A HYPOTHESIS TESTING APPROACH TO BLIND DS-CDMA USER IDENTIFICATION AND DETECTION VIA CODE-CONSTRAINED SUCCESSIVE CANCELLATION

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ABSTRACT

A code-constrained inverse filter criterion based approach was recently presented in Tugnait & Li (IEEE Trans. SP, July 2001) for blind detection of a desired user in asynchronous short-code DS-CDMA (direct sequence code division multiple access) systems over multipath channels. The method proposed therein works well for low-to-moderate loading; however, it can converge to an undesired user under high loading. In this paper we augment this method with a binary hypothesis testing approach for extracted user identification (whether the desired user is acquired/extracted) and for successive user cancellation (if an undesired user is extracted, cancel its contribution and repeat). An illustrative simulation example is presented.

1. INTRODUCTION

In this paper we consider blind detection (i.e. no training sequence) of a desired user signal, given knowledge of its spreading code, in the presence of MUI, ISI and user asynchronism (lack of knowledge of user transmission delays, including that of the desired user). Past work on blind detection of short-code DS-CDMA signals include [1], [3], [6] and references therein. In this paper our focus is on extraction of a desired user's signal. Unlike [6] and [7], we do not assume synchronization with the desired user's signal. In [1] we investigated maximization of the normalized fourth cumulant of inverse filtered (equalized) data w.r.t. the equalizer coefficients subject to the equalizer lying in a subspace associated with the desired user's code sequence. Properly initialized, constrained maximization can lead to extraction of the desired user's signal whereas arbitrarily initialized unconstrained maximization leads to the extraction of any one of the existing users. In [7] we augmented the method of [1] with extracted user identification (whether the desired user is acquired/extracted) and successive user cancellation (if an undesired user is extracted, cancel its contribution and repeat) procedures. The methods of [7] are heuristic in that the thresholds for user identification were selected empirically. In this paper we propose a binary (statistical) hypothesis testing approach where the thresholds are selected to achieve CFAR (constant false alarm rate). This entails analysis of the test statistic under the null hypothesis that the desired user has been extracted.

Successive user cancellation to enhance system capacity has been widely used, see [4], [5] and references therein; also the discussion in [7]. We assume much less prior information than [4] and [5]. [3] has considered the inverse filter criterion with normalized fourth cumulant cost without any code constraints. Using the approach of [2], ref. [3] proposes extraction of one user at a time. Unlike [3] where spreading codes of all users are known, we assume the knowledge of only the desired user's spreading code. Also unlike [3] the spreading codes do not have to be maximal length PN sequences.

2. SYSTEM MODEL

Consider an asynchronous short-code DS-CDMA system with M users and N chips per symbol with the j-th user's spreading code denoted by $\mathbf{c}_j = [c_j(0), \cdots, c_j(N-1)]^T$. Consider a baseband discrete-time model representation. Let $s_j(k)$ denote the j-th user's k-th symbol. The sequence $\{s_j(k)\}$ is zero-mean, independently and identically distributed (i.i.d.), differentially encoded either 4-QAM $\forall j$ or binary $\forall j$. For different j's, $\{s_j(k)\}$'s are mutually independent. In the presence of a linear dispersive channe, let $g_j(n)$ denote the j-th user's effective channel impulse response (IR) assuming zero transmission delay, sampled at the chip interval T_c. Let

$$h_j(n) = \sum_{m=0}^{N-1} c_j(m) g_j(n-m-d_j), \tag{1}$$

where $h_j(n)$ represents the effective signature sequence of user j (i.e. code $c_j(n)$ "distorted" due to multipath etc.) and d_j $(0 \le d_j < N)$ is the (effective) transmission delay (mod N) of user j in chip intervals. Define a $[(d+1)N] \times [2N]$ code matrix $\mathbf{C}_j^{(d)}$, a Toeplitz matrix, with its first column given by $\mathbf{c}_j$ with dN trailing zeros [1],[7]. If we collect N chip-rate measurements of received signal (from all users) into N-vector $\mathbf{y}(k)$, then we obtain, at the symbol rate, the MIMO model (additive white Gaussian noise $\mathbf{w}(k)$ is defined in a manner similar to $\mathbf{y}(k)$):

