DECISION-FEEDBACK MULTIPLE-SYMBOL DIFFERENTIAL DETECTION OF DIFFERENTIAL SPACE-TIME MODULATION IN CONTINUOUSLY FADING CHANNELS

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ABSTRACT

Linear prediction (LP)-based decision-feedback differential detection (DFDD) only works for diagonal differential space-time modulation (DSTM) when fading is changing fast and continuously. For other constellations it suffers bad performance. In this paper, we propose DFDD based on multiple-symbol differential detection (MSDD) for DSTM to cope with continuous fading. A key observation is that the correlation matrix of the received signal can be expressed in terms of DSTM matrices corresponding to the sent information symbols. In this way decision feedback can be inserted into the MSDD metric, yielding a DF-MSDD receiver while maintaining almost the same performance as MSDD.

I. INTRODUCTION

Differential space-time modulation (DSTM) is an extension of the standard single-antenna differential modulation scheme to multiple-antenna systems, which allows noncoherent detection and promises significant performance gain in fading channels. Tarokh and Jafarkhani [1] proposed a scheme on the basis of Alamouti's orthogonal design for two transmit antennas [2]. Using the powerful tool of the group theory, Hughes [3] and Hochwald and Sweldens [4] presented powerful design that can handle an arbitrary number M of transmit antennas. An appealing feature of the group design is that matrix multiplication may be replaced by addition and table look-up. In particular, Hochwald and Sweldens' diagonal DSTM greatly simplified the design of constellations.

Differential detection (DD) for DSTM suffers an irreducible error floor in time-selective fading channels. To mitigate the flooring effect, a number of receiver structures outperforming the differential detector have been developed. Schober and Lampe [5] proposed multiple-symbol differential detection (MSDD) for DSTM, of which the computational complexity is exponential in the observation length. To overcome the computation burden, decision-feedback differential detection (DFDD) based on linear prediction (LP) has been proposed [5], [6], [7], [8].

Diagonal constellations were assumed in many works on DFDD for DSTM [5], [6]. In this case, receiver design and associated performance analysis are simplified considerably. Specifically, the structure of the linear predictor is the same as that for DPSK, i.e., a time-invariant linear filter subject to an adverse effect that the Doppler frequency shift is multiplied by M. Others [7], [8] did consider nondiagonal constellations, mainly in Alamouti's two-antenna orthogonal design, but made an assumption that the fading process is invariant during a DSTM supersymbol. Though the accuracy of this assumption is acceptable in slowly fading channels, the temporal variation is no longer negligible in fast fading channels.

In this paper, we propose a DFDD receiver based on MSDD. The paper is organized as follows. The system model is given in Sect. II. DF-MSDD is introduced in Sect. III. The bit error rate (BER) performance is analyzed in Sect. IV. Numerical results are presented in Sect. V.

II. SYSTEM MODEL

Consider a multiple-antenna communication system over a flat-fading channel, where data are sent from M transmit antennas to N receive antennas. In DSTM, signals are grouped into an $M \times M$ matrix $\mathbf{S}[k]$ whose row indices represent different antennas and column indices represent time instants $kM, \ldots, kM + M - 1$. The matrices are properly normalized so that the average power of each column is one. The total transmitted power, therefore, does not depend on the number of transmit antennas.

A unitary DSTM system with M transmit antennas and a rate R contains $L = 2^{RM}$ different signals. Each signal is an $M \times M$ unitary matrix drawn from a set $\mathcal{G} = \{\mathbf{G}_0, \mathbf{G}_1, \cdots, \mathbf{G}_{L-1}\}$, $\mathbf{G}_l^H \mathbf{G}_l = \mathbf{I}_M$ [1], [3], [4]. Every RM bits to be transmitted at time instant kM are mapped to a matrix $\mathbf{G}[k]$. Before transmission takes place, the matrices $\mathbf{G}[k]$ are differentially encoded in a fashion similar to DPSK

$$\mathbf{S}[k] = \mathbf{S}[k-1]\mathbf{G}[k], \quad \mathbf{S}[0] = \mathbf{A}, \tag{1}$$

where $\mathbf{A}$ is an initially transmitted unitary matrix.

