ROBUST BLIND MULTIUSER DETECTION FOR SYNCHRONOUS CDMA SYSTEMS

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ABSTRACT

The performance of blind multiuser detection methods is known to degrade in the presence of mismatches between the actual and the presumed desired user signatures. Such mismatches may occur in practical situations due to an imperfect knowledge of the channel impulse response.

In this paper, we propose a new robust approach to blind multiuser detection in the presence of unknown arbitrary-type mismatches of the desired user signature. The formulations of our robust multiuser receivers are based on the explicit modeling of uncertainties in the covariance matrix of the desired user signature/data covariance matrix and optimization of the worst-case performance. The proposed methods have a computational complexity comparable to that of the traditional blind multiuser detection algorithms, while offer an improved robustness and faster convergence rates.

1. INTRODUCTION

Linear receivers for multiuser detection have been widely considered in the literature as simple suboptimal solutions [1]-[4]. One of the most popular solutions among linear receivers is the minimum output energy (MOE) receiver [4].

Recently, blind multiuser detection techniques (which do not require any training) attracted a great interest. These methods are entirely based on the spreading code of the desired user and are able to detect its symbols from the received data without any knowledge of the channel or spreading codes of other users. For example, a well-known Capon estimator has been adopted in [5] for blind multiuser detection. However, the performance of the Capon multiuser detector can degrade severely at low signal-to-noise ratios (SNRs) and short data lengths.

In [4] and [6], two robust blind multiuser receivers have been presented that exploit the idea of the MOE receiver. Interestingly, the methods of [4] and [6] are essentially similar because both of them lead to *diagonal loading* of the covariance matrix of the received data. However, as shown in [7], the robustness of the approach of [4] may be insufficient. Motivated by this fact, the authors of [7] have proposed another solution to the robust blind multiuser detection problem. Their approach uses the worst-case performance optimization to improve the robustness of the MOE receiver. Unfortunately, the method of [7] does not provide any closed-form solution and is not suitable for efficient on-line implementations.

In this paper, we apply the idea of the worst-case performance optimization to blind multiuser detection and develop two closed-form robust multiuser receivers in the presence of an arbitrary mismatch in the desired user signature. Unlike the existing blind multiuser receivers, the proposed techniques can be applied to random time-varying channel scenarios where the channel impulse response, and, correspondingly, the desired user signature are subject to substantial fluctuations during the observation interval.

2. BACKGROUND

Consider a K-user synchronous CDMA system [1]. The received continuous-time baseband signal can be modeled as [7]

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=1}^{K} A_k b_k(m) s_k(t - mT_s) + v(t) \qquad (1)$$

where A_k is the received signal amplitude of the kth user, $b_k(m)$ is the mth data symbol of this user, $s_k(t)$ is its signature waveform, T_s is the symbol period, and $v(t)$ is the zero-mean additive random noise process with the variance σ^2. Let us consider the short spreading code case assuming that the chip sequence period is the same as the symbol period [2]. Furthermore, we assume that for each user, the data symbols are independent random variables which are equally likely drawn from a finite alphabet.

We model the channel for each user as an FIR filter whose impulse response is much shorter than the symbol period T_s, so that the effect of inter-symbol-interference (ISI) can be neglected [4]. However, the duration of the channel impulse response is assumed to be comparable to the chip period T_c, so that there is a substantial inter-chip-interference (ICI) [6]. Using these assumptions, the signature waveform of the kth user is given by [3]

$$s_k(t) = \sum_{l=0}^{L-1} c_k(l) g_k(t - lT_c) \qquad (2)$$

where $\mathbf{c}_k = [c_k(0), c_k(1), \ldots, c_k(L-1)]^T$ is the user spreading code vector, $g_k(t)$ is its chip waveform convolved with the channel impulse response, L is the spreading factor (the number of chips per symbol), $T_c = T_s/L$ is the chip period, and $(\cdot)^T$ stands for the transpose. In an ICI-free scenario, $g_k(t)$ spans only one chip period, while in practice, due to the channel dispersion, $g_k(t)$ can span several chip periods and this may cause ICI.

