MULTI-USER DETECTION FOR RANDOM PERMUTATION-BASED MULTIPLE ACCESS

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ABSTRACT

This paper deals with the multi-user detection in the context of random permutations. Random permutations, as a special class of discrete Linear Periodic Time Varying filters, are a tool to generate multiple access, for which it is important to develop appropriate receiver detectors. In this paper, the case of the synchronous model is considered, for which two detectors are developped: the matched-filter detector and the decorrelating detector. Theoretical probabilities of error are given, which are confirmed by Monte-Carlo simulations. A comparison with decorrelating techniques for CDMA systems is provided, along with a discussion on the choice of the permutations.

1. INTRODUCTION

Linear Periodic Time Varying (LPTV) filters can be used for multiple access purposes [5]. Within the wide set of LPTV filters, Periodic Clock Changes (PCC) are a special set which transforms an input signal $x(t)$ into an output signal $x(t-f(t))$, where $f(t)$ is a periodic function [3]. Using PCC, multiple access can be achieved with spreading effects [4], [5]. In this paper a particular discrete PCC (the permutation) is considered. The basic principle of random permutation-based Multiple Access (MA) have been studied in previous works [4], [5]. A challenging issue of multiple-access techniques consists of defining appropriate detectors at the receiver, which allow to retrieve the bits of the different users with a low probability of error. For CDMA systems, many detectors have then been proposed in the literature (see for instance [1], [2], [6] and references therein). In this paper, the Multi User Detection (MUD) problem is investigated for random permutation-based MA systems. Section II deals with the basic principles of the MA scheme based on permutations; the synchronous signal model considered in this paper is then presented, when the channel is an Additive White Gaussian Noise (AWGN) channel. Section III is devoted to the development and the analysis of the so-called "matched-filter" detector. The theoretical probability of error is given, along with an approximate probability, which is based on the assumption of a Gaussian Multiple-Access Interference (MAI). Section IV of the paper presents the MUD detector based on a decorrelating technique. Simulation results are reported in section V. Theoretical derivations are first validated by Monte-Carlo simulations. The performance of the matched-filter detector and the decorrelating detector are then compared according to the near-far effect. Furthermore, a comparison between decorrelating detectors for random permutation systems and for CDMA systems is conducted. Gold codes are chosen for the CDMA codes. Figures show a good performance of permutation- based MA compared to Gold codes. Finally, the problem of the choice of the permutations is discussed in section VI.

2. PROBLEM FORMULATION

2.1. Permutations and spreading effects

Let $\mathbf{Z} = \{Z(n),\ n \in \mathbb{Z}\}$ denote a stationary random sequence. Let $B = (B_1, B_2, .., B_P)$ be a random permutation uniformly distributed over $\{1, .., P\}$. The random sequence $\mathbf{Z}$ is divided into blocks of P samples $\left(\left(X_n^k\right)_{n=1,\dots,P}\right)_{k\in\mathbb{Z}}$, where $X_n^k = Z((k-1)P+n)$. The terms of each block $\left(X_n^k\right)_{n=1,\dots,P}$ are permuted according to permutation B, yielding the new block $\left(Y_n^k\right)_{n=1,\dots,P}$ where $Y_n^k = X_{B_n}^k$. Denote $\mathbf{Y} = \left(\left(Y_n^k\right)_{n=1,\dots,P}\right)_{k\in\mathbb{Z}}$ as the resulting process. It can be shown that such transformation is a particular case of a Periodic Clock Change [3]. If $\mathbf{Z}$ is the result of the sampling of a data signal with N samples per symbol, the spectrum of $\mathbf{Y}$ is obtained by spreading the spectrum of $\mathbf{Z}$ by a factor N. Thus, N is the equivalent spreading factor. This spreading property can then be used for multiple access communications [5].

