DIFFERENTIAL SPACE-TIME MODULATION WITH TRANSMIT-BEAMFORMING FOR CORRELATED MIMO FADING CHANNELS*

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ABSTRACT

While the knowledge of each channel realization is not available in a system with differential space-time modulation, channel correlation can be easily estimated without training at the receiver, and exploited by the transmitter to enhance the error probability performance. We develop a transmission scheme that combines transmit-beamforming with differential space-time modulation based on orthogonal space-time block coding. Error probability is analyzed for both correlated and independent channels. Based on the error probability analysis, we derive power loading coefficients to improve performance.

1. INTRODUCTION

Recent advances in wireless communications show that multi-antenna transmission systems can support high data rates with low error probability. Without any channel state information (CSI) at the transmitter, space-time coding offers an effective counter-measure against fading, and thereby reduces the error probability. Whenever (even partial) CSI is available at the transmitter, it should be exploited to further improve the performance of multi-antenna transmission systems. Since in most cases, the transmitter cannot acquire the CSI perfectly, utilization of partial CSI at the transmitter has received considerable attention recently.

A general statistical model of partial CSI is presented in [8, 12, 14]. Based on this model, a linear transformation was applied to orthogonal space-time block coding (STBC) to enhance the SER performance [8]. Two cases of partial CSI, termed mean feedback and covariance feedback, were studied to maximize channel capacity [12]; capacity maximization based on covariance feedback was also investigated for multiple input multiple output (MIMO) systems in [6]. The optimal beamforming and STBC that minimizes symbol error probability were derived in [13] and [14], based on channel mean and correlation, respectively. Note that all these works require CSI at the receiver.

In this paper, we consider differential space-time modulation based on orthogonal STBC which was also investigated in [3, 7] without partial CSI for independent, identically distributed (iid) fading channels. Since practical multi-antenna systems may exhibit strong correlation among fading channels associated with different transmit antennae on the downlink [6, 10], we will focus on correlated channels. While knowledge of each channel realization is not available in a differential space-time transmission system, channel correlations can be estimated at the receiver without training, and fed back to the transmitter. Based on error probability analysis, we will exploit the channel correlations at the transmitter to combine differential space-time modulation with transmit-beamforming, and thereby enhance the error probability performance.

*Work in this paper was supported by the ARL/CTA grant no. DAAD19-01-2-011.

2. SIGNAL MODEL

Consider a multi-antenna transmission system comprising N_T transmit antennae, and a single receive antenna, signaling over a Rayleigh flat-fading environment. Suppose that the base station (BS) and the mobile user assume the roles of transmitter and receiver, respectively. Let h_m denote the channel coefficient between the mth transmit and the receive antenna, which is a complex Gaussian random variable with zero-mean. In a wireless environment where the BS is elevated and unobstructed, it has been shown that in practical systems with reasonable antenna spacing, channel gains associated with different transmit antennae exhibit strong correlations [6]. Letting $\mathbf{h} := [h_1, \ldots, h_{N_T}]^T$, we define the channel correlation matrix as $\mathbf{R}_h := E[\mathbf{h}\mathbf{h}^{\mathcal{H}}]$, where the superscript $^{\mathcal{H}}$ (^{T}) denotes Hermitian (transposition).

The P symbols transmitted in the tth block are first collected in an $N \times N$ space-time code matrix $\mathbf{S}_t = (1/P) \sum_{p=1}^{P} (\mathbf{\Phi}_p s_{t,p}^R + j \mathbf{\Psi}_p s_{t,p}^I),\ t > 0$ [3], where $s_{t,p}^R$ and $s_{t,p}^I$ are real and imaginary parts of the complex symbol $s_{t,p}$, respectively, i.e., $s_{t,p} = s_{t,p}^R + j s_{t,p}^I$, and $N \times N$ matrices $\mathbf{\Phi}_p$ and $\mathbf{\Psi}_p$ satisfy the orthogonal conditions given in [3]. Drawing $s_{t,p}$ from M-PSK constellations, and letting $|s_{t,p}| = 1$, it can be shown that the matrix $\mathbf{S}_t$ is unitary, i.e., $\mathbf{S}_t^{\mathcal{H}} \mathbf{S}_t = \mathbf{I}_N$. The $N \times N$ code matrix $\mathbf{C}_t$ for differential space-time modulation is then written recursively as [3, 4, 5]