Using the assumption that there is no ISI, we obtain that $\mathbf{s}_k(t) = 0$ for $t < 0$ or $t > T_s$. Then, sampling (1) at $t =$

Supported in parts by the Natural Sciences and Engineering Research Council (NSERC) of Canada, Communications and Information Technology Ontario (CITO), Premier Research Excellence Award Program of the Ministry of Energy, Science, and Technology (MEST) of Ontario, and Wolfgang Paul Award Program of the Alexander von Humboldt Foundation.

$nT_s + pT_c$ for $p = 0, 1, 2, ..., L-1$ and using the vector notation, we have

$$\mathbf{x}(n) = \sum_{k=1}^{K} A_k b_k(n)\mathbf{s}_k + \mathbf{v}(n) \qquad (3)$$

where $\mathbf{x}(n)$, $\mathbf{s}_k$ and $\mathbf{v}(n)$ are the data vector, the signature vector of the kth user, and the noise vector, respectively. Assuming without loss of generality that the first user is the desired one, let us rewrite (3) as

$$\mathbf{x}(n) = \mathbf{s}_{\mathrm{d}}(n) + \mathbf{i}(n) + \mathbf{v}(n) \qquad (4)$$

where

$$\mathbf{s}_{\mathrm{d}}(n) \triangleq A_1 b_1(n)\mathbf{s}_1 \qquad (5)$$

contains the desired user data, while

$$\mathbf{i}(n) \triangleq \sum_{k=2}^{K} A_k b_k(n)\mathbf{s}_k \qquad (6)$$

contains the multiuser interference (MUI).

In the MOE method [4], the receiver coefficient vector $\mathbf{f}$ is designed to minimize the output power subject to the constraint which ensures that the receiver response to the desired user is *distortionless*. If the desired user signature vector $\mathbf{s}_1$ is known, the MOE multiuser receiver can be designed by solving

$$\min_{\mathbf{f}} \mathbf{f}^H \mathbf{R}_{\mathrm{x}} \mathbf{f} \quad \text{s.t.} \quad \mathbf{f}^H \mathbf{s}_1 = 1 \qquad (7)$$

where $(\cdot)^H$ stands for the Hermitian transpose. In the finite-sample case, the data covariance matrix $\mathbf{R}_{\mathrm{x}}$ is estimated as

$$\hat{\mathbf{R}}_{\mathrm{x}} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}(n)^H \qquad (8)$$

where N is the number of data vectors. Using $\hat{\mathbf{R}}_{\mathrm{x}}$ instead of $\mathbf{R}_{\mathrm{x}}$ in (7), the solution to this problem can be written as

$$\mathbf{f}_{\mathrm{MOE}} = (\mathbf{s}_1^H \hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{s}_1)^{-1}\hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{s}_1 \qquad (9)$$

In practice, the precise knowledge of the desired user signature $\mathbf{s}_1$ may be unavailable. In this case, ignoring the effect of the unknown channel, one can use $\mathbf{c}_1$ instead of $\mathbf{s}_1$ [4] and the MOE multiuser receiver can be rewritten as

$$\mathbf{f}_{\mathrm{MOE}} = (\mathbf{c}_1^H \hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{c}_1)^{-1}\hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{c}_1 \qquad (10)$$

The receiver (10) is very sensitive to even a slight mismatch between $\mathbf{c}_1$ and $\mathbf{s}_1$, see [4], [6], and [7]. To improve the robustness of the MOE receiver, diagonal loading can be used in which $\hat{\mathbf{R}}_{\mathrm{x}}$ is replaced by $\gamma\mathbf{I} + \hat{\mathbf{R}}_{\mathrm{x}}$ where γ is the loading factor. Using this approach, the robust diagonally loaded MOE receiver can be written as [4], [6]

$$\mathbf{f}_{\mathrm{DL-MOE}} = \frac{(\gamma\mathbf{I} + \hat{\mathbf{R}}_{\mathrm{x}})^{-1}\mathbf{c}_1}{\mathbf{c}_1^H(\gamma\mathbf{I} + \hat{\mathbf{R}}_{\mathrm{x}})^{-1}\mathbf{c}_1} \qquad (11)$$