$$\mathbf{y}(k) = \sum_{j=1}^{M} \sum_{l=0}^{L_j} \mathbf{h}_j(l) s_j(k-l) + \mathbf{w}(k) \tag{2}$$

where

$$\mathbf{h}_j(l) = [h_j(lN), \cdots, h_j(lN+N-1)]^T, \tag{3}$$

and $L_j + 1$ is the length of the j-th user's vector IR. It follows that for any $d \ge 0$,

$$\mathbf{h}_j^{(d)} := \begin{bmatrix} \mathbf{h}_j^H(0) & \mathbf{h}_j^H(1) & \cdots & \mathbf{h}_j^H(d) \end{bmatrix}^H = \mathbf{C}_j^{(d)} \mathbf{g}_j \tag{4}$$

where the superscript H denotes the complex conjugate transpose (Hermitian) operation,

$$\mathbf{g}_j := [\ g_j(-d_j) \quad g_j(-d_j+1) \quad \cdots \quad g_j(2N-d_j-1)\ ]^T, \tag{5}$$

$\mathbf{h}_j^{(d)}$ is $(d+1)N$-vector, $\mathbf{g}_j$ is $2N$-vector and we assume that $g_j(l) = 0$ for $l > N$ (in addition to $g_j(l) = 0$ for $l < 0$), i.e. the multipath delays can be of at most one symbol duration (N chips). Not all elements in $\mathbf{g}_j$ are nonzero. It follows that $\mathbf{h}_j(l) = 0$ for $l \ge 3$.

3. CODE-CONSTRAINED INVERSE FILTER CRITERION (CC-IFC) [1]

Consider an $N \times 1$ vector equalizer $\{\mathbf{f}(i)\}_{i=0}^{L_e-1}$ of length L_e symbols (NL_e chips) operating on the data $\mathbf{y}(n)$ (see (2)) to yield

$$e(n) = \sum_{i=0}^{L_e-1} \mathbf{f}^H(i) \mathbf{y}(n-i) \tag{6}$$

where $\mathbf{f}(i)$ is $N \times 1$. Define

$$\widetilde{\mathbf{f}}^H := \begin{bmatrix} \mathbf{f}^H(0) & \mathbf{f}^H(1) & \cdots & \mathbf{f}^H(L_e-1) \end{bmatrix}. \tag{7}$$

Let $\text{cum}_4(e)$ denote the fourth-order cumulant of a complex-valued scalar zero-mean random variable e, defined as

$$\text{cum}_4(e) := E\{|e|^4\} - 2[E\{|e|^2\}]^2 - |E\{e^2\}|^2. \tag{8}$$

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Ref. [1] considers maximization of the inverse filter cost

$$J_{42}(\widetilde{\mathbf{f}}) := \frac{|\text{cum}_4(e(n))|}{[E\{|e(n)|^2\}]^2} \tag{9}$$

for designing the linear equalizer. It is shown in [2] that under certain mild sufficient conditions, when (9) is maximized w.r.t. $\{\mathbf{f}(i)\}_{i=0}^{L_e-1}$ using a stochastic gradient algorithm, then (6) reduces to

$$e(n) \;=\; \alpha s_{j_0}(n-n_0), \tag{10}$$

where complex $\alpha \neq 0$, $0 \leq n_0 \leq L_e - 1 + L_j$ is some integer, j_0 indexes some user out of the given M users, i.e., the equalizer output is a possibly scaled and shifted version of one of the users. The problem is that there is no control over which user is extracted.

