Differential Space-Time Modulation with Full Spatio-Spectral Diversity and Arbitrary Number of Transmit Antennas in ISI Channels

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***Abstract*— We present herein a differential space-time-frequency (DSTF) modulation scheme for systems with an *arbitrary* number of transmit antennas over frequency-selective fading channels. The proposed DSTF scheme employs a concatenation of a specially designed spectral encoder and a differential encoder/mapper that yield the maximum spatio-spectral diversity advantage and significant coding gain. To reduce the decoding complexity, the differential encoder is designed with a unitary structure that decouples the maximum likelihood (ML) detection in space and time; meanwhile, the spectral encoder utilizes a new *linear constellation decimation (LCD)* coding scheme that encodes across a minimally required subchannels and, as a result, has the least decoding complexity among all full-diversity codes. Numerical results show that the proposed DSTF scheme compares favorably with several existing differential space-time schemes for frequency-selective channels.**

I. Introduction

Differential space-time coding (DSTC), which circumvents the challenging task of multi-channel estimation in time-varying channels, has generated significant interest recently [1]–[3]. Current DSTC schemes are designed primarily for flat-fading channels. One possible wideband extension is to use DSTC with orthogonal frequency division multiplexing (OFDM) on each subcarrier across the transmit antennas (e.g., [4]). Such an extension, however, does not exploit additional degrees of freedom offered by multipath propagation in wideband systems. Hence, it achieves only spatial diversity, but no spectral diversity inherent in wideband systems.

We present herein a novel *differential space-time-frequency (DSTF)* modulation scheme for systems with an *arbitrary* number of transmit antennas in frequency-selective channels. The DSTF scheme employs a concatenation of a specially designed spectral encoder and a differential encoder that are designed to maximize the spatio-spectral diversity and coding gain.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote the transpose, conjugate, and conjugate transpose, respectively; $\mathbf{I}_M$ is the $M \times M$ identity matrix; $\mathbf{0}$ (respectively, $\mathbf{1}$) is a vector or matrix with all zero (resp., one) elements; $\otimes$ denotes the Kronecker product; finally, $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix.

II. System Description

Fig. 1 depicts a baseband DSTF system with $N_t \geq 2$ transmit antennas (Tx's) and $N_r = 1$ receive antenna (Rx). At the transmitter, the information stream is first serial-to-parallel (S/P) converted to form vectors $\mathbf{d}(n) \triangleq [d(nP), \ldots, d(nP+P-1)]^T$. The *spectral encoder* $\mathcal{M}_{\rm s}\{\cdot\}$ maps $\mathbf{d}(n)$ to $P \times 1$ vectors $\mathbf{s}(n)$. The differential encoder $\mathcal{M}_{\rm d}\{\cdot\}$ takes as input N_s consecutive spectrally encoded vectors, $\mathbf{s}(nN_s), \ldots, \mathbf{s}(nN_s+N_s-1)$, and outputs the following DSTF code matrix:

This work was supported by the New Jersey Commission on Science and Technology and the Center for Wireless Network Security at Stevens Institute of Technology.

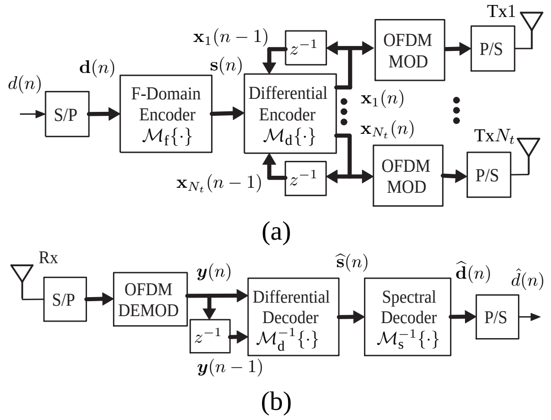

Fig. 1. A baseband differential space-time-frequency (DSTF) system with two transmit antennas and one receive antenna. (a) Transmitter. (b) Receiver.