3. EXTENDED FORMULATION OF THE MOE RECEIVER

Let us find $\mathbf{f}$ that minimizes the output power subject to the constraint which requires that the power contribution of the desired user be a positive constant, i.e.,

$$\min_{\mathbf{f}} \mathbf{f}^H \hat{\mathbf{R}}_{\mathrm{x}} \mathbf{f} \quad \text{s.t.} \quad \mathrm{E}\{|\mathbf{f}^H \mathbf{s}_{\mathrm{d}}(n)|^2\} = \text{ const} \qquad (12)$$

Note that $\mathrm{E}\{|\mathbf{f}^H \mathbf{s}_{\mathrm{d}}(n)|^2\} = |A_1|^2 \mathrm{E}\{|b_1(n)|^2\}\mathbf{f}^H \mathbf{R}_{\mathrm{s}}\mathbf{f}$ where $\mathbf{R}_{\mathrm{s}} \triangleq \mathrm{E}\{\mathbf{s}_1\mathbf{s}_1^H\}$. We stress here that in the case of deterministic (quasi-static) channels $\mathtt{rank}\{\mathbf{R}_{\mathrm{s}}\} = 1$, while in the random (time-varying) channel case the rank of $\mathbf{R}_{\mathrm{s}}$ can be higher than one. Noting that the value of the constant in (12) doest not affect the probability of error at the output of the symbol detector, we can rewrite (12) as

$$\min_{\mathbf{f}} \mathbf{f}^H \hat{\mathbf{R}}_{\mathrm{x}} \mathbf{f} \quad \text{s.t.} \quad \mathbf{f}^H \mathbf{R}_{\mathrm{s}} \mathbf{f} = 1 \qquad (13)$$

The solution to (13) can be found using the Lagrange multiplier method. The so-obtained vector of the receiver coefficients can be explicitly written as

$$\mathbf{f}_{\mathrm{opt}} = \mathcal{P}\{\hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{R}_{\mathrm{s}}\} \qquad (14)$$

where $\mathcal{P}\{\cdot\}$ is the operator which yields the *principal eigenvector* of a matrix (i.e., the eigenvector that corresponds to its maximal eigenvalue). Note that the solution (14) does not change if we multiply $\mathbf{R}_{\mathrm{s}}$ or $\hat{\mathbf{R}}_{\mathrm{x}}$ by an arbitrary constant. As any eigenvector can be normalized in an arbitrary way, from (13) we obtain that the resulting vector $\mathbf{f}_{\mathrm{opt}}$ should be normalized to satisfy the constraint $\mathbf{f}_{\mathrm{opt}}^H \mathbf{R}_{\mathrm{s}} \mathbf{f}_{\mathrm{opt}} = 1$ in (13). However, the multiplication of the receiver coefficient vector by any positive constant does not affect the probability of error at the output of the symbol detector. Hence, such a normalization is immaterial.

In the deterministic channel case, we have $\mathbf{R}_{\mathrm{s}} = \mathbf{s}_1\mathbf{s}_1^H$, and, therefore, (14) can be rewritten as

$$\mathbf{f}_{\mathrm{opt}} = \mathcal{P}\{\hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{R}_{\mathrm{s}}\} = \mathcal{P}\{\hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{s}_1\mathbf{s}_1^H\} = \beta\hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{s}_1 \qquad (15)$$

where β should be chosen as $\beta = (\mathbf{s}_1^H \hat{\mathbf{R}}_{\mathrm{x}}^{-1}\mathbf{s}_1)^{-1}$ to satisfy the constraint $\mathbf{f}_{\mathrm{opt}}^H \mathbf{s}_1 = 1$. Therefore, (14) reduces in this case to the conventional MOE receiver (9).