$\mathcal{G}$ may or may not be a group, depending on the design methodology. The orthogonal design based on Alamouti's two-antenna code [2]

$$\frac{1}{\sqrt{2}}\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \tag{2}$$

normally yields nongroup constellations, except when $R = 1$

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[3]. In [1] a block of R information bits selects a point from the 2^R-ary PSK signal set $\{\exp(j2\pi m/2^R): 0 \le m \le 2^R - 1\}$, and a second block selects another point. The two points are transmitted as x_1 and x_2, respectively.

Hughes constructed optimal group constellations for two antennas [3], and the approach may be extended to more antennas. Hughes' and Alamouti's constellations are generally nondiagonal. By restricting $\mathcal{G}$ to be Abelian, Hochwald and Sweldens [4] proposed diagonal signals of the form

$$\mathbf{G}_l = (\mathbf{G}_1)^l, \quad \mathbf{G}_1 = \mathrm{diag}\left[e^{j2\pi u_0/L}, \cdots, e^{j2\pi u_{M-1}/L}\right]$$

where $u_m \in \{0, 1, \ldots, L-1\}$.

The mobile radio link is assumed subject to time-correlated fading in accordance with the Jakes model [9]. The fading processes $h_{nm}(t)$ for $n = 0, \ldots, N-1$, $m = 1, \ldots, M-1$ are complex normal $\mathcal{CN}(0,1)$ and spatially independent. The autocorrelation of a generic fading process $h(t)$ is given by $\phi[i] = E[h(t)h^*(t+iT)] = J_0(2\pi f_d Ti)$, where $J_0(.)$ is the zeroth-order Bessel function of the first kind, f_d is the maximum Doppler frequency shift, and T is the symbol duration. Likewise, the noises $n_n(t)$ for $n = 0, \ldots, N-1$ are assumed to be independent across both time and space, and are identically $\mathcal{CN}(0, \sigma^2)$ distributed, where $\sigma^2 = E[|n_n[t]|^2]$ is the noise variance. Because of the power normalization, the average bit SNR at each receive antenna is $E_b/N_0 = 1/(R\sigma^2)$.

Let $\mathbf{s}_m[k]$ be the mth column of $\mathbf{S}[k]$, $\mathbf{H}_m[k]$ be the N-by-M matrix of channel coefficients seen by $\mathbf{s}_m[k]$, and $\mathbf{N}[k]$ be an N-by-M noise matrix. Then the received data are given by

$$\mathbf{Y}[k] = \mathcal{H}[k]\mathcal{S}[k] + \mathbf{N}[k], \tag{3}$$

where the N-by-M^2 matrix $\mathcal{H}[k]$ is obtained by stacking $\mathbf{H}_m[k]$:

$$\mathcal{H}[k] = \left[\mathbf{H}_0[k], \mathbf{H}_1[k], \cdots, \mathbf{H}_{M-1}[k]\right],$$

and $\mathcal{S}[k]$ is a stretched version of $\mathbf{S}[k]$, which is no longer square, but has dimension M^2-by-M:

$$\mathcal{S}[k] = \mathrm{blkdiag}\left[\mathbf{s}_0[k], \mathbf{s}_1[k], \cdots, \mathbf{s}_{M-1}[k]\right]. \tag{4}$$

To reduce the error floor associated with differential demodulation in the presence of temporal fading correlation, LP-DFDD makes use of V previous observations in decision [5]-[8]:

$$\hat{\mathbf{G}}[k] = \arg\max_{\mathbf{G}_l \in \mathcal{G}} \mathrm{Re}\left[Tr\left(\mathbf{G}_l \mathbf{Y}^H[k] \sum_{v=1}^{V} p_v \mathbf{Y}[k-v] \prod_{\mu=1}^{v-1} \hat{\mathbf{G}}[k-\mu]\right)\right] \tag{5}$$

where V is the prediction order, p_v for $v = 1, \ldots, V$ are the predictor taps [5], and $\prod_{\mu=1}^{v-1} \hat{\mathbf{G}}[k-\mu] \triangleq \hat{\mathbf{G}}[k-(v-1)] \cdots \hat{\mathbf{G}}[k-1]$ reflects the feedback of previously detected matrices, and is equal to $\mathbf{I}$ if $v = 1$. The standard differential detection corresponds to $V = 1$. For diagonal DSTM, the strategy (5) is optimum in the framework of DFDD provided that the tap vector $\mathbf{p} = [p_1, \ldots, p_V]$ is derived from the Wiener-Hopf equations for the fading-plus-noise process [5]. In existing works, (5) was employed for nondiagonal DSTM as well. However, our recent work showed that LP-DFDD is only optimum if the group $\mathcal{G}$ is diagonal. If $\mathcal{G}$ is nondiagonal, LP-DFDD leads to performance degradation in fast fading, though it causes little impairment in slow fading.