$$\boldsymbol{\mathcal{X}}(n) \triangleq \begin{bmatrix} \mathbf{x}_1(nN_d) & \ldots & \mathbf{x}_1(nN_d+N_d-1) \\ \vdots & \vdots & \vdots \\ \mathbf{x}_{N_t}(nN_d) & \ldots & \mathbf{x}_{N_t}(nN_d+N_d-1) \end{bmatrix}. \quad (1)$$

The $P \times 1$ vector $\mathbf{x}_i(nN_d+l) \triangleq [x_i(nN_d+l;0), \ldots, x_i(nN_d+l;P-1)]^T$ is next OFDM modulated on P subcarriers, parallel-to-serial (P/S) converted, and transmitted from Txi during the lth OFDM symbol interval. At the receiver, the received data is S/P converted and OFDM demodulated to output $\mathbf{y}(n) \triangleq [y(n;0), \ldots, y(n;P-1)]^T$, where $y(n;p)$ denotes the sample corresponding to the pth subcarrier of the nth OFDM symbol. The differential decoder $\mathcal{M}_{\rm d}^{-1}\{\cdot\}$ takes as inputs $2N_d$ data vectors $\mathbf{y}((n-1)N_d), \ldots, \mathbf{y}(nN_d+N_d-1)$, performs differential decoding, and outputs estimates $\hat{\mathbf{s}}(nN_s), \ldots, \hat{\mathbf{s}}(nN_s+N_s-1)$. Finally, the spectral decoder $\mathcal{M}_{\rm s}^{-1}\{\cdot\}$ performs decoding. The channel between Txi and the Rx is modeled as a finite impulse response (FIR) filter with coefficients $\{h_i(l)\}_{l=0}^{L}$, where L denotes the channel order. The frequency response at the pth subchannel is $H_i(p) \triangleq \sum_{l=0}^{L} h_i(l) \exp(-\jmath 2\pi lp/P)$. Since OFDM converts the frequency-selective channel into a set of frequency-flat channels, we have

$$y(n;p) = \textstyle\sum_{i=1}^{N_t} H_i(p) x_i(n;p) + w(n;p), \quad (2)$$

where $w(n;p)$ denotes the zero-mean complex white Gaussian noise with variance $N_0/2$ per dimension.

The problem of interest to this work is to design $\mathcal{M}_{\rm d}\{\cdot\}$ and $\mathcal{M}_{\rm s}\{\cdot\}$ for wideband differential transmission that yields the maximum spatio-spectral diversity gain as well as significant coding gain.

III. DIFFERENTIAL MODULATION

The proposed differential scheme makes use of *block orthogonal designs*. To introduce necessary notation, the idea of complex orthogonal designs [5] is briefly reviewed. A (generalized) complex orthogonal design of *size* N_t in variables $s_0, \ldots, s_{N_s-1}$ is an $N_t \times N_d$ matrix $\mathbf{C}$, formed by entries 0, $\pm s_0, \pm s_0^*, \ldots, \pm s_{N_s-1}, \pm s_{N_s-1}^*$ and their linear combinations, that satisfies $\mathbf{C}\mathbf{C}^H = \alpha(|s_0|^2 + \cdots + |s_{N_s-1}|^2)\mathbf{I}_{N_t}$ for some positive constant α [5]. To facilitate differential modulation, we classify complex orthogonal designs into two categories: *square* and *non-square* complex orthogonal designs. Square designs are those with $N_t = N_d$, which exist for powers of two, i.e., $N_t = 2, 4, 8, \ldots$. Square designs also form the base of *non-square* designs with $N_d \neq N_t$ (and necessarily $N_d > N_t$ [5]). Specifically, any non-square complex orthogonal design can be obtained by taking the first N_t rows of the corresponding base square design [5]. The ratio N_s/N_d is called the *rate* of the design. It is known that full-rate (i.e., $R_{\rm d} \triangleq N_s/N_d = 1$) complex orthogonal designs exist only for $N_t = 2$. The best rate known for $N_t = 3$ and 4 is $R_{\rm d} = 3/4$, whereas for $N_t > 4$ it is $R_{\rm d} = 1/2$ [5], [6].