Note that the traditional formulation in (7) assumes that the desired user signature $\mathbf{s}_1$ is a deterministic vector which does not change during the observation time. However, in the time-varying channel case (where the channel impulse response varies within the observation interval) the desired user signature vector can fluctuate. Hence, the conventional MOE receiver is not applicable to scenarios with random channels where the receiver (14) has to be used.

4. ROBUST BLIND MULTIUSER DETECTION

The solution (14) assumes that $\mathbf{R}_{\mathrm{s}}$ is exactly known. In practice, because of imperfect knowledge of the channel impulse response, there is always a certain mismatch between the *presumed* matrix $\mathbf{R}_{\mathrm{s}}$ and its *actual* value $\tilde{\mathbf{R}}_{\mathrm{s}}$. Therefore, we have

$$\tilde{\mathbf{R}}_{\mathrm{s}} = \mathbf{R}_{\mathrm{s}} + \mathbf{E}_{\mathrm{s}} \qquad (16)$$

where $\mathbf{E}_{\mathrm{s}}$ is an unknown complex error matrix.

Assume that the norm of $\mathbf{E}_s$ can be bounded by some known constant $\varepsilon > 0$:

$$\|\mathbf{E}_s\| \le \varepsilon \tag{17}$$

where $\|\cdot\|$ is the Frobenius norm. In practice, ε can be easily determined by finding an upper bound on the ICI. For example, exploiting some coarse knowledge about the channel (such as its approximate delay spread, etc.), one can evaluate the maximum possible amount of the ICI, and find a proper value of the parameter ε.

To incorporate robustness against an arbitrary norm-bounded mismatch between $\mathbf{R}_s$ and $\tilde{\mathbf{R}}_s$, we modify the MOE problem (13) to guarantee that for all possible $\tilde{\mathbf{R}}_s$, the desired user power at the output of the receiver is larger than a constant value, that is, $\mathbf{f}^H(\mathbf{R}_s + \mathbf{E}_s)\mathbf{f} \ge 1$ for all $\|\mathbf{E}_s\| \le \varepsilon$. This constraint guarantees that the desired user power will be not less than one for the *worst-case mismatch* which corresponds to the smallest value of $\mathbf{f}^H(\mathbf{R}_s + \mathbf{E}_s)\mathbf{f}$. Therefore, the proposed design should improve the receiver robustness.

Using this idea, we can write the robust formulation of the MOE problem as

$$\min_{\mathbf{f}} \mathbf{f}^H\hat{\mathbf{R}}_x\mathbf{f} \quad \text{s.t.} \quad \mathbf{f}^H(\mathbf{R}_s + \mathbf{E}_s)\mathbf{f} \ge 1 \ \text{ for all } \ \|\mathbf{E}_s\| \le \varepsilon \tag{18}$$

Note that the constraint in (18) can be replaced by

$$\min_{\|\mathbf{E}_s\| \le \varepsilon} \mathbf{f}^H(\mathbf{R}_s + \mathbf{E}_s)\mathbf{f} = 1 \tag{19}$$

The proof of this fact can be found in [8].

Now, we use the following lemma to simplify (19):

Lemma 1: For any fixed $\mathbf{f}$ and $\mathbf{R}_s$,

$$\min_{\|\mathbf{E}_s\| \le \varepsilon} \mathbf{f}^H(\mathbf{R}_s + \mathbf{E}_s)\mathbf{f} = \mathbf{f}^H(\mathbf{R}_s - \varepsilon\mathbf{I})\mathbf{f} \tag{20}$$