III. DF-MSDD

Rather than using linear prediction, we could insert decision feedback in MSDD. This is achieved by means of a simplification in the expression of the received signal correlation matrix. The new expression depends only on uncoded signal matrices $\mathbf{G}[i]$, $i \in \mathbb{Z}$.

For a fair comparison, let the observation interval span $V + 1$ supersymbols. Stacking the variables involved in MSDD yields the notations

$$\bar{\mathcal{S}}[k] = \mathrm{diag}\left[\mathcal{S}[k-V], \cdots, \mathcal{S}[k-1], \mathcal{S}[k]\right],$$
$$\bar{\mathcal{H}}[k] = \left[\mathcal{H}[k-V], \cdots, \mathcal{H}[k-1], \mathcal{H}[k]\right],$$
$$\bar{\mathbf{Y}}[k] = \left[\mathbf{Y}[k-V], \cdots, \mathbf{Y}[k-1], \mathbf{Y}[k]\right],$$
$$\bar{\mathbf{N}}[k] = \left[\mathbf{N}[k-V], \cdots, \mathbf{N}[k-1], \mathbf{N}[k]\right].$$

We thus have the signal model

$$\bar{\mathbf{Y}}[k] = \bar{\mathcal{H}}[k]\bar{\mathcal{S}}[k] + \bar{\mathbf{N}}[k] \tag{6}$$

The MSDD decides in favor of $\bar{\mathcal{S}}[k]$ that maximizes the conditional probability density function [5]

$$f(\bar{\mathbf{Y}}[k] \mid \bar{\mathcal{S}}[k]) = \frac{\exp(-Tr(\bar{\mathbf{Y}}[k]\mathbf{R}_{\bar{\mathcal{S}}}^{-1}[k]\bar{\mathbf{Y}}^H[k]))}{(\pi^{M(V+1)}\mathrm{Det}(\mathbf{R}_{\bar{\mathcal{S}}}[k]))^N} \tag{7}$$

where $\mathbf{R}_{\bar{\mathcal{S}}}[k]$ is the autocorrelation matrix for $\bar{\mathbf{Y}}[k]$ in the single receive antenna scenario when $\bar{\mathcal{S}}[k]$ is transmitted. $\mathbf{R}_{\bar{\mathcal{S}}}[k]$ can be expressed as

$$\begin{aligned}\mathbf{R}_{\bar{\mathcal{S}}}[k] &= E\left[\bar{\mathbf{Y}}^H[k]\bar{\mathbf{Y}}[k] \mid \bar{\mathcal{S}}[k], N=1\right] \\ &= \bar{\mathcal{S}}^H[k](\mathbf{R}_h \otimes \mathbf{I}_M)\bar{\mathcal{S}}[k] + \sigma^2\mathbf{I}_{M(V+1)}\end{aligned} \tag{8}$$

where $\otimes$ denotes the Kronecker product, and $\mathbf{R}_h$ is the $M(V+1)$-by-$M(V+1)$ autocorrelation matrix of a fading process, whose (i, j)th entry is given by $\phi[j-i]$.

Though it is possible to feedback coded symbols $\hat{\mathcal{S}}[k-V], \cdots, \hat{\mathcal{S}}[k-1]$ to (8), a more convenient way is to handle the information symbols $\hat{\mathbf{G}}[k-V], \cdots, \hat{\mathbf{G}}[k-1]$ directly. No differential decoding is needed after detection in this way. This can be done by expressing $\mathbf{R}_{\bar{\mathcal{S}}}[k]$ in terms of the information supersymbols.