Back to the DSTF system, let $\boldsymbol{\mathcal{S}}(n)$ be $\bar{N}_t P \times N_d$ matrices formed from these vectors through the following block complex orthogonal design

$$\boldsymbol{\mathcal{S}}(n) \triangleq N_s^{-1/2} \sum_{k=0}^{N_s-1} [\mathbf{A}_k \otimes \mathbf{s}(nN_s+k) + \mathbf{B}_k \otimes \mathbf{s}^*(nN_s+k)], \tag{3}$$

where $\mathbf{A}_k$ and $\mathbf{B}_k$ are $\bar{N}_t \times N_d$ matrices associated with a complex orthogonal design of size $\bar{N}_t$ [5], [7], which is identical to the size of a base square complex orthogonal design. In particular, $\bar{N}_t = N_t$, if $N_t = 2, 4, 8, \ldots$; $\bar{N}_t = 4$, if $N_t = 3$; $\bar{N}_t = 8$, if $N_t = 5, 6, 7$; so on and so forth. Let $\bar{\boldsymbol{\mathcal{X}}}(-1) = \sqrt{E_s}\mathbf{I}_{\bar{N}_t} \otimes \mathbf{1}_{P\times 1}$ be the initial DSTF code matrix, where $\sqrt{E_s}$ is an energy scaling factor, and [cf. (1)]

$$\bar{\boldsymbol{\mathcal{X}}}(n) \triangleq \begin{bmatrix} \mathbf{x}_1(nN_d) & \ldots & \mathbf{x}_1(nN_d+N_d-1) \\ \vdots & \vdots & \vdots \\ \mathbf{x}_{\bar{N}_t}(nN_d) & \ldots & \mathbf{x}_{\bar{N}_t}(nN_d+N_d-1) \end{bmatrix}. \tag{4}$$

The proposed differential encoding scheme proceeds *as if there were $\bar{N}_t$ transmit antennas*:

$$\bar{\boldsymbol{\mathcal{X}}}(n) = \bar{\boldsymbol{\mathcal{D}}}_x(n-1)\boldsymbol{\mathcal{S}}(n),\ n = 0, 1, \ldots, \tag{5}$$

where the $\bar{N}_t P \times \bar{N}_t P$ matrix $\bar{\boldsymbol{\mathcal{D}}}_x(n)$ is similarly defined as $\bar{\boldsymbol{\mathcal{X}}}(n)$ in (4) with each subvector $\mathbf{x}_i(nN_d+l)$ replaced by the corresponding diagonal matrix $\mathbf{D}_{x_i}(nN_d+l) \triangleq \mathrm{diag}\{\mathbf{x}_i(nN_d+l)\}$. Throughout the paper, we assume that the coded symbols $\mathbf{s}(n)$ are drawn from a constant-modulus constellation $\mathcal{A}_s$ (e.g., PSK) with unit-energy elements. This assumption, along with the orthogonal design (3), suggest that $\bar{\boldsymbol{\mathcal{D}}}_x(n)$ is a (scaled) unitary matrix with $\bar{\boldsymbol{\mathcal{D}}}_x(n)\bar{\boldsymbol{\mathcal{D}}}_x^H(n) = E_s\mathbf{I}_{N_d P}$. Since we have N_t rather than $\bar{N}_t$ antennas, we cannot proceed to transmit $\bar{\boldsymbol{\mathcal{X}}}(n)$. Instead, the following matrix is transmitted $\boldsymbol{\mathcal{X}}(n) = \mathbf{T}\bar{\boldsymbol{\mathcal{X}}}(n)$, where $\mathbf{T} \triangleq [\mathbf{I}_{N_t P}, \mathbf{0}_{N_t P\times(\bar{N}_t - N_t)P}]$. This can be thought of as having $\bar{N}_t - N_t$ *virtual* transmit antennas associated with *zero* channel response, which are used to "transmit" the last $(\bar{N}_t - N_t)P$ rows of $\bar{\boldsymbol{\mathcal{X}}}(n)$.