Proof: See [8]. □

Using (19) and Lemma 1, the original robust multiuser detection problem (18) can be rewritten as

$$\min_{\mathbf{f}} \mathbf{f}^H\hat{\mathbf{R}}_x\mathbf{f} \quad \text{s.t.} \quad \mathbf{f}^H(\mathbf{R}_s - \varepsilon\mathbf{I})\mathbf{f} = 1 \tag{21}$$

The optimal solution to (21) can be obtained using the Lagrange multiplier method and is given by (see [8] for details[1])

$$\mathbf{f}_{\text{rob}} = \mathcal{P}\{\hat{\mathbf{R}}_x^{-1}(\mathbf{R}_s - \varepsilon\mathbf{I})\} \tag{22}$$

We see that the robust multiuser receiver problem (18) has a simple *closed-form* solution (22) that applies *negative diagonal loading* to the presumed covariance matrix of the desired user signature.

In the simple case when the channel is assumed to be deterministic (quasi-static) and unknown, we can use $\mathbf{c}_1\mathbf{c}_1^H$ instead of $\mathbf{R}_s$ and (22) can be rewritten as

$$\mathbf{f}_{\text{rob}} = \mathcal{P}\{\hat{\mathbf{R}}_x^{-1}(\mathbf{c}_1\mathbf{c}_1^H - \varepsilon\mathbf{I})\} \tag{23}$$

Our robust multiuser receiver can be further extended as follows. In (18), we only considered a mismatch in the desired user signature covariance matrix $\mathbf{R}_s$ while $\hat{\mathbf{R}}_x$ was assumed to be a good estimate of $\mathbf{R}_x$. In practical applications, the latter condition is not always true and, in such cases, mismatches in both $\mathbf{R}_s$ and $\mathbf{R}_x$ need to be considered. Then, (18) can be extended as

$$\begin{aligned}&\min_{\mathbf{f}} \max_{\|\mathbf{E}_x\| \le \gamma} \mathbf{f}^H(\hat{\mathbf{R}}_x + \mathbf{E}_x)\mathbf{f} \\ &\text{s.t.} \ \ \mathbf{f}^H(\mathbf{R}_s + \mathbf{E}_s)\mathbf{f} \ge 1 \ \text{ for all } \ \|\mathbf{E}_s\| \le \varepsilon\end{aligned} \tag{24}$$

where γ is some (preliminary known) level of uncertainty in $\hat{\mathbf{R}}_x$ and the matrix $\mathbf{E}_x$ takes into account all mismatches that may be caused, for example, by data/channel non-stationarity, short data length effects, and quantization errors.

To solve (24), we use the following lemma:

Lemma 2: For any fixed $\mathbf{f}$ and $\hat{\mathbf{R}}_x$,

$$\max_{\|\mathbf{E}_x\| \le \gamma} \mathbf{f}^H(\hat{\mathbf{R}}_x + \mathbf{E}_x)\mathbf{f} = \mathbf{f}^H(\hat{\mathbf{R}}_x + \gamma\mathbf{I})\mathbf{f} \tag{25}$$

Proof: See [8]. □

Using (20) and (25), the problem (24) can be rewritten in a much simpler equivalent form

$$\min_{\mathbf{f}} \mathbf{f}^H(\hat{\mathbf{R}}_x + \gamma\mathbf{I})\mathbf{f} \quad \text{s.t.} \quad \mathbf{f}^H(\mathbf{R}_s - \varepsilon\mathbf{I})\mathbf{f} = 1 \tag{26}$$

Similar to the problem (21), the solution to (26) can be expressed in a closed form and is given by

$$\mathbf{f}_{\text{rob}} = \mathcal{P}\{(\hat{\mathbf{R}}_x + \gamma\mathbf{I})^{-1}(\mathbf{R}_s - \varepsilon\mathbf{I})\} \tag{27}$$

It follows from (27) that the solution to (24) naturally combines two different types of diagonal loading, where the *positive* diagonal load $\gamma\mathbf{I}$ is applied to $\hat{\mathbf{R}}_x$, while the *negative* load $-\varepsilon\mathbf{I}$ is applied to $\mathbf{R}_s$.