It has been shown in [1] that in order to extract the desired user ($j_0 = 1$) with desired delay ($n_0 = d$), the linear equalizer should belong to the null space of a matrix $\mathcal{A}$ which is a function of the desired user's code matrix $\mathbf{C}_1^{(d)}$ and the data correlation matrix. It is a $[N(L_e-2)] \times [NL_e]$ matrix given by

$$\mathcal{A} = \mathcal{U}^{(1)H}\mathcal{T}^{(d)}\mathcal{R}_{yy} \tag{11}$$

where $\mathcal{R}_{yy}$ is the $[NL_e] \times [NL_e]$ data correlation matrix with ij-th block element $\mathbf{R}_{yy}(j-i) = E\{\mathbf{y}(k+j-i)\mathbf{y}^H(k)\}$,

$$\mathcal{T}^{(d)} := \begin{bmatrix} \mathcal{T}_d & 0 \\ 0 & I_{N(L_e-1-d)} \end{bmatrix} = [NL_e] \times [NL_e] \;\text{ matrix,} \tag{12}$$

I_K denotes a $K \times K$ identity matrix,

$$\mathcal{T}_d := \begin{bmatrix} 0 & \cdots & 0 & I_N \\ 0 & \cdots & I_N & 0 \\ \vdots & \ddots & \vdots & \vdots \\ I_N & \cdots & 0 & 0 \end{bmatrix} = [N(d+1)] \times [N(d+1)], \tag{13}$$

$$\mathcal{C}_1^{(d)} := \begin{bmatrix} \mathbf{C}_1^{(d)} \\ 0 \end{bmatrix} = [NL_e] \times [2N] \;\text{ matrix} \tag{14}$$

and columns of $\mathcal{U}^{(1)}$ denote an orthonormal basis for the orthogonal complement of $\mathcal{C}_1^{(d)}$. Since $\mathcal{C}_1^{(d)}$ is of full column rank, $\mathcal{U}^{(1)}$ is an $[NL_e] \times [NL_e - 2N]$ matrix (it can be obtained via an SVD (singular value decomposition) of $\mathcal{C}_1^{(d)}$). **Thus, the desired solution satisfies (15) in addition to maximizing (9) (in fact, in addition to being a stationary point of (9))** where

$$\mathcal{A}\widetilde{\mathbf{f}} = 0. \tag{15}$$

By [1] and [2] there exists an equalizer that maximizes (9) as well satisfies (15).

In [1] a projection algorithm for constrained maximization of (9) subject to (15) has been considered. After convergence of the constrained maximization procedure, unconstrained maximization of (9) (without enforcing (15)) is carried out.

4. CHANNEL ESTIMATOR STATISTICS

4.1. True $\{s_j(k)\}$ Available

Using (2), it follows that

$$\mathbf{h}_j(l) = \frac{1}{\sigma_s^2}E\{\mathbf{y}(k+l)s_j^*(k)\},\; l = 0,\cdots,L_j \tag{16}$$

where $\sigma_s^2 = E\{|s_j(k)|^2\}$. If we had access to $\{s_j(k)\}$, the estimate $\widehat{\mathbf{h}}_j(l)$ of $\mathbf{h}_j(l)$ would be

$$\widehat{\mathbf{h}}_j(l) = \frac{1}{T\sigma_s^2}\sum_{k=1}^{T}\mathbf{y}(k+l)s_j^*(k). \tag{17}$$

Define the effective signature vector $\mathbf{h}_j$ and its estimate $\widehat{\mathbf{h}}_j$

$$\mathbf{h}_j := \frac{1}{\sigma_s^2}E\left\{\begin{bmatrix} \mathbf{y}(k) \\ \vdots \\ \mathbf{y}(k+L_j) \end{bmatrix} s_j^*(k)\right\}, \tag{18}$$

$$\widehat{\mathbf{h}}_j := \frac{1}{T\sigma_s^2}\sum_{k=1}^{T}\begin{bmatrix} \mathbf{y}(k) \\ \vdots \\ \mathbf{y}(k+L_j) \end{bmatrix} s_j^*(k). \tag{19}$$