Let $\mathbf{h}_{{\rm f},i} \triangleq [H_i(0), \ldots, H_i(P-1)]^T$ and $\mathbf{D}_{h_{{\rm f},i}} \triangleq \mathrm{diag}\{\mathbf{h}_{{\rm f},i}\}$. In vector form, the received signal can be written as [cf. (2)]:

$$\mathbf{y}(nN_d+l) = \textstyle\sum_{i=1}^{\bar{N}_t} \mathbf{D}_{x_i}(nN_d+l)\mathbf{h}_{{\rm f},i} + \mathbf{w}(nN_d+l). \tag{6}$$

Let $\boldsymbol{y}(n) \triangleq [\mathbf{y}^T(nN_d), \ldots, \mathbf{y}^T(nN_d+N_d-1)]^T$ and $\boldsymbol{w}(n)$ be similarly formed from $\{\mathbf{w}(nN_d+l)\}$. We can write (6) collectively as $\boldsymbol{y}(n) = \bar{\boldsymbol{\mathcal{D}}}_x^T(n)\bar{\boldsymbol{h}}_{\rm f} + \boldsymbol{w}(n)$, where $\bar{\boldsymbol{h}}_{\rm f} \triangleq [\mathbf{h}_{{\rm f},1}^T, \ldots, \mathbf{h}_{{\rm f},\bar{N}_t}^T]^T$. An equivalent form of (5) is $\bar{\boldsymbol{\mathcal{D}}}_x(n) = \bar{\boldsymbol{\mathcal{D}}}_x(n-1)\boldsymbol{\mathcal{D}}_s(n)$, where [see (3)] $\boldsymbol{\mathcal{D}}_s(n) = \frac{1}{\sqrt{N_s}}\sum_{k=0}^{N_s-1}[\mathbf{A}_k \otimes \mathbf{D}_s(nN_s+k) + \mathbf{B}_k \otimes \mathbf{D}_s^*(nN_s+k)]$, with $\mathbf{D}_s(n) \triangleq \mathrm{diag}\{\mathbf{s}(n)\}$. Note that $\boldsymbol{\mathcal{D}}_s(n)$ is unitary by construction. Hence,

$$\begin{aligned} \boldsymbol{y}(n) =& \boldsymbol{\mathcal{D}}_s^T(n)\bar{\boldsymbol{\mathcal{D}}}_x^T(n-1)\bar{\boldsymbol{h}}_{\rm f} + \boldsymbol{w}(n) \\ \triangleq& \boldsymbol{\mathcal{D}}_s^T(n)\boldsymbol{y}(n-1) + \boldsymbol{v}(n), \end{aligned} \tag{7}$$

where $\boldsymbol{v}(n) \triangleq \boldsymbol{w}(n) - \boldsymbol{\mathcal{D}}_s^T(n)\boldsymbol{w}(n-1)$ formed by i.i.d. complex Gaussian entries with zero-mean and variance N_0 per dimension. Hence,

$$\begin{aligned} \boldsymbol{y}(n) =& N_s^{-1/2} \sum_{k=0}^{N_s-1}\sum_{l=0}^{N_d-1} \Big\{[\mathbf{a}_{k,l} \otimes \mathbf{D}_y((n-1)N_d+l)] \\ & \times \mathbf{s}(nN_s+k) + [\mathbf{b}_{k,l} \otimes \mathbf{D}_y((n-1)N_d+l)] \\ & \times \mathbf{s}^*(nN_s+k)\Big\} + \boldsymbol{v}(n), \end{aligned}$$

where $\mathbf{a}_{k,l}$ and $\mathbf{b}_{k,l}$ are $N_d \times 1$ vectors formed from the lth row of $\mathbf{A}_k$ and $\mathbf{B}_k$, respectively. The above equation is referred to as the *fundamental differential receiver equation* for $N_t \geq 2$. The variance of $\boldsymbol{v}(n)$ is twice that of $\mathbf{w}(n)$, which translates to a 3 dB loss of SNR.

Due to the unitary structure of the proposed differential encoder, the ML detection of the space-time multiplexed vectors $\{\mathbf{s}(nN_s+k)\}_{k=0}^{N_s-1}$ is decoupled. In particular, we have the following result.