In the simpler deterministic channel case, (27) can be simplified as

$$\mathbf{f}_{\text{rob}} = \mathcal{P}\{(\hat{\mathbf{R}}_x + \gamma\mathbf{I})^{-1}(\mathbf{c}_1\mathbf{c}_1^H - \varepsilon\mathbf{I})\} \tag{28}$$

5. SIMULATIONS

We model a 7-user CDMA system which uses Gold codes of length $L = 31$ as user spreading codes. All users are synchronized and have the BPSK modulation. At the receiver, the spreading codes of the users are distorted by an additive random Gaussian vector drawn from $\mathcal{N}(\mathbf{0}, \delta^2\mathbf{I})$. For each user, such a random vector is added to the spreading code vector to simulate the effect of the ICI, see [7]. The interferers are assumed to have the interference-to-noise ratio (INR) equal to 20 dB.

The performances of the following techniques are compared in terms of the bit error rate (BER):

- the *benchmark* MOE algorithm (9) which corresponds to the ideal case when the desired user signature $\mathbf{s}_1$ is known exactly (this algorithm does not correspond to the practical situation considered but is included in our simulations for comparison reasons only),
- the conventional MOE receiver (10),
- the diagonally loaded multiuser receiver (11),
- the robust algorithm (23),
- the robust algorithm (28).

[1] Note that if $\mathbf{R}_s - \varepsilon\mathbf{I}$ is negative definite then (21) does not have any solution. Therefore, ε must be smaller than the maximal eigenvalue of $\mathbf{R}_s$.

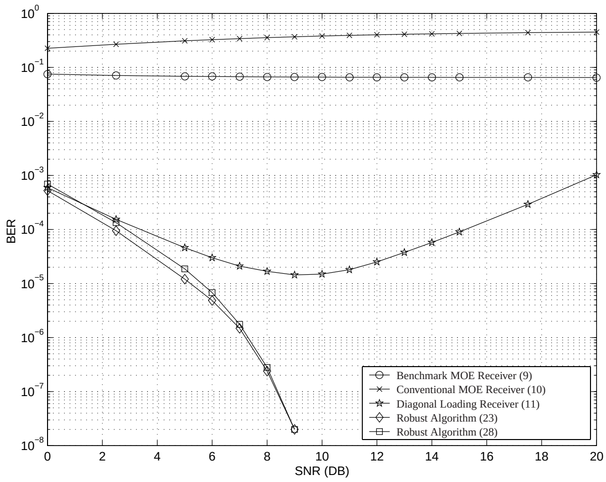

Fig. 1. Bit error rate versus the SNR. First example.

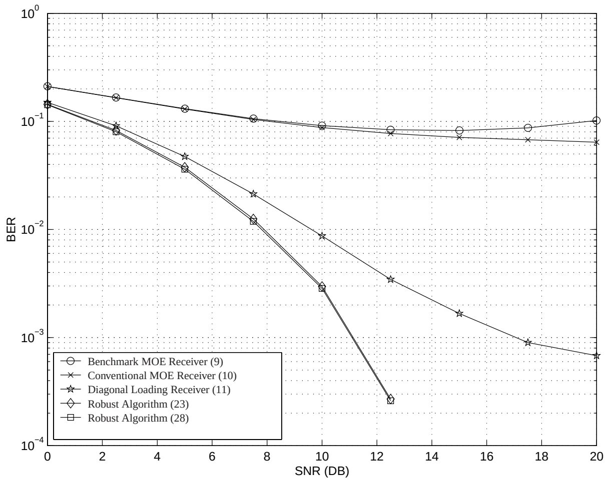

Fig. 2. Bit error rate versus the SNR. Second example.

We have chosen $\varepsilon = 22.5$ which gives nearly the best performance for the robust multiuser receivers tested. Furthermore, the parameter γ in the diagonal loading receiver (11) and in our robust multiuser receiver (27) is chosen as $50\sigma^2$.