Asymptotically (as $T \to \infty$), $\widehat{\mathbf{h}}_j$ follows a complex normal distribution with mean value $E\{\widehat{\mathbf{h}}_j\} = \mathbf{h}_j$ [8, p. 228]. The covariances

$$\text{cov}(\widehat{\mathbf{h}}_j, \widehat{\mathbf{h}}_j) := E\{\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H\} - \mathbf{h}_j\mathbf{h}_j^H, \tag{20}$$

$$\text{cov}(\widehat{\mathbf{h}}_j, \widehat{\mathbf{h}}_j^*) := E\{\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^T\} - \mathbf{h}_j\mathbf{h}_j^T \tag{21}$$

are given as follows. For 4-QAM $\{s_j(k)\}$ (having $E\{|s_j(k)|^2\} = 2$ and $\text{cum}_4(s_j(k)) = -4$) it can be shown that

$$\text{cov}(\widehat{\mathbf{h}}_j, \widehat{\mathbf{h}}_j) = \frac{1}{T}\left(\frac{\mathbf{R}_{yy}}{\sigma_s^2} - \mathbf{h}_j\mathbf{h}_j^H\right) =: T^{-1}\mathbf{R}_h, \tag{22}$$

$$\text{cov}(\widehat{\mathbf{h}}_j, \widehat{\mathbf{h}}_j^*) =: T^{-1}\mathbf{R}_v$$

$$= \frac{1}{T}\sum_{l=-L_j,\; l\neq 0}^{L_j}\begin{bmatrix} \mathbf{h}_j(l) \\ \vdots \\ \mathbf{h}_j(L_j+l) \end{bmatrix}\begin{bmatrix} \mathbf{h}_j(-l) \\ \vdots \\ \mathbf{h}_j(L_j-l) \end{bmatrix}^T \tag{23}$$

where

$$\mathbf{R}_{yy} := E\left\{\begin{bmatrix} \mathbf{y}(k) \\ \vdots \\ \mathbf{y}(k+L_j) \end{bmatrix}\begin{bmatrix} \mathbf{y}(k) \\ \vdots \\ \mathbf{y}(k+L_j) \end{bmatrix}^H\right\}. \tag{24}$$

In particular, when $L_j = 1$, (23) is reduced to

$$\mathbf{R}_v = \begin{bmatrix} \mathbf{0} & \mathbf{h}_j(1)\mathbf{h}_j^T(0) \\ \mathbf{h}_j(0)\mathbf{h}_j^T(1) & \mathbf{0} \end{bmatrix}. \tag{25}$$

Note that $\widehat{\mathbf{h}}_j$, $\mathbf{h}_j$, $\mathbf{R}_h$ and $\mathbf{R}_v$ are all possibly complex-valued with $\widehat{\mathbf{h}}_j = \widehat{\mathbf{h}}_r + j\widehat{\mathbf{h}}_i$, $\mathbf{h}_j = \mathbf{h}_r + j\mathbf{h}_i$, $\mathbf{R}_h = \mathbf{R}_{h,r} + j\mathbf{R}_{h,i}$ and $\mathbf{R}_v = \mathbf{R}_{v,r} + j\mathbf{R}_{v,i}$ where $j := \sqrt{-1}$. Define real-valued quantities

$$\widehat{\mathbf{r}} := \begin{bmatrix} \widehat{\mathbf{h}}_r \\ \widehat{\mathbf{h}}_i \end{bmatrix},\; \mathbf{r} = \begin{bmatrix} \mathbf{h}_r \\ \mathbf{h}_i \end{bmatrix},\; \Gamma := \begin{bmatrix} \frac{\mathbf{R}_{h,r}+\mathbf{R}_{v,r}}{2} & \frac{\mathbf{R}_{v,i}-\mathbf{R}_{h,i}}{2} \\ \frac{\mathbf{R}_{v,i}+\mathbf{R}_{h,i}}{2} & \frac{\mathbf{R}_{h,r}-\mathbf{R}_{v,r}}{2} \end{bmatrix}. \tag{26}$$

By [8, Thm. 14, p. 228], asymptotically (as $T \to \infty$),

$$\sqrt{T}\left(\widehat{\mathbf{r}} - \mathbf{r}\right) \sim \mathcal{N}(0, \Gamma) \tag{27}$$

where $\mathcal{N}$ denotes a normal distribution.