Theorem 1: [7] The ML detection of the N_s coded vectors $\{\mathbf{s}(nN_s+k)\}_{k=0}^{N_s-1}$ based on two adjacent vectors $\boldsymbol{y}(n-1)$ and $\boldsymbol{y}(n)$ decouples into N_s individual detections:

$$\hat{\mathbf{s}}^{\rm ML}(nN_s+k) = \arg\max_{\mathbf{s}_k\in\boldsymbol{\mathcal{B}}_s} \Re\big\{\mathbf{z}^H(nN_s+k)\boldsymbol{\Omega}_y^{1/2}(n-1)\mathbf{s}_k\big\},$$

where $\boldsymbol{\mathcal{B}}_s \subseteq \mathcal{A}_s^{P\times 1}$ denotes a valid codebook (of the spectral encoder $\mathcal{M}_{\rm s}\{\cdot\}$), $\mathbf{z}(nN_s+k) \triangleq \boldsymbol{\Omega}_y^{-1/2}(n-1)$

$\sum_{l=0}^{N_d-1}\Big\{[\mathbf{a}_{k,l}^T\otimes\mathbf{D}_y^H((n-1)N_d+l)]\boldsymbol{y}(n)+[\mathbf{b}_{k,l}^T\otimes\mathbf{D}_y((n-1)N_d+l)]\boldsymbol{y}^*(n)]\Big\}$ and $\mathbf{\Omega}_y(n-1)\triangleq\sum_{l=0}^{N_d-1}\mathbf{D}_y((n-1)N_d+l)\mathbf{D}_y^H((n-1)N_d+l)$.

IV. Spectral Encoding

We assume *(correlated) Rayleigh fading* channels and leave extensions to other channel models elsewhere. Specifically, assume **A1:** The channel vectors $\mathbf{h}_i\triangleq[h_i(0),\ldots,h_i(L)]^T$, $i=1,2$, are zero-mean complex Gaussian with non-singular covariance matrix $\mathbf{R}_h\triangleq E\{\mathbf{h}\mathbf{h}^H\}$, where $\mathbf{h}\triangleq[\mathbf{h}_1^T,\ldots,\mathbf{h}_{N_t}^T]^T$. To minimize decoding complexity, we are interested in *short* codes that encode across a minimally required number of subchannels for full diversity, meanwhile achieving a coding gain as large as possible. The idea is to transmit coded symbols in well separated subchannels by *subcarrier interleaving (SI)*. SI has recently been introduced to achieve full spectral diversity in systems with *one* transmit antenna [8]. Let $\mathcal{I}\triangleq\{0,1,\ldots,P-1\}$ collect the indices of all subcarriers. Briefly stated, SI is a partition of $\mathcal{I}$ into M non-overlapping subsets $\mathcal{I}^{(m)}\triangleq\{p_{m,0},p_{m,1},\ldots,p_{m,Q_m-1}\}$, where Q_m is the number of subcarriers in the mth subset. For channels satisfying **A1**, we need $Q_m\ge L+1$ to achieve the maximum spectral diversity [8]. We choose the minimum $Q_m=L+1$ so that the decoding complexity is minimized. Among other alternatives, the following SI scheme is conceptually simple [8]: $\mathcal{I}^{(m)}=\big\{m,\ \ M+m,\ \ \ldots,\ \ LM+m\big\}$, where $M\triangleq P/(L+1)$, and P is assumed a multiple of $L+1$.

The input-output relation, when SI is utilized, for the mth subcarrier subset, $m=0,\ldots,M-1$, is given by [7]

$$\begin{aligned}\mathbf{z}_m(nN_s+k)&=N_s^{-1/2}\mathbf{\Omega}_{y,m}^{1/2}(n-1)\mathbf{s}_m(nN_s+k)\\&\quad+\boldsymbol{\mu}_m(nN_s+k),\ k=0,\ldots,N_s-1\end{aligned}\tag{8}$$

where $\mathbf{z}_m(nN_s+k)\in\mathbb{C}^{(L+1)\times1}$, $\mathbf{\Omega}_{y,m}(n-1)\in\mathbb{C}^{(L+1)\times(L+1)}$, $\mathbf{s}_m(nN_s+k)\in\boldsymbol{\mathcal{B}}_{s,m}$, and $\boldsymbol{\mu}_m(nN_s+k)\in\mathbb{C}^{(L+1)\times1}$ are quantities associated with the mth subcarrier subset, similarly defined as their counterparts in Theorem 1. The probability of erroneously choosing $\mathbf{s}_m(nN_s+k)$ as $\mathbf{s}'_m(nN_s+k)$ by the ML detector is upper-bounded by (henceforth, we drop indices for brevity) [7]:

$$\mathrm{P}(\mathbf{s}_1\to\mathbf{s}_2)\le[E_s/(8N_0)]^{-r_e}\Big[\det(\mathbf{R}_h)\prod_{l=1}^{r_e}\lambda_l\Big]^{-1}\tag{9}$$

where $r_e\triangleq\operatorname{rank}(\mathbf{\Phi}_e)\le N_t(L+1)$, $\mathbf{\Phi}_e\triangleq N_s^{-1}\mathbf{I}_{N_t}\otimes(\boldsymbol{\mathcal{F}}_m^H\mathbf{D}_e^*\mathbf{D}_e\boldsymbol{\mathcal{F}}_m)$, and $\{\lambda_l\}_{l=1}^{r_e}$ are the r_e non-zero eigenvalues of $\mathbf{\Phi}_e$. Here, $\mathbf{D}_e\triangleq\operatorname{diag}(\mathbf{e})$, $\mathbf{e}\triangleq\mathbf{s}-\mathbf{s}'$, and $\boldsymbol{\mathcal{F}}_m\in\mathbb{C}^{(L+1)\times(L+1)}$ is formed by rows $m,m+M,\ldots,m+LM$ of the P-point FFT matrix $\boldsymbol{\mathcal{F}}\in\mathbb{C}^{P\times(L+1)}$: $[\boldsymbol{\mathcal{F}}]_{p,q}\triangleq\exp(-\jmath2\pi(p-1)(q-1)/P)$. Following the routines, $G_\mathrm{d}\triangleq\min_{\forall\mathbf{e}\neq\mathbf{0}}r_e$ is called the *diversity advantage*, which determines the slope of the symbol error rate vs. SNR at high SNR on a log-log scale and must be maximized first; $G_\mathrm{c}\triangleq\min_{\forall\mathbf{e}\neq\mathbf{0}}\big[\det(\mathbf{R}_h)\prod_{l=1}^{r_e}\lambda_l\big]^{1/r_e}$ is the *coding advantage* over an uncoded system. We summarize the optimum G_d and G_c as follows:

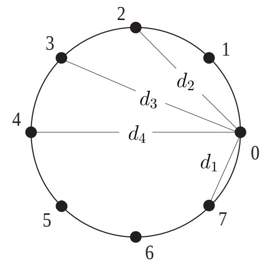

Fig. 2. 8-PSK constellation.

Theorem 2: [7] Under condition **A1**, the maximum diversity advantage of the DSTF system is $G_{\mathrm{d,max}}=N_t(L+1)$, which is achieved iff the code $\mathbf{s}$ has a uniform Hamming distance of $L+1$. Any maximum-diversity achieving code has a coding advantage given by $G_{\mathrm{c,max}}=N_s^{-1}(L+1)[\delta_{\min}^{2N_t}\det(\mathbf{R}_h)]^{1/[N_t(L+1)]}$, where $\delta_{\min}$ denotes the *minimum product distance* of the code: $\delta_{\min}=\min_{\forall\mathbf{e}\neq\mathbf{0}}|\det(\mathbf{D}_e)|$.

For notational brevity, we will drop the subcarrier subset index m. To achieve a code rate of R_s bps/Hz, we need a codebook with $N_\mathrm{c}\triangleq2^{R_s(L+1)}$ distinct codewords of length $L+1$ (which is the minimally required code length for full diversity), with coded symbols drawn from an M_c-PSK constellation $\mathcal{A}_s$. Let $\mathbf{s}_i\triangleq[s_{i,0},\ldots,s_{i,L}]^T$ denote the ith codeword, and $\boldsymbol{\mathcal{B}}_s\triangleq[\mathbf{s}_0,\ldots,\mathbf{s}_{N_\mathrm{c}-1}]_{(L+1)\times N_\mathrm{c}}$ denotes the codebook. To ensure that $\boldsymbol{\mathcal{B}}_s$ has a uniform Hamming distance, the constellation size M_c must be no less than N_c; otherwise, there exist at least one pair of codewords that share a coded symbol at the same location, which decreases the minimum Hamming distance to less than $L+1$. We choose $M_\mathrm{c}=N_\mathrm{c}$ to minimize the decoding complexity. Let us label the constellation points in $\mathcal{A}_s$ as $0,1,\ldots,M_\mathrm{c}-1$ (e.g., the 8-PSK shown in Fig. 2) and form the sequence $\vec{c}\triangleq[0,1,\ldots,M_\mathrm{c}-1]$. The uniform Hamming distance requirement mandates that each row of $\boldsymbol{\mathcal{B}}_s$ be a permutation of $\vec{c}$, and any code formed by permutations has a uniform Hamming distance of $L+1$. However, there are a total of $(N_\mathrm{c}!)^L$ such permutation codes, all achieving the full diversity. An exhaustive search for codes with the best product distance quickly becomes infeasible even for relatively small N_c and L.

