiating with respect to β, and then letting $\beta = 2/\sqrt{p}$ we obtain:

$$\begin{aligned} \mathcal{J}_1(s) = &\frac{p\sqrt{p}\, e^{-\frac{\delta}{2p}}}{2\rho^3} \left[-(\delta+2p)\, e^{\frac{\delta-\rho}{2p}} E_1\left(\frac{\delta-\rho}{2p}\right)\right. \\ &\left. - (\delta-2p)\, e^{\frac{\delta+\rho}{2p}} E_1\left(\frac{\delta+\rho}{2p}\right) + \rho\delta\right], \end{aligned} \tag{19}$$

for $\mathcal{R}e\{s\} < \sigma/p + 2/\sqrt{p}$. Now, if we substitute (17) and (19) into (13), we obtain the MGF of $\Gamma(X,Y)$, and consequently, after replacing X with $\gamma_{1,k}$ and Y with $\gamma_{2,k}$ we obtain (6).

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