4.2. Estimated $\{s_j(k)\}$

At the conclusion of the two optimizations of Sec. 3 (constrained followed by unconstrained), one has extracted a user resulting in an output specified by (10). If the constrained maximization works well then we have $j_0 = 1$ (desired user) and $n_0 \in \{d, d-1\}$ (see [1]). Suppose that we quantize $e(n)$ into $\widetilde{s}_{j_0}(n-n_0)$. Then we have

$$\widetilde{s}_{j_0}(n-n_0) = \alpha_q s_{j_0}(n-n_0) \tag{28}$$

where $|\alpha_q| = 1$ and α_q takes one of two values for binary $s_{j_0}(k)$ and one of four values for 4-QAM $s_{j_0}(k)$. Following (17), we estimate the channel for user j_0 via cross-correlation of $\mathbf{y}(k)$ and $\widetilde{e}(n) := \widetilde{s}_{j_0}(n-n_0)$ as

$$\widehat{\mathbf{h}}_{j_0}(l) = \frac{1}{T\sigma_s^2}\sum_{k=1}^{T}\mathbf{y}(k+l)\widetilde{e}^*(k+d-l). \tag{29}$$

If we choose $0 \le l \le 3$ in (29), then $\widehat{\mathbf{h}}_{j_0}(l)$ (and $\mathbf{h}_{j_0}(l+n_0-d)$) contains the entire effective signature of user j_0 for $n_0 \in \{d, d-1\}$ since $\mathbf{h}_{j_0}(l) = 0$ for $l < 0$ and $l \ge 3$ (by the model assumptions of Sec. 2).

Define the m-vector

$$\widetilde{\mathbf{h}}_j^{(m,k)} := [h_j(k), h_j(k+1), \cdots, h_j(m+k-1)]^T \tag{30}$$

and the m-vector $\widehat{\widetilde{\mathbf{h}}}_j^{(m,k)}$ as in (30) except for $h_j(l)$ replaced with $\widehat{h}_j(l)$. If we choose $0 \le k \le d_j$ and $m+k \ge 3N$, then the entire effective signature of user j is included in $\widetilde{\mathbf{h}}_j^{(m,k)}$. Further define the n-vector

$$\widetilde{\mathbf{g}}_j^{(n,k)} := [\underbrace{0 \cdots 0}_{k\ zeros},\ g_j(0),\ \cdots,\ g_j(n-k-1)]^T \tag{31}$$

and the $m \times n$ code-matrix $\widetilde{\mathbf{C}}_j^{(m,n,k)}$, a Toeplitz matrix with first column given by $\mathbf{c}_j$ with k leading zeros and $m-N-k$ trailing zeros [7, Sec. 5.2]. Then it follows that

$$\widetilde{\mathbf{h}}_j^{(dN,0)} = \widetilde{\mathbf{C}}_j^{(dN,N,d_j)}\widetilde{\mathbf{g}}_j^{(N,0)} = \widetilde{\mathbf{C}}_j^{(dN,N+d_j,0)}\widetilde{\mathbf{g}}_j^{(N+d_j,d_j)} \tag{32}$$

and

$$\widetilde{\mathbf{h}}_j^{(dN,d_j+(d-n_0)N)} = \widetilde{\mathbf{C}}_j^{(dN,N,0)}\widetilde{\mathbf{g}}_j^{(N,0)}. \tag{33}$$