2.2. Synchronous multi-user signal model

Let K denote the number of users. User k transmits the stream of L bits $\underline{b}_k \triangleq [b_k(1), \dots, b_k(L)]^T$, where $b_k(l) \in \{-1; +1\}$, $l \in \{1, \dots, L\}$. Let T and $m(t)$ be the bit duration and the waveform of bit $+1$, respectively. It is assumed in this paper that $m(t)$ uses an antipodal signaling (for instance, NRZ or Biphase codes can be considered). Let $\underline{m} = [m_1, \dots, m_N]^T$ denote a sampling of the waveform $m(t)$. According to section 2.1, N represents the spreading factor. For brievity, it is assumed without loss of generality that $\|\underline{m}\|^2 = \underline{m}^T\underline{m} = 1$. The stream of modulated bits corresponding to user k can then be expressed as $[b_k(1)\underline{m}^T, \dots, b_k(L)\underline{m}^T]^T = M^T\underline{b}_k$, where $M = \underline{m} \otimes I_L$ is the $L \times (NL)$ matrix defined by

$$M = \begin{bmatrix} \underline{m}^T & 0 & \cdots & \\ 0 & \ddots & \ddots & \\ & \cdots & & \underline{m}^T \end{bmatrix}$$

($\otimes$ denotes the Kronecker product, and I_n is the identity matrix of order n). Each user is assigned a particular periodic clock change, i.e. a random permutation on the set $\{1, \dots, NL\}$ is applied to the stream $M^T\underline{b}_k$. Denote P_k as the $(NL) \times (NL)$ permutation matrix associated to user k. Hence, user k transmits the scrambled sequence $P_k M^T \underline{b}_k$. Since the waveform $m(t)$ is binary, the continuous signal transmitted by the kth user can be written as:

$$\sum_{j=1}^{NL} \left(P_k M^T \underline{b}_k\right)_j r\left(t - \frac{j-1}{N}T - \tau_k\right)$$

where $(v)_j$ denotes the jth component of any vector v, $r(t)$ is the indicator function over $[0; T/N]$, and $\tau_k \in [0; T]$ is the offset of user k. In this paper, the synchronous model is considered, so that the bit epochs of all users are aligned at the receiver. Consequently, $\tau_k = 0 \; \forall k \in \{1, \ldots, K\}$, and the continuous K-user received signal can be written as :

$$y(t) = \sum_{k=1}^{K} A_k \sum_{j=1}^{NL} \left(P_k M^T \underline{b}_k\right)_j r\left(t - \frac{j-1}{N} T\right) + n(t)$$

where $i)$ $y(t)$ is the continuous received signal; $ii)$ $A_k > 0$ is the received amplitude of kth user's signal; $iii)$ $n(t)$ is an AWGN.

The multi-user detection problem consists then of estimating the bits of the different users, given the received signal $y(t)$. The continuous signal is first passed through a filter bank, yielding the variables:

$$\begin{aligned} y_j &\triangleq \int y(t) r\left(t - (j-1)T/N\right) dt \,, \; j = 1, \ldots, NL \\ &= \sum_{k=1}^{K} A_k \left(P_k M^T \underline{b}_k\right)_j + n_j \end{aligned}$$

It can be shown as in [6] that the vector variable $\underline{y} \triangleq [y_1, y_2, \ldots, y_{NL}]^T$ is a sufficient statistic for the bits $(b_k(j))_{\substack{1 \le j \le L \\ 1 \le k \le K}}$. Consequently, the detectors proposed in this paper are based on the statistic $\underline{y}$, which can be expressed as

$$\underline{y} = \sum_{k=1}^{K} A_k P_k M^T \underline{b}_k + \underline{n} \qquad (1)$$

where the noise term $\underline{n} \triangleq [n_1, n_2, \ldots, n_{NL}]^T$ is a zero-mean Gaussian vector with covariance matrix $\sigma^2 I_{NL}$.

3. MATCHED-FILTER DETECTOR

3.1. Detection Scheme

Given vector $\underline{y}$, the basic idea to retrieve the bits of the kth user consists of: $i)$ performing the inverse kth permutation: now, since P_k is a permutation matrix, the matrix of the inverse permutation is simply P_k^T; $ii)$ passing the data through a filter matched to the waveform $m(t)$, i.e. multiplying the obtained discrete data by M. Consequently, the matched-filter detector for the kth user consists of defining the variable:

$$\underline{z}_k \triangleq M P_k^T \underline{y}$$

It is then straightforward to prove that $\underline{z}_k$ can be expressed as

$$\underline{z}_k = A_k \underline{b}_k + \sum_{l \neq k} A_l R_{kl} \underline{b}_l + M P_k^T \underline{n} \qquad (2)$$

where $R_{kl} \triangleq \left(P_k M^T\right)^T \left(P_l M^T\right)$ (note in particular that $R_{kk} = I_{NL}$ and $R_{kl}^T = R_{lk}$). The elements of R_{kl} represent the correlations between the bits of the kth and the lth users. Since $A_k > 0$, the decision rule for the jth bit of the kth user is defined by:

$$\widehat{b}_k(j) \triangleq sign\left(z_k(j)\right)$$

3.2. Performance

It can be shown using Bayes' formula that the bit-error-rate (BER) for $b_k(j)$ is given by:

$$\begin{aligned} P_k^{mf}(j) = \frac{1}{2^{(K-1)L+1}} & \sum_{\underline{q}_1 \in \{-1;+1\}^L} \cdots \sum_{\underline{q}_K \in \{-1;+1\}^L}_{l \neq k} \\ & Q\left(\frac{1}{\sigma}\left(A_k - \sum_{l \neq k} A_l \left(R_{kl}\underline{q}_l\right)_l\right)\right) \\ & + Q\left(\frac{1}{\sigma}\left(A_k + \sum_{l \neq k} A_l \left(R_{kl}\underline{q}_l\right)_l\right)\right) \end{aligned} \qquad (3)$$

where $Q(x) \triangleq \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$. The computational cost of formula (3) grows exponentially with K and L, and cannot hardly be used in practice. Instead, using the Central Limit theorem, for large K and/or L, the variable $\sum_{l \neq k} A_j \left(R_{kl}\underline{b}_l\right)_j$ can be considered as a zero-mean Gaussian variable, with variance $\sum_{l \neq k} A_l^2 \left(R_{kl} R_{kl}^T\right)_{j,j}$. Thus, the BER for $b_k(j)$ can be approximated by

$$P_k^{mf}(j) \simeq Q\left(A_k / \left(\sigma^2 + \sum_{l \neq k} A_l^2 \left(R_{kl} R_{kl}^T\right)_{j,j}\right)^{1/2}\right)$$

It must be pointed out that, due to the permutations, the bits of the same user do not behave equally. Therefore, the BER is not constant for all bits of a given user. Moreover, because of the MAI, the error probability is not zero in the absence of addtive noise, except in the particular case where the permutations are orthogonal, i.e. $M P_k^T P_l M^T = 0$ for $l \neq k$ (in that case, the matched-filter detector is optimal). Consequently, this detector exhibits the near-far problem behavior, i.e. its performance can decrease dramatically when the energies of the other users increase.

4. DECORRELATING DETECTOR

4.1. Detection Scheme

It has been shown in the previous section that, even for high SNR's, the BER may remain high due to the MAI. The objective consists now of removing this MAI in order to obtain null BER in absence of additive noise. In MUD literature, such a technique is referred to as decorrelating detection (see [6] and references therein). In this paper, a decorrelation detector for model (1) is investigated. Eq. (2) can be expressed in a matrix form as:

$$\underline{z}_k = [A_1 R_{k1} \;\; A_2 R_{k2} \; \cdots \; A_K R_{kK}] \, \underline{b} + M P_k^T \underline{n}$$

where $\underline{b} \triangleq \left[\underline{b}_1^T, \ldots, \underline{b}_K^T\right]^T$. Denoting $\underline{z} \triangleq \left[\underline{z}_1^T, \ldots, \underline{z}_K^T\right]^T$, one obtains:

$$\underline{z} = RA\underline{b} + \widetilde{\underline{n}}$$

where R is the block matrix whose block (k, l) is R_{kl} (note that R is symmetric since $R_{kl}^T = R_{lk}$); $A = diag([A_1, \ldots, A_K]^T \otimes 1_L)$ is a diagonal matrix and $\widetilde{\underline{n}} \triangleq \Pi \underline{n}$ with $\Pi \triangleq \left[P_1 M^T, \ldots, \; P_K M^T\right]^T$.