By collecting the differentially encoded supersymbols in a matrix

$$\mathbf{C}[k] \triangleq \left[\mathbf{I}, \mathbf{G}[k-(V-1)], \cdots, \prod_{i=1}^{V-1}\mathbf{G}[k-i], \prod_{i=0}^{V-1}\mathbf{G}[k-i]\right],$$

it can be shown that $\mathbf{R}_{\bar{S}}[k]$ can expressed by reduced-dimension matrices as

$$\mathbf{R}_{\bar{S}}[k] = \mathbf{R}_h \circ (\mathbf{C}^H[k]\mathbf{C}[k]) + \sigma^2 \mathbf{I}_{M(V+1)}, \quad (9)$$

where ∘ is the Hadamard product. It is clear that $\mathbf{R}_{\bar{S}}[k]$ only depends on V information supersymbols $\mathbf{G}[k-(V-1)],\cdots,\mathbf{G}[k]$. Hence, we write $\mathbf{R}_{\bar{S}}[k] = \mathbf{R}_{\mathbf{C}}[k]$ to better reflect its sole dependence on **C**[k].

Now we are in a position to introduce decision-feedback symbols $\hat{\mathbf{G}}[k-(V-1)],\cdots,\hat{\mathbf{G}}[k-1]$ in $\mathbf{C}[k]$, and include the trial matrices $\mathbf{G}_l$ to define

$$\tilde{\mathbf{C}}_l[k] \triangleq \left[\mathbf{I}, \hat{\mathbf{G}}[k-(V-1)], \cdots, \prod_{i=1}^{V-1}\hat{\mathbf{G}}[k-i], \left(\prod_{i=1}^{V-1}\hat{\mathbf{G}}[k-i]\right)\mathbf{G}_l\right].$$

After taking the logarithm of (7), we obtain the DF-MSDD rule

$$\hat{\mathbf{G}}[k] = \arg\min_{0\le l\le L-1}\left\{Tr(\bar{\mathbf{Y}}[k]\mathbf{R}^{-1}_{\tilde{\mathbf{C}}_l}[k]\bar{\mathbf{Y}}^H[k] + N\ln(\mathrm{Det}(\mathbf{R}_{\tilde{\mathbf{C}}_l}[k]))\right\}. \quad (10)$$

The second term is needed because $\mathrm{Det}(\mathbf{R}_{\tilde{\mathbf{C}}_l}[k])$ generally depends on the sent codeword.

For comparison, the optimum MSDD performs an exhaustive search over L^V trial sequences [5]

$$\hat{\mathbf{C}}[k] = \arg\min_{0\le l\le L^V-1}\left\{Tr(\bar{\mathbf{Y}}[k]\mathbf{R}^{-1}_{\tilde{\mathbf{C}}_l}[k]\bar{\mathbf{Y}}^H[k] + N\ln(\mathrm{Det}(\mathbf{R}_{\tilde{\mathbf{C}}_l}[k]))\right\}$$

It represents a performance limit of DF-MSDD.

The DF-MSDD structure, in general, does not lead to a linear prediction receiver for nondiagonal $\mathcal{G}$. Nonetheless, if $\mathcal{G}$ is diagonal, it is equivalent to the linear prediction receiver given in (5).

IV. ERROR ANALYSIS

In this section, we give an error analysis of DF-MSDD with correct feedback and a comparison with the optimum MSDD. The impact of error propagation caused by erroneous feedback will be assessed in the next section. Usually, but not always, the effect of erroneous feedback is to increase the BER by a factor of two.

We start with the evaluation of the pairwise error probability (PEP). It turns out that when $\mathcal{G}$ is nondiagonal, the uniform error probability criterion is generally not satisfied. That is, the symbol error probability depends on which codeword is sent. Therefore, it is necessary to condition on the event $\Omega = (\mathbf{G}[k-(V-1)],\cdots,\mathbf{G}[k])$ when evaluating the PEP.