$$\mathbf{C}_t = \mathbf{S}_t \mathbf{C}_{t-1}, \quad t > 0, \tag{1}$$

with $\mathbf{C}_0 = \mathbf{I}_N$. Since $\mathbf{S}_t$ is unitary, matrix $\mathbf{C}_t$ is unitary too, by design. For an arbitrary N_T, we may not be able to find a square code matrix $\mathbf{S}_t$ with $N = N_T$ [3]. In this case, we can construct a square code matrix $\mathbf{S}_t$ with $N > N_T$, find $\mathbf{C}_t$ using (1), and transmit the first N_T columns of $\mathbf{C}_t$ from N_T transmit antennae as in [3]. Mathematically, letting $\mathbf{\Theta} = [\mathbf{I}_{N_T} \;\; \mathbf{0}_{N_T \times (N-N_T)}]^T$, the matrix codeword transmitted over N_T antennae in the tth block is $\mathbf{C}_t\mathbf{\Theta}$.

When the channel coefficients associated with different transmit antennae are correlated, a modulation scheme exploiting the channel correlation at the transmitter is well motivated. To this end, we will transmit the codeword $\mathbf{C}_t\mathbf{\Theta}$ along the eigenvectors of the channel correlation matrix $\mathbf{R}_h$ with proper power loaded on each eigenvector. This transmission scheme, termed eigen-beamforming, was introduced in [14] for *coherent* STBC over correlated fading channels. The eigen-decomposition of matrix $\mathbf{R}_h$ can be written as $\mathbf{R}_h = \mathbf{U\Lambda U}^{\mathcal{H}}$, where the diagonal matrix $\mathbf{\Lambda}$ contains the ordered eigenvalues of $\mathbf{R}_h$, and the unitary matrix $\mathbf{U}$ consists of the corresponding eigenvectors. Then, the transmitted signal in the tth block is given by the $N \times N$ matrix $\mathbf{X}_t = \sqrt{P\mathcal{E}_s}\mathbf{C}_t\mathbf{\Theta D U}^{\mathcal{H}}$, where the diagonal matrix $\mathbf{D}$ contains power loading coefficients which will be specified later in Section 3.3. We have the following constraint on $\mathbf{D}$: $\sum_{i=1}^{N_t}[\mathbf{D}]_{i,i}^2 = 1$. Using this constraint, we can verify that $\mathrm{Tr}(\mathbf{X}_t\mathbf{X}_t^{\mathcal{H}}) = P\mathcal{E}_s$, where $\mathcal{E}_s$ stands for the energy per transmitted symbol. We can also write $\mathbf{X}_t$ as a recursion, initialized by $\mathbf{X}_0 = \sqrt{P\mathcal{E}_s}\mathbf{\Theta D U}^{\mathcal{H}}$, as [c.f. (1)]

$$\mathbf{X}_t = \mathbf{S}_t\mathbf{X}_{t-1}, \quad t > 0. \tag{2}$$

Comparing (1) with (2) reveals that the fundamental differential transmission equation is not changed by the loaded transmit eigen-beamforming matrices $\mathbf{DU}^{\mathcal{H}}$. From (2), it is seen that we need to beamform and power load only in the first transmitted block $\mathbf{X}_0$; and after the first block, signals will be automatically transmitted along eigen-beams without any beamforming operation. The received signal in the tth block can be written in an $N \times 1$ vector $\mathbf{y}_t$ as

$$\mathbf{y}_t = \mathbf{X}_t\mathbf{h} + \mathbf{w}_t, \tag{3}$$

where $\mathbf{w}_t$ contains complex additive white Gaussian noise (AWGN) with mean zero, and variance $N_0/2$ per dimension. We assume that $\mathbf{h}$ remains invariant over two consecutive blocks, and we will detect $\mathbf{S}_t$ based on $\mathbf{y}_{t-1}$ and $\mathbf{y}_t$.

Using the fundamental differential receiver equation $\mathbf{y}_t = \mathbf{S}_t\mathbf{y}_{t-1} + \mathbf{w}_t - \mathbf{S}_t\mathbf{w}_{t-1}$, we can detect codeword $\mathbf{S}_t$ as $\hat{\mathbf{S}}_t = \arg\max_{\mathbf{S}} \mathrm{Re}(2\mathbf{y}_{t-1}^{\mathcal{H}}\mathbf{S}^{\mathcal{H}}\mathbf{y}_t)$ [3, 4]. Let $z_{p,R} := \mathrm{Re}(2\mathbf{y}_{t-1}^{\mathcal{H}} \mathbf{\Phi}_p^{\mathcal{H}}\mathbf{y}_t)$, $z_{p,I} := \mathrm{Re}(-j2\mathbf{y}_{t-1}^{\mathcal{H}}\mathbf{\Psi}_p^{\mathcal{H}}\mathbf{y}_t)$, and $z_p := z_{p,R} + jz_{p,I}$. Due to the structure of $\mathbf{S}_t$, the detector for the codeword $\mathbf{S}_t$ reduces to a symbol-by-symbol detector [3]