The maximum length (in chips) over which $h_j(l)$ may be nonzero is $2N$ (by Sec. 2). Therefore, for some $k \in \{0, 1, \cdots, 2N-1\}$, $\widehat{\widetilde{\mathbf{h}}}_j^{(2N,k)}$ contains the entire effective signature of user j. Note that $\widehat{\widetilde{\mathbf{h}}}_j^{(2N,k)}$ represents a consecutive $2N$-chip segment of $\widehat{\widetilde{\mathbf{h}}}_j^{(4N,0)}$ for any $0 \le k \le 2N-1$. Let

$$k_0 := \arg\left\{\min_{0\le k\le 2N-1} \|\widehat{\widetilde{\mathbf{h}}}_j^{(2N,k)}\|^2\right\}. \tag{34}$$

Then $\widehat{\widetilde{\mathbf{h}}}_j^{(2N,k_0)}$ "corresponds" to (19) with $L_j = 1$; its asymptotic statistics are those of $\widehat{\mathbf{h}}_j$ in (19) with $L_j = 1$. Therefore, under (28), (27) applies when we use $\widehat{\widetilde{\mathbf{h}}}_j^{(2N,k_0)}$ instead of (19). By (33),

$$\widetilde{\mathbf{h}}_j^{(2N,k_0)} = \widetilde{\mathbf{C}}_j^{(2N,N,0)}\widetilde{\mathbf{g}}_j^{(N,0)}. \tag{35}$$

5. USER IDENTIFICATION AND SUCCESSIVE CANCELLATION

After applying the method of Sec. 3, one has extracted a user resulting in an output specified by (10). How do we know if this user is the desired user. Using the method of Sec. 4.2 we can estimate the associated channel impulse response whose statistics are give by the results of Sec. 4.1. The properties of the estimated channel can be used to devise a CFAR test, as discussed in Sec. 5.1. In Sec. 5.2, we review and modify the successive cancellation method of [7] for desired user detection.

5.1. Desired User Identification

Desired user identification may be accomplished by checking if the estimated channel of the extracted user j_0 has the structure (35) corresponding to the desired user $j_0 = 1$. Let the columns of $(2N) \times N\ \widetilde{\mathcal{U}}_{2N,N,0}^{(1)}$ form an orthonormal basis for the orthogonal complement of $\widetilde{\mathbf{C}}_1^{(2N,N,0)}$. If the extracted user is $j_0 = 1$, then

$$\widetilde{\mathcal{U}}_{2N,N,0}^{(1)H}\widetilde{\mathbf{h}}_j^{(2N,k_0)} = \mathbf{0}, \tag{36}$$

else it is nonzero. Let $L_j = 1$, and $\widehat{\mathbf{h}}_j$ and $\mathbf{h}_j$ in Sec. 4.1 correspond to $\widehat{\widetilde{\mathbf{h}}}_j^{(2N,k_0)}$ and $\widetilde{\mathbf{h}}_j^{(2N,k_0)}$, respectively, in Sec. 4.2; define $\widehat{\mathbf{r}}$ and $\mathbf{r}$ accordingly. Let $\widetilde{\mathcal{U}}_{2N,N,0}^{(1)} = \mathcal{U}_r + j\mathcal{U}_i$ and define

$$\mathbf{V} := \begin{bmatrix} \mathcal{U}_r & \mathcal{U}_i \\ -\mathcal{U}_i & \mathcal{U}_r \end{bmatrix}.$$

Eqn. (36) is then equivalent to

$$\mathbf{V}^T\mathbf{r} = 0. \tag{37}$$

It then follows from (27) and (37) that asymptotically

$$\widehat{\mathbf{u}} := \sqrt{T}\mathbf{V}^T\widehat{\mathbf{r}} \sim \mathcal{N}\left(0, \mathbf{V}^T\Gamma\mathbf{V}\right) =: \mathcal{N}(0, \Lambda). \tag{38}$$

where $\widehat{\mathbf{u}}$ is $(2N) \times 1$. Thus asymptotically

$$\widehat{\mathbf{u}}^T\Lambda^{-1}\widehat{\mathbf{u}} \sim \chi^2(2N). \tag{39}$$