Conditioned on Ω, $\hat{\mathbf{C}}_l[k]$ takes values on L matrices when $\mathbf{C}[k]$ is transmitted. The MS-DFDD will make a wrong decision $\mathbf{G}_{l'}$ if

$$D = Tr\left(\bar{\mathbf{Y}}[k]\mathbf{R}^{-1}_{\tilde{\mathbf{C}}_{l'}}[k]\bar{\mathbf{Y}}^H[k]\right) - Tr\left(\bar{\mathbf{Y}}[k]\mathbf{R}^{-1}_{\tilde{\mathbf{C}}_l}[k]\bar{\mathbf{Y}}^H[k]\right) < Nc,$$

where $c \triangleq \ln(\mathrm{Det}(\mathbf{R}_{\tilde{\mathbf{C}}_l}[k])) - \ln(\mathrm{Det}(\mathbf{R}_{\tilde{\mathbf{C}}_{l'}}[k]))$. For now, let $N = 1$ so as to simplify the manipulation. In this case, an error occurs if

$$D_0 = \bar{\mathbf{Y}}[k]\mathbf{R}^{-1}_{\tilde{\mathbf{C}}_{l'}}[k]\bar{\mathbf{Y}}^H[k] - \bar{\mathbf{Y}}[k]\mathbf{R}^{-1}_{\tilde{\mathbf{C}}_l}[k]\bar{\mathbf{Y}}^H[k] < c.$$

Defining $\mathbf{F} = \mathbf{R}^{-1}_{\tilde{\mathbf{C}}_{l'}}[k] - \mathbf{R}^{-1}_{\tilde{\mathbf{C}}_l}[k]$, we may express D_0 by a quadratic form $D_0 = \bar{\mathbf{Y}}[k]\mathbf{F}\bar{\mathbf{Y}}^H[k]$. The characteristic function of D_0 can be determined in terms of the eigenvalues of $\mathbf{R}_{\tilde{\mathbf{C}}_l}\mathbf{F}$

$$\Phi_{D_0}(s) = \mathrm{Det}^{-1}(\mathbf{I} + s\mathbf{R}_{\bar{\mathbf{Y}}|\tilde{\mathbf{C}}_l}\mathbf{F}) = \left[\prod_{m=0}^{M(V+1)-1}(1+s\lambda_m)\right]^{-1}$$

Since the fading processes are statistically independent between receive antennas, the characteristic function of D is simply given by $\Phi_{D_0}(s)$ raised to the Nth power. Consequently, the conditional PEP in the presence of N receive antennas is given by

$$P\{\mathbf{G}_l \to \mathbf{G}_{l'} \mid \Omega\} = \frac{1}{2\pi j}\int_{\varepsilon-j\infty}^{\varepsilon+j\infty}\frac{e^{sNc}\Phi^N_{D_0}(s)}{s}ds \qquad \varepsilon > 0, \quad (11)$$

which can be calculated efficiently via the Gauss-Chebyshev quadrature [10].

Then we invoke the union-bound technique to give an upper bound on the error probability. Finally, an average over all the L^V possible events is performed, as the uniform error probability criterion is not fulfilled. This gives the union bound of the BER for DF-MSDD

$$P_b^{\text{MS-DFDD}} = \frac{1}{L^V RM}\sum_{\Omega}\sum_{l'=0,l'\ne l}^{L-1} d^1_{l,l'}P\{\mathbf{G}_l \to \mathbf{G}_{l'} \mid \Omega\} \quad (12)$$

where $d^1_{l,l'}$ represents the Hamming distance between the two message codewords associated with $\mathbf{G}_l$ and $\mathbf{G}_{l'}$. The outer sum of (12) has complexity exponential in V, but a good approximation of the BER may be obtained by averaging over a moderate number of randomly selected sequences instead of the complete set.

It is possible to obtain significant insights into how well DF-MSDD works through a performance comparison with the MSDD. In slowly fading channels, we have derived the seemingly surprising relation that

$$P_b^{\text{DF-MSDD}} \approx \frac{1}{2}P_b^{\text{MSDD}}$$

when correct symbols are fed back. Since the BER will usually be doubled when detected symbols are fed back, DF-MSDD has almost the same performance as MSDD in slowly fading channels. As fading gets faster, $P_b^{\text{DF-MSDD}}$ will be gradually higher than P_b^{MSDD} when correct symbols are fed back. Nevertheless, the performance of DF-MSDD can be maintained at the same level as MSDD.