$$\hat{s}_{t,p} = \arg\max_{s} \mathrm{Re}(z_p s^*). \tag{4}$$

For QPSK, $s = (\pm 1 \pm j)/\sqrt{2}$, eq. (4) becomes

$$\hat{s}_{t,p}^R = \mathrm{sign}(z_{p,R}), \;\; \hat{s}_{t,p}^I = \mathrm{sign}(z_{p,I}). \tag{5}$$

3. PERFORMANCE AND POWER LOADING

3.1. Exact BER of BPSK and QPSK constellations

Consider the decision variable $z_{p,R}$ in (5). If $s_{t,p}^R$ takes ± 1 values with equal probability, the BER of $s_{t,p}^R$ is given by $P_b(e) = P(z_{p,R} < 0|s_{t,p}^R = 1)$. It can be shown that this error probability is the same for $s_{t,p}^R$ and $s_{t,p}^I$, $\forall p$; thus, it also stands for the overall BER. The decision variable $z_{p,R}$ can also be expressed as

$$z_{p,R} = \mathbf{y}_{t-1}^{\mathcal{H}}\mathbf{\Phi}_p^{\mathcal{H}}\mathbf{y}_t + \mathbf{y}_t^{\mathcal{H}}\mathbf{\Phi}_p\mathbf{y}_{t-1}^{\mathcal{H}} = \mathbf{y}^{\mathcal{H}}\bar{\mathbf{\Phi}}_p\mathbf{y}, \tag{6}$$

where $\bar{\mathbf{\Phi}}_p := \begin{bmatrix} \mathbf{0} & \mathbf{\Phi}_p^{\mathcal{H}} \\ \mathbf{\Phi}_p & \mathbf{0} \end{bmatrix}$. It is seen from (6) that $z_{p,R}$ is a quadratic form of the complex Gaussian random vector $\mathbf{y}$. Thus, the Laplace transform of the pdf of $z_{p,R}$ is given by [9, p. 595]

$$\phi(\omega) := \frac{1}{\det(\mathbf{I} + \omega\mathbf{R}_{\mathrm{y}}\bar{\mathbf{\Phi}}_{\mathrm{p}})} = \frac{1}{\prod_{i=1}^{2N}(1+\omega\lambda_i)}, \tag{7}$$

where λ_i is the eigenvalue of matrix $\mathbf{A} := \mathbf{R}_y\bar{\mathbf{\Phi}}_p$. Then, the error probability can be found as [2]

$$P_b(e) = -\sum_{\omega_i > 0} \mathrm{Res}[\phi(\omega)/\omega; \omega_i], \tag{8}$$

where ω_i is a pole of $\phi(\omega)/\omega$, and $\mathrm{Res}[f(x); x_i]$ denotes the residue of $f(x)$ at x_i. From (7), we have $\omega_i = -1/\lambda_i$, if $\lambda_i \neq 0$. To evaluate the BER in (8), we need to find the eigenvalues of $\mathbf{A}$. In [1], we prove the following proposition and corollary.

Proposition 1 *The matrix $\mathbf{A}$ is similar to*

$$\mathbf{G} := \begin{bmatrix} \lambda_{B,1}\mathbf{D}_h & \lambda_{B,2}(\mathbf{D}_h + N_0\mathbf{I}_N) \\ \lambda_{B,1}(\mathbf{D}_h + N_0\mathbf{I}_N) & \lambda_{B,2}\mathbf{D}_h \end{bmatrix}, \tag{9}$$

where $\mathbf{D}_h := P\mathcal{E}_s\mathbf{\Theta D}^2\mathbf{\Lambda\Theta}^{\mathcal{H}}$; $\lambda_{B,1} = (\alpha + j\sqrt{|\alpha^2 - 4|})/2$ and $\lambda_{B,2} = (\alpha - j\sqrt{|\alpha^2 - 4|})/2$ are two eigenvalues of the matrix $\mathbf{B}_1 := \mathbf{C}_t^{\mathcal{H}}\mathbf{\Phi}_p\mathbf{C}_{t-1}$.