Assuming that R is invertible, the decorrelation simply consists of multiplying z by R^{-1} to remove the multiple–access terms, i.e.[1]:

$$\underline{\zeta} \triangleq R^{-1}\underline{z} = A\underline{b} + R^{-1}\widetilde{\underline{n}}$$

[1] Note that R and R^{-1} are $(NKL) \times (NKL)$ matrices, but can be saved as sparse matrices, since they only contain $K^2 NL$ non-zero elements. Hence, they do not imply particular memory inconvenience.

The $(k-1)L+j$ -th component of $\underline{\zeta}$ corresponds to the bit $b_k(j)$:

$$\underline{\zeta}_{(k-1)L+j} = A_k b_k(j) + \left(R^{-1}\underline{\widetilde{n}}\right)_{(k-1)L+j} \tag{4}$$

Thus, the decision rule for the decorrelating detector is given by

$$\widehat{b}_k(j) \triangleq sign\left(\underline{\zeta}_{(k-1)L+j}\right) \tag{5}$$

4.2. Performance

It can be noted from (4) and (5) that the decision on $b_k(j)$ is not corrupted by the MAI: the only possible errors on the decision are due to the additive noise term $\left(R^{-1}\underline{\widetilde{n}}\right)_{(k-1)L+j}$. Therefore, the BER for $b_k(j)$ is given by

$$P_k^d(j) = Q\left(A_k / \left(var(\left(R^{-1}\underline{\widetilde{n}}\right)_{(k-1)L+j})\right)^{1/2}\right)$$

Since $\underline{\widetilde{n}} = \Pi \underline{n}$ and $\Pi\Pi^T = R$, one concludes that $\underline{\widetilde{n}} \sim \mathcal{N}(0, \sigma^2 R)$. Hence, $R^{-1}\underline{\widetilde{n}} \sim \mathcal{N}(0, \sigma^2 R^{-1} R \left(R^{-1}\right)^T)$, i.e. $R^{-1}\underline{\widetilde{n}} \sim \mathcal{N}(0, \sigma^2 R^{-1})$, which yields:

$$P_k^d(j) = Q\left(A_k / \left(\sigma\left(\left(R^{-1}\right)_{(k-1)L+j,(k-1)L+j}\right)^{1/2}\right)\right)$$

When compared to the matched-filter detector, it is important to note that $i)$ the BER vanishes in absence of noise ($P_k^d(j) \to 0$ when $\sigma \to 0$); $ii)$ the BER's for the kth user only depend on the amplitude A_k, i.e. on its own power: consequently, there is no near-far effect[2].

5. SIMULATION RESULTS

A large amount of simulations have been performed to validate the theoretical derivations. Fig. 1 presents the theoretical and experimental BER's obtained with the matched-filter detector and the decorrelating detector. The number of users is $K = 4$, the spreading factor is $N = 8$, and the number of bits per user is $L = 4$; all users have equal energies $A_k = 1$, and the random permutations are non-orthogonal. The simulation results have been obtained using 2000 Monte-Carlo runs. It can be seen that simulations confirm the theoretical derivations (obviously, for low BER's, the curves do not coincide that well, since the number of runs is not large enough to accurately estimate such error probabilities). Hence, in the following figures, only the theoretical results are drawn. As expected in section 4.1, it can be observed that for "high" SNR's (here $SNR \gtrsim 2$) the decorrelating detector gives much better results compared to the matched-filter detector, since the non-zero MAI has been removed. As explained above, even for equal energies, the error probability is not constant for different bits and/or different users, due to the random permutations which give different correlations between bits of all users.

[2] Note that the synchronous model is mainly considered in the downlink channel, where in general the near-far effect is negligible. However, in future works, this property of the decorrelating detector will be shown to persist in the asynchronous case, i.e. in the uplink channel, where this phenomenon is very important.

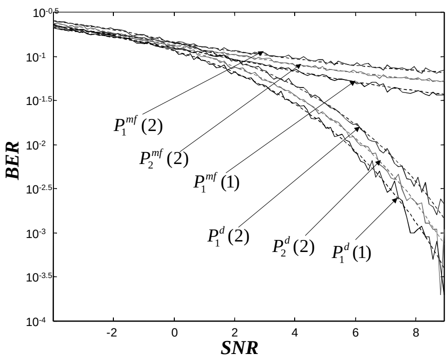

Fig. 1: Theroretical and simulated BER's of Matched Filter ($P_k^{mf}(j)$) and Decorrelating ($P_k^d(j)$) detectors for constant users'energies.