Let $\widehat{\Lambda}$ denote a consistent (in probability) estimator of Λ obtained by using sample averaging in (24) and replacing $\mathbf{h}_j(l)$ in (22) and (25) with $\widehat{\mathbf{h}}_j(l)$. Then asymptotically [9, Lemma B.4]

$$\widehat{\mathbf{u}}^T\widehat{\Lambda}^{-1}\widehat{\mathbf{u}} \sim \chi^2(2N) \tag{40}$$

where $\chi^2(K)$ denotes central chi-square distribution with K degrees-of-freedom. In practice we use the Moore-Penrose pseudo-inverse $\widehat{\Lambda}^{\#}$ instead of $\widehat{\Lambda}^{-1}$ in (40).

Let the null hypothesis H_0 correspond to the extraction of the desired user and the alternative H_1 denote its complement. Based on (36)-(40), a CFAR test with probability of false alarm P_{FA} is given by

$$\text{cost} := \widehat{\mathbf{u}}^T\widehat{\Lambda}^{\#}\widehat{\mathbf{u}} \underset{H_0}{\overset{H_1}{\gtrless}} \tau \tag{41}$$

where τ is picked so that $P\{X \ge \tau\} = P_{FA}$ when $X \sim \chi^2(2N)$.

5.2. Successive User Cancellation

We now apply the methods of Secs. 3, 4 and 5.1 iteratively to extract the desired users $j = 1$. Suppose that number of active users in the system (or an upper bound on it) is P. The following algorithm is proposed:

(0) Set $p = 1$.

(I) Given the data $\mathbf{y}(k)$, design the equalizer $\widetilde{\mathbf{f}}$ to extract the desired user using the code-constrained optimization followed by unconstrained enhancement discussed in Sec. 3 and [1].

   (a) If the cost $J_{42} \le \beta_1$ (see (9)) for some $0 < \beta_1 < 1$ (assuming 4-QAM signals for which maximum $J_{42} = 1$) at the conclusion of constrained optimization, we take it as lack of convergence. In this case, go to unconstrained optimization (follow [2]) without using the results of constrained optimization.

   (b) If the cost $J_{42} \le \beta_2$ for some $0 < \beta_2 < 1$ at the conclusion of unconstrained optimization, we take it as lack of convergence. In this case, quit; otherwise denote the extracted user by j_0.

(II) Estimate the channel as $\widehat{\mathbf{h}}_{j_0}(l)$ for $l = 0, 1, 2, 3$ via (29). Estimate k_0 as in (34).

(III) Calculate the cost at stage p (denoted by cost_p) using (41). If $\text{cost}_p < \tau$, then declare the extracted user as the desired user and quit, else continue.

(IV) Calculate the contribution $\widehat{\mathbf{y}}_{j_0}(k)$ of the extracted user to the data as in [7] and remove it from the data as $\mathbf{y}(k) \leftarrow \mathbf{y}(k) - \widehat{\mathbf{y}}_{j_0}(k)$.

(V) Set $p \leftarrow p + 1$.

(a) **Acquisition Mode:** If $p \leq P$, go to step (I), else quit declaring that the desired user is not present in the received signal.

(b) If $p > P$ (and therefore, the desired has not been extracted in any of the preceding P stages), find

$$p_o := \min_{1 \leq p \leq P} \text{cost}_p. \tag{42}$$

We declare the extracted user at stage p_o to be the desired user. We call this the **minimum cost** approach.