Corollary 1 *With $D_{hi} := [\mathbf{D}_h]_{i,i}$, the $2N$ eigenvalues of $\mathbf{A}$ are given by*

$$\lambda_i = \frac{1}{2}\left(\alpha D_{hi} \pm \sqrt{\alpha^2 D_{hi}^2 + 4(N_0^2 + 2D_{hi}N_0)}\right), \quad i = 1, \ldots, N. \tag{10}$$

Given these eigenvalues, we are ready to evaluate the BER using (8). Differential space-time modulation based on orthogonal STBC was studied for iid channels in [3, 7], but exact BER analysis was not provided. The BER analysis presented here is applicable to both correlated and independent Rayleigh fading channels. If beamforming and power loading are not used, Proposition 1 still holds true; thus, we can calculate the eigenvalues of $\mathbf{G}$ numerically, which in turn enables us to evaluate the BER. If beamforming and equal power loading are employed, then the matrix $\mathbf{G}$ has eigenvalues identical to those without beamforming and power loading. Therefore, these two cases have the same error probability performance, which implies that beamforming alone does not improve the performance. This motivates appropriate power loading to reduce the error probability.

3.2. Approximate SER of M-PSK constellations

It is difficult to obtain the exact BER or SER in closed form for any M-PSK other than BPSK and QPSK. Thus, we will also derive an approximate SER for any M-PSK constellation, which is useful for the power loading algorithm. If $\tilde{\mathbf{h}} := \mathbf{D}\mathbf{U}^{\mathcal{H}}\mathbf{h}$ and $\tilde{\mathbf{C}}_t := \mathbf{C}_t\mathbf{\Theta}$, then $z_{p,R}$ and $z_{p,I}$ in (4) can be expressed as $z_{p,R} = 2\mathcal{E}_s|\tilde{\mathbf{h}}|^2 s_p^R + w_1 + w_3 + w_5$ and $z_{p,I} = 2\mathcal{E}_s|\tilde{\mathbf{h}}|^2 s_p^I + w_2 + w_4 + w_6$, where $w_1 := 2\mathrm{Re}[\mathbf{w}_{t-1}^{\mathcal{H}}\mathbf{\Phi}_p^{\mathcal{H}}\mathbf{S}_t\tilde{\mathbf{C}}_{t-1}\tilde{\mathbf{h}}]$, $w_3 := 2\mathrm{Re}[\tilde{\mathbf{h}}^{\mathcal{H}}\tilde{\mathbf{C}}_{t-1}^{\mathcal{H}}\ \mathbf{\Phi}_p^{\mathcal{H}}\mathbf{w}_t]$, $w_5 := 2\mathrm{Re}[\mathbf{w}_{t-1}^{\mathcal{H}}\mathbf{\Phi}_p^{\mathcal{H}}\mathbf{w}_t]$, and w_2, w_4, w_6 are defined in a way similar to w_1, w_3, w_5, respectively, with $\mathbf{\Phi}_p$ replaced by $j\mathbf{\Psi}_p$. When the SNR is high, w_5 and w_6 are negligible; thus, $z_p = z_{p,R} + jz_{p,I}$ is well approximated by

$$\tilde{z}_p = 2\mathcal{E}_s|\tilde{\mathbf{h}}|^2 s_p + w, \qquad (11)$$

where $w = w_1 + w_3 + j(w_2 + w_4)$. Based on (11), it is argued in [3] that differential modulation incurs 3-dB penalty in SNR compared with orthogonal STBC with coherent detection, since noise power is doubled. However, it is not clear if we can use SER formulas for coherent modulation to calculate an approximate SER for differential modulation based on (11), since the phase of $\tilde{z}_p$ is disturbed by the noise w, and we do not know whether the real and the imaginary parts of w are correlated or not. The following fact, which is proven in [1], enables us to obtain an approximate SER.

Fact 1 $\mathrm{Re}(w)$ *and* $\mathrm{Im}(w)$ *are uncorrelated, and have identical variance* $\sigma^2 = 4|\tilde{\mathbf{h}}|^2 N_0$.