The near-far problem is considered in fig. 2, where model parameters are identical to those of fig. 1, except that the amplitudes are not constant, but equal to $[10; 1; 5; 8]$. It can be noted that the matched-filter detector performance for a particular user highly depends on its relative amplitude facing those of the other users. Hence, the BER's are very different between all users, and the higher the amplitude, the lower the error probability is. On the contrary, for the decorrelating detector, the performance does not depend on the relative amplitude: here, user 2 has lower BER than users 3 and 4, while it has the lowest amplitude, which confirms the abscence of near-far effect for this method (again, the different performances between users are only due to the involved permutations). Note also that for low SNR's, the matched-filter detector can perform better (here, for bit 1 of user 1) since in that case, the errors are essentially due to the additive noise.

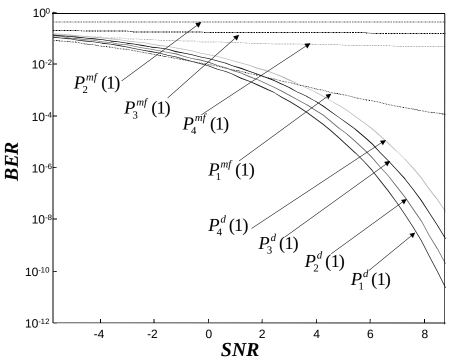

Fig. 2: BER's of Matched Filter ($P_k^{mf}(j)$) and Decorrelating ($P_k^d(j)$) detectors for different users'energies.

Finally, fig. 3 and 4 present a comparison between the decorrelating detector for random permutation-based multiple access and the decorrelating detector for CDMA using Gold codes (see [6]). Since the performances for both methods highly depend on the particular permutation or code sets, it would not have been very significant to show error probabilitites obtained for only few permutations or codes. Instead, fig. 3 and 4 superimpose error probability curves obtained for 100 random permutations and 100 code sets, respectively. The parameters are identical to those of fig. 1. The Gold codes have length 7, which is comparable to $N = 8$ for the random permutations. Thus, the spreading factor is equal to 8 for permutations and 7 for Gold codes. Here, a set of Gold codes is defined by a particular choice of 4 codes among 9 possible codes, along with a random delay from 0 to 6 chips. These figures show that these multiple access techniques behave similarly using decorrelation. However, it seems that the permutation method gives more constant error probabilities, i.e. the results are more similar from a permutation to another than results obtained with Gold codes from one code set to another. It must be noted that larger spreading factors (e.g., 32 or 64) give similar results.

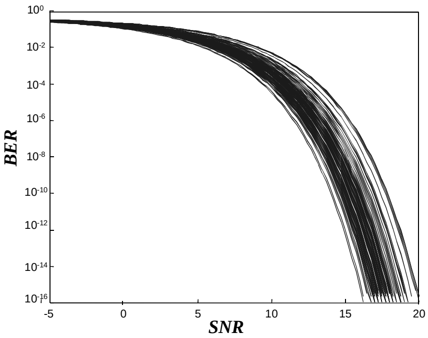

Fig. 3: BER's of Decorrelating detector for different permutation sets.

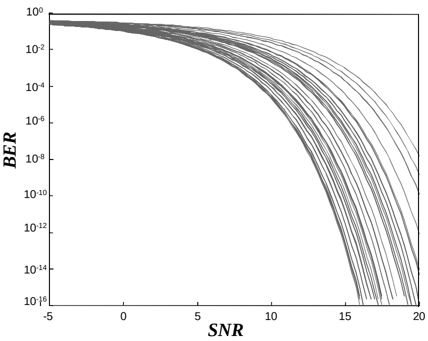

Fig. 4: BER's of Decorrelating detector for different Gold code sets.