To facilitate code constructions, we introduce the idea of *constellation decimation*, which effectively imposes a linear structure on the code. The linear structure makes the analysis of distance property and search for good codes significantly easier. Specifically, let $\vec{c}[k]$ be the kth element of $\vec{c}$. Denote by $\vec{c}_q\triangleq\{\vec{c}_q[0],\vec{c}_q[1],\ldots,\vec{c}_q[M_\mathrm{c}-1]\}$ the *qth decimation* of $\vec{c}$, $q=1,2,\ldots,M_\mathrm{c}$, where $\vec{c}_q[k]\triangleq\vec{c}[qk\pmod{M_\mathrm{c}}]$, $k=0,1,\ldots,M_\mathrm{c}-1$. It is noted that q and M_c have to be relatively prime so that the decimated sequence will be a permutation of $\vec{c}$.

A *linear constellation decimation (LCD)* code $\boldsymbol{\mathcal{B}}_s$ is an $(L+1)\times M_\mathrm{c}$ matrix, each row of which is obtained by a proper decimation of $\vec{c}$. We use the notation $\boldsymbol{\mathcal{B}}_s=\langle q_0,q_1,\ldots,q_L\rangle$ to signify that LCD code $\boldsymbol{\mathcal{B}}_s$ is obtained by using decimation factors q_j, $j=0,1,\ldots,L$, for the jth row of $\boldsymbol{\mathcal{B}}_s$. Consider two

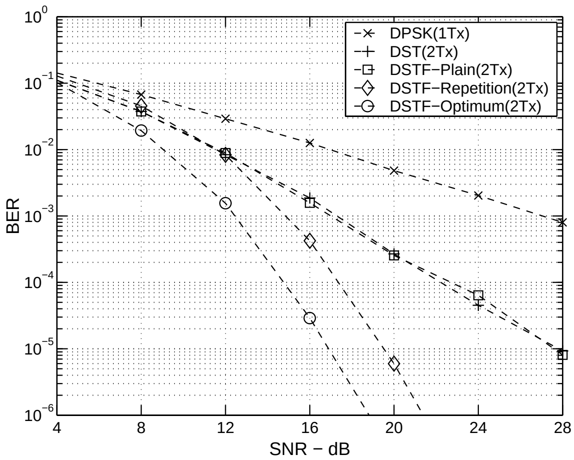

Fig. 3. Bit error rate (BER) with $N_t = 2$ transmit antennas transmission rate $R = 1$ bit/s/Hz.

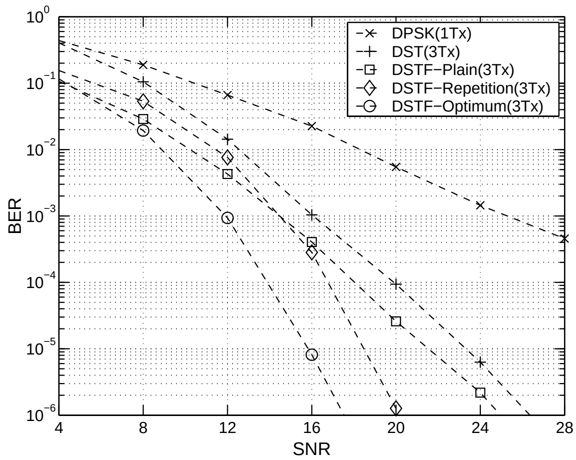

Fig. 4. Bit error rate (BER) with $N_t = 3$ transmit antennas transmission rate $R = 3/4$ bits/s/Hz.