V. NUMERICAL RESULTS

In this section, numerical results on the performance of DSTM with DF-MSDD reception are presented and compared with MSDD and LP-DFDD. We concentrate on rate-1 DSTM using one receive antenna. The observation

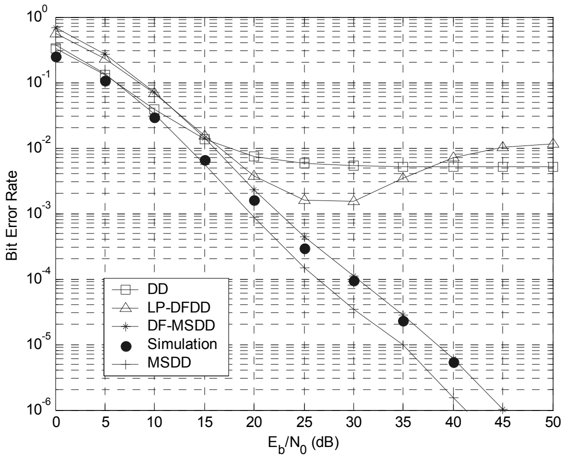

Fig. 1. Performance of two-antenna nondiagonal DSTM with DF-MSDD reception. $f_dT = 0.03$, $V = 3$, $N = 1$.

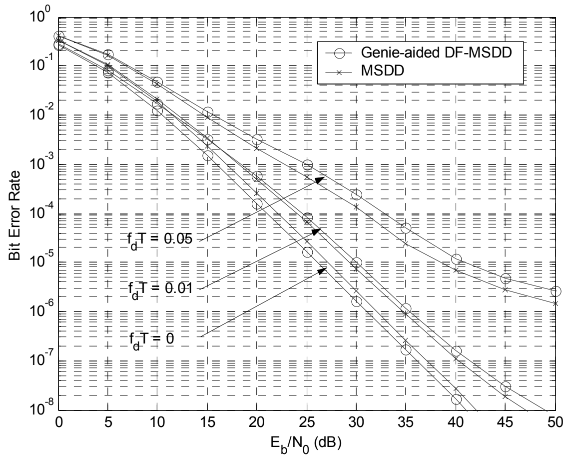

Fig. 2. Performance comparison of two-antenna nondiagonal DSTM with genie-aided DF-MSDD and MSDD reception as fading gets faster. $V = 3$, $N = 1$.

interval of both MSDD and DFDD spans 4 supersymbols, i.e., $V = 3$. To save time, we average over randomly selected ten codewords, instead of a total of L^V in the outer summation of (12). The calculation of the BER for MSDD is based on the $2(V+1)$ main error patterns.

Figure 1 displays the union bounds on the BER of two-antenna DSTM over a fast fading channel with $f_dT = 0.03$. We show the performance of DSTM based on Alamouti's two-antenna code for BPSK signaling, with standard DD, LP-DFDD, DF-MSDD and MSDD. The BER of MS-DFDD is approximated by $2P_b^{\text{DF-MSDD}}$, i.e., by doubling the genie-aided BER. We can see that LP-DFDD suffers very bad performance for Alamouti's two-antenna code. Its BER curve grows up at high SNR and eventually flattens at a BER level even higher than differential detection. In contrast, DF-MSDD performs quite satisfactorily. Its BER appears to differ from that of MSDD only by a constant factor when the SNR is high. The simulation points for DF-MSDD with actual decision feedback are provided, which agree with the theoretical curve fairly well at high SNR. At low SNR, the BER is overestimated, partially because the union bound is loose here. Therefore, we are convinced that the effect of erroneous feedback on DF-MSDD is marginal.

Figure 2 shows the performance of genie-aided DF-MSDD and MSDD as fading gets faster. In a static-fading channel, i.e., $f_dT = 0$, the relation $P_b^{\text{DF-MSDD}} = P_b^{\text{MSDD}}/2$ is well valid. However, $P_b^{\text{DF-MSDD}} > P_b^{\text{MSDD}}$ in fast fading channel with $f_dT = 0.03$ and 0.05. In all the cases considered, DF-MSDD has performance close to MSDD.

VI. CONCLUSIONS

We have proposed DF-MSDD for DSTM in continuously fading channels. The idea is to insert decision-feedback symbols into the MSDD metric. Performance analysis showed DF-MSDD and MSDD have close BER. The impact of error propagation was assessed by computer simulation and turned out to be marginal. The proposed DF-MSDD receiver circumvents the limitation of the linear prediction receiver while maintaining the low-complexity feature.

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