In the first example, we consider a deterministic channel, i.e., the random Gaussian distortions (which simulate the effect of the ICI) are fixed in each trial. We assume that $\delta = 0.2$ and use $N = 100$ symbols to obtain the sample covariance matrix $\hat{\mathbf{R}}_x$. Fig. 1 shows the BER of the multiuser detectors tested versus the SNR of the desired user. As can be seen from this figure, when increasing the SNR, the BERs of our robust receivers decrease much faster than the BERs of the other receivers tested. Furthermore, the BER of the diagonal loading based MOE multiuser receiver does not decrease monotonically. Interestingly, for $N = 100$, even the benchmark MOE receiver does not provide satisfactory performance. This is obviously an effect of the short data length.

In the second example, we consider a scenario with a random channel. In this example, the additive random Gaussian distortions (which simulate the effect of the ICI) change from one data vector to another. The other conditions are the same as in the first example.

Fig. 2 displays the BER of the multiuser receivers tested versus the SNR for $\delta = 0.2$ and $N = 100$. As it can be seen from this figure, in the random channel case the proposed robust receivers substantially outperform the other receivers tested. These improvements are especially substantial at high SNRs.

In summary, our simulation examples demonstrated that the proposed blind multiuser receivers consistently enjoy better performance in terms of BER as compared to the MOE receiver and the robust diagonal loading based receiver. These improvements are remarkable in both the cases of a deterministic (quasi-static) channel and random (time-varying) channel.

6. CONCLUSIONS

In this paper, a novel approach to robust blind multiuser detection in synchronous CDMA systems has been proposed. Our multiuser receivers are based on an explicit modeling of arbitrary (yet norm-bounded) uncertainties in the covariance matrices of the desired user signature and of the received data, and use the worst-case performance optimization to yield simple closed-form diagonal loading-based solutions. In contrast to the existing blind multiuser receivers, the proposed techniques can be applied to random time-varying scenarios where the channel impulse response, and, consequently, the desired user signature are subject to substantial fluctuations during the observation interval.

Simulation results show that our multiuser detectors substantially outperform the traditional algorithms in scenarios with mismatches in the desired user signature.

7. REFERENCES

[1] R. Lupas and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. Inf. Theory,* vol. 35, pp. 123-136, Jan. 1989.

[2] M. Honig and M. K. Tsatsanis, "Adaptive techniques for multiuser CDMA receivers," *IEEE Signal Processing Magazine*, vol. 17, pp. 49-61, May 2000.

[3] M. K. Tsatsanis and G. B. Giannakis, "Optimal decorrelating receivers for DS-CDMA systems: a signal processing framework," *IEEE Trans. Signal Processing,* vol. 44, pp. 3044-3055, Dec. 1996.

[4] M. Honig, U. Madhow, and S. Verdú, "Blind adaptive multiuser detection," *IEEE Trans. Inf. Theory,* vol. 41, pp. 944-960, July 1995.

[5] M. K. Tsatsanis, "Performance analysis of minimum variance CDMA receivers," *IEEE Trans. Signal Processing,* vol. 46, pp. 3014-3022, Nov. 1998.

[6] Z. Tian, K. L. Bell, and H. L. Van Trees, "Robust constrained linear receivers for CDMA wireless systems," *IEEE Trans. Signal Processing*, vol. 49, pp. 1510-1522, July 2001.

[7] S. Cui, Z.-Q. Luo, and Z. Ding, "Robust blind multiuser detection against CDMA signature mismatch," in *Proc. ICASSP'01,* May 2001, Salt Lake City, UT, USA, pp. 2297-2300.

[8] S. Shahbazpanahi and A. B. Gershman, "Robust blind multiuser detection for synchronous CDMA systems using worst-case performance optimization," submitted to *IEEE Trans. Inf. Theory*, 2002.