6. CHOICE OF PERMUTATIONS

It has been pointed out previously that different permutations give different error probabilities, even when all other parameters remain fixed. Consequently, since the number of permutations is finite (although very large when N and/or L are large), it exists at least one set of optimal permutations for a given set of parameters. More precisely, for the bit j of user k, the BER is minimized for the matched-filter detector when $\sum_{l\neq k} A_l^2 \left(R_{kl}R_{kl}^T\right)_{j,j}$ is minimized, and for the decorrelating detector when $\left(R^{-1}\right)_{(k-1)L+j,(k-1)L+j}$ is minimized. For the matched-filter detector, the minimum BER is then achieved for all bits of all users when the K permutations are orthogonal, i.e. $MP_k^T P_l M^T = 0$ for $l \neq k$. In that case, this detector is optimal since there is no MAI: indeed, the MUD problem is then consituted by K single-user detection problems for which the matched-filter detector is optimal. Now, the choice of such a set of permutations is not obvious, since there are $(NL)!$ possible permutations and then $(NL)!!/((NL)!)K!$ possible choices. Thus, MAI may exist, and the error probabilty can increase dramatically. The decorrelating detector seems then better fitted to MUD for such multiple-access technique. For this detector, the correlation matrix R must first be checked to be invertible. Given the form of R and the number of possible permutations, the probability of having R singular appears quite low: thus, the invertibility of R is not a critical point and is almost always verified (in fact, in the many simulations which have been performed, R was never found to be singular). Now, the choice of a set of permutations which yield minimum $\left(R^{-1}\right)_{(k-1)L+j,(k-1)L+j}$ for all $k \in \{1,\ldots,K\}$ and $j \in \{1,\ldots,L\}$ is a challenging issue. This could be achieved numerically by minimizing an appropriate contrast function using Monte Carlo Markov Chains algorithms (e.g., the simulated annealing algorithm), or other stochastic algorithms such as genetic algorithm. This problem is beyond the scope of this paper and will not be investigated further. An interesting alternative solution would consist of deriving lower and upper bounds for the error probability on the set of all possible set of permutations. This is also an open problem.

7. CONCLUSION

In this paper, random permutations, which are a special case of discrete Periodic Clock Changes, are used as a particular technique for multiple access. It is therefore important to develop efficient methods adapted to such technique in order to detect the different users' signals arriving at the receiver. This paper presents two detectors for the synchronous model: the matched-filter detector and the decorrelating detector. The first detector simply consists, for a given user, of performing the inverse permutation and the filtering matched to the bit waveform. With the second detector, the MAI is removed by applying the inverse correlation matrix to the data obtained after inverse permutations and matched-filtering. This detector shows much better performance for high SNR's, while for low SNR's both detector behaves quite similarly. In particular, an important advantage of the decorrelating detector is that it does not exhibit any near-far effect. For both detectors, the theoretical error probabilities have been derived and confirmed by Monte-Carlo simulations. The decorrelating detector for random permutations has also been compared to the equivalent detector developped for CDMA systems. Both multiple access techniques give similar results. However, this comparison deserves to be more deeply analysed for different set of parameters, in particular according to the number of users and the spreading factor. This study will be considered in future works. Moreover, the development of other detectors (e.g., the MMSE detector) and the case of the asynchronous model are currently under investigation.

8. REFERENCES

[1] "Multiuser Detection Techniques with Application to Wired and Wireless Communications Systems I", IEEE J. on Selected Areas in Comm., Vol. 19, n° 8, August 2001.

[2] "Multiuser Detection Techniques with Application to Wired and Wireless Communications Systems II", IEEE J. on Selected Areas in Comm., Vol. 20, n° 2, February 2002.

[3] B. Lacaze, "Stationary clock changes on stationary processes", Signal processing, 55 (2) (1996) pp. 191-205.

[4] B. Lacaze and D. Roviras, "Effect of random permutations applied to random sequences and related applications", Signal Processing, 82 (2002) pp. 821-831.

[5] D. Roviras, B. Lacaze and N. Thomas, "Effects of Dicrete LPTV on Stationary Signals", Proc. of ICASSP 2002, Orlando, USA, may 2002.

[6] S. Verdu, Multiuser Detection, Cambridge University Press, Cambridge, 1998.