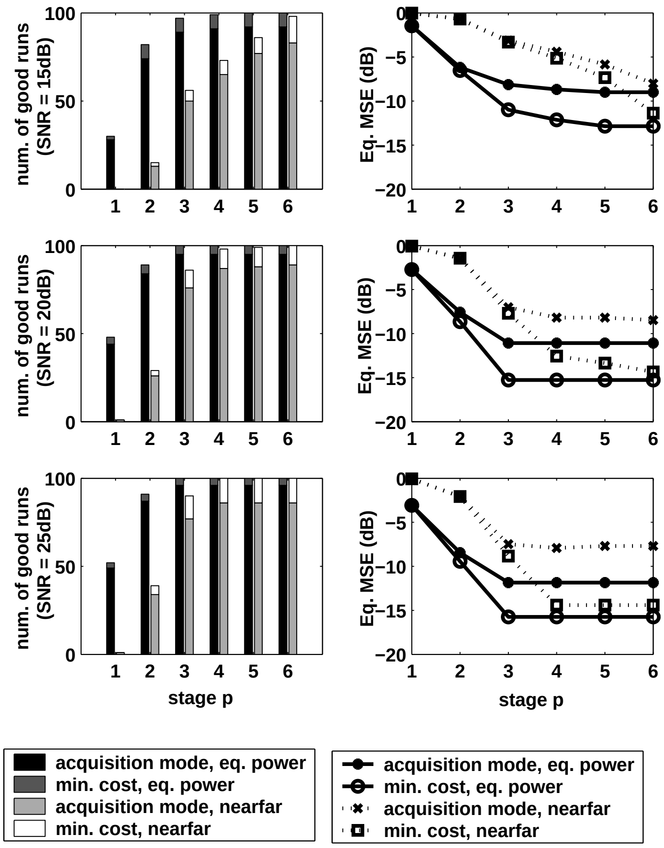

Fig. 1. Desired user identification and equalization: $N = 8$, $M = 6$, record length $T = 1000$ symbols. Based on 100 Monte Carlo runs.

Remark 1. The method corresponding to p_o in step (V-b) above is based on the assumption that the desired user is present in the received signal; the rationale for it is as in [7].

Remark 2. We are in the **acquisition mode** when the presence of the desired user in the received signal is uncertain.

Remark 3. Our proposed method is predicated on the assumption that one of the active users is extracted by maximizing (9). Steps (Ia) and (Ib) check for this heuristically: for 4-QAM signals under no noise, global maximum of J_{42} is 1. If one is far away from this, most likely none of the users has been extracted. For simulations presented in Sec. 6, we took $\beta_1 = 0.9$, $\beta_2 = 0.75$ and $P_{FA} = 0.01$.

6. SIMULATION EXAMPLE

We consider the case of 6 users, each transmitting 4-QAM signals, and short-codes with 8 chips per symbol. The spreading codes were randomly generated binary (± 1, with equal probability) sequences. The multipath channels for each user have 4 paths with transmission delays uniformly distributed over one symbol interval, and the remaining 3 multipaths having mutually independent delays (w.r.t. the first arrival) uniformly distributed over one symbol interval. All four multipath amplitudes are complex Gaussian with zero-mean and identical variance. The channels for each user were randomly generated in each of the 100 Monte Carlo runs (i.e. they were different in different runs). Complex white zero-mean Gaussian noise was added to the received signal from the 6 users. The SNR refers to the symbol SNR of the desired user, which was user 1, and it equals the energy per symbol divided by N_0 (= one-sided power spectral density of noise $= 2E\{\|\mathbf{w}(k)\|^2\}/N$). In the equal-power case (0dB MUIs), all users have the same power; in the near-far case (10dB MUIs), the desired user power is 10 dB below that of other users. The record length for equalizer design and cost calculations was taken to be 1000 symbols, and for equalization MSE (mean-square error) calculations was 3000 symbols per run.

Equalizer of length (L_e) 5 symbols and desired delay $d = 3$ was designed for desired user detection. Initialization was done as in [1]. Fig. 1 shows the results of user identification and equalization for processing gain $N = 8$, number of active users $M = 6$ and varying SNRs. The min. cost approach refers to the usage of Step (V-b) whereas the acquisition mode approach refers to usage of Step (V-a). In Fig. 1 good runs refer to extraction of the desired user and bad runs refer to extraction of an undesired user. The equalization MSE is the normalized MSE where the MSE is divided by $E\{|s_1(k)|^2\}$.

It is seen from Fig. 1 that proposed approach works well in that successive cancellation considered considerably improves desired user's detection performance.

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