LCD codes $L = 2$ (i.e., 3-ray channel), $\mathcal{A}_s = 8$-PSK as shown in Fig. 2, and rate $R_\mathrm{s} = 1$ bps/Hz:

$$\begin{aligned}\boldsymbol{\mathcal{B}}_s^{\langle 1,1,1\rangle} &= \begin{bmatrix} 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7\\ 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7\\ 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7\end{bmatrix},\\ \boldsymbol{\mathcal{B}}_s^{\langle 1,3,5\rangle} &= \begin{bmatrix} 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7\\ 0, & 3, & 6, & 1, & 4, & 7, & 2, & 5\\ 0, & 5, & 2, & 7, & 4, & 1, & 6, & 3\end{bmatrix}.\end{aligned} \tag{10}$$

Effectively, $\boldsymbol{\mathcal{B}}_s^{\langle 1,1,1\rangle}$ coincides with a repetition code. It can be quickly verified that both codes have a uniform Hamming distance $L + 1 = 3$. The minimum product distances of the two codes are $\delta_{\text{min}}^{\langle 1,1,1\rangle} = d_1^3$ and $\delta_{\text{min}}^{\langle 1,3,5\rangle} = d_3 d_1^2$ (cf. Fig. 2), respectively. By Theorem 2, $\mathbf{S}^{\langle 1,3,5\rangle}$ achieves a coding gain of $10\log_{10}\left(\delta_{\text{min}}^{\langle 1,3,5\rangle}/\delta_{\text{min}}^{\langle 1,1,1\rangle}\right)^{2/(L+1)} \approx 2.55$ dB relative to the repetition code. In fact, $\mathbf{S}^{\langle 1,3,5\rangle}$ can be shown (by a quick computer search) to be the optimum LCD code with the largest product distance.

Finally, we comment on the transmission rate of the proposed DSTF system. For a DSTF system using a spectral encoder with code rate of R_s bits per coded symbol, the overall transmission rate R is defined as $R \triangleq R_\mathrm{d} R_\mathrm{s}$, where we recall that R_d is the rate of the orthogonal design used for differential encoding (see Section III). Interested readers are referred to [7] for additional details on LCD codes.

V. Simulation Results

Consider an OFDM system with $P = 48$ subcarriers. The transmitter may have multiple Tx's, but the receiver has only one Rx. The channel coefficients are assumed complex Gaussian with zero-mean and variance $N_0 = 1/(L+1)$, where $L = 2$ (i.e., 3-ray Rayleigh channels). The following schemes are compared: **1) DPSK:** Differential OFDM with one Tx and standard DPSK applied on each subcarrier, which yields no diversity and serves as a benchmark for other diversity systems. **2) DST:** Differential space-time coded OFDM with multiple Tx's and the unitary differential space-time code [3] applied on each subcarrier. **3) DSTF-Plain:** DSTF with mutiple Tx's but no spectral encoding (thus the word *plain*). **4) DSTF-Repetition:** DSTF with mutiple Tx's and repetition code for spectral encoding. **5) DSTF-Optimum:** DSTF with multiple Tx's and the optimum LCD code $\langle 1, 3, 5\rangle$ for spectral encoding.

For the diversity schemes (e.g., DST and DSTF), we consider both $N_t = 2$ and 3 Tx's. All DSTF schemes involve a rate loss when $N_t = 3$, due to the use of the rate R_d-$3/4$ orthogonal design for $N_t = 3$ (see Section III). To make fair comparison, a rate-3/4 convolutional code is used for all non-DSTF schemes when $N_t = 3$ to ensure all schemes are compared at the same spectral efficiency. The rate-3/4 code is obtained by puncturing a rate-1/2 convolutional code. Fig. 3 depicts the BER vs. SNR (defined as E_s/N_0) when $N_t = 2$ and the transmission rate $R = 1$ b/s/Hz, whereas Fig. 4 depicts the results when $N_t = 3$ and the transmission rate $R = 3/4$ b/s/Hz. It is seen that DSTF-Optimum compares favorably with all other schemes.

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