Based on this fact, we see that the signal model (11) is the same as that of M-PSK modulation with maximum ratio combining (MRC) [11, p. 266]. Furthermore, SER of M-PSK can be calculated by [11, p. 271]

$$P_s(e) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \mathcal{M}\left(-\frac{g_{\mathrm{PSK}}}{\sin^2\theta}\right) d\theta, \qquad (12)$$

where $g_{\mathrm{PSK}} := \sin^2(\pi/M)$, and $\mathcal{M}(\cdot)$ is the moment generating function (MGF) of the random variable $\mathcal{E}_s|\tilde{\mathbf{h}}|^2/(2N_0)$. Since the instantaneous SNR of $\tilde{z}_p$ is $\mathcal{E}_s|\tilde{\mathbf{h}}|^2/(2N_0)$, the approximate SER derived from (11) is 3 dB worse than that of coherent modulation with the same transmitted power.

3.3. Power Loading

Similar to [8, 14], we will minimize the Chernoff bound of SER given in (12), which in turn will reduce the actual error probability. From (12), we can find the Chernoff bound of $P_s(e)$ as follows $P_{s,bound}(e) = (M-1)/(M\prod_{i=1}^{N_T}[1 + g_{\mathrm{PSK}}\mathcal{E}_s\lambda_{R,i}[\bar{\mathbf{D}}]_{i,i}/(2N_0)])$ [14], where $\lambda_{R,i}$ is the eigenvalue of $\mathbf{R}_h$, and $\bar{\mathbf{D}} := \mathbf{D}^2$. To select power loading coefficients, we minimize $P_{s,bound}(e)$ with respect to $\bar{\mathbf{D}}$ with the constraints $[\bar{\mathbf{D}}]_{i,i} \geq 0$, and $\mathrm{Tr}(\bar{\mathbf{D}}) = 1$. This optimization problem has been formulated and solved in [14]. The solution is $[\bar{\mathbf{D}}]_{i,i} = \frac{1}{\bar{N}_T} + \frac{2N_0}{g_{\mathrm{PSK}}\mathcal{E}_s}\left(\frac{1}{\bar{N}_T}\sum_{l=1}^{\bar{N}_T}\frac{1}{\lambda_{R,l}} - \frac{1}{\lambda_{R,i}}\right)$ [14], where $\bar{N}_T$ $(0 < \bar{N}_T \leq N_T)$ is the number of beams that transmit signals, given the transmitted power budget $\mathcal{E}_s$. For the selection of $\bar{N}_T$ and detailed description of the power loading algorithm, we refer the reader to [14].

While the works in [3, 7] show that orthogonal STBC can be modified to facilitate differential modulation and detection, our work here reveals that the loaded eigen-beamforming derived in [14] for coherent STBC, can also be used in a differential STBC setup.

4. SIMULATIONS AND NUMERICAL RESULTS

We consider a linear array of $N_T = 4$ antennae at the transmitter, and $N_R = 1$ antenna at the receiver. The N_T transmit antennae are equispaced by d. We assume that the direction of arrival is perpendicular to the transmitter antenna array. Let λ be the wavelength of the transmitted signal, and Δ denote the angle spread. When Δ is small, the channel correlation can be calculated as $[\mathbf{R}_h]_{m,n} \approx \frac{1}{2\pi}\int_0^{2\pi}\exp[-j\,2\pi(m-n)d\Delta\sin\theta/\lambda]d\theta$ [10]. In our analysis and simulations, we will consider two channels: channel 1 has $d = 0.5\lambda$ and $\Delta = 5°$, while channel 2 has $d = 0.5\lambda$ and $\Delta = 25°$. Channels are normalized so that $\mathrm{Tr}(\mathbf{R}_h) = N_T$. For channel 1, the eigenvalues of $\mathbf{R}_h$ are in $\mathbf{\Lambda}_1 = \mathrm{diag}(3.81849, 0.18079, 0.00071, 0.00001)$; and for channel 2, we have $\mathbf{\Lambda}_2 = \mathrm{diag}(1.790, 1.741, 0.454, 0.015)$. QPSK constellations will be adopted. In all plots, the SNR is defined as $\mathrm{SNR} := \mathcal{E}_s/N_0$. We denote power loading in Section 3.3 as the optimal power loading in the sense that it minimizes the Chernoff bound of the approximate SER.

Fig. 1 compares simulated against exact BER. In simulations, $\mathbf{R}_h$ is assumed perfectly known at the transmitter. The correlation matrix of channel 1 has two very small eigenvalues: if we use equal power loading, the transmitted power along two eigenvectors corresponding to these two small eigenvalues is wasted in the SNR region of practical interest. Hence, the optimal power loading outperforms the equal power loading by more than 3 dB in the SNR region of interest. Channel 2 is less correlated; thus, the performance gap between optimal power loading and equal power loading is smaller, but still noticeable. Recall that the performance of differential modulation with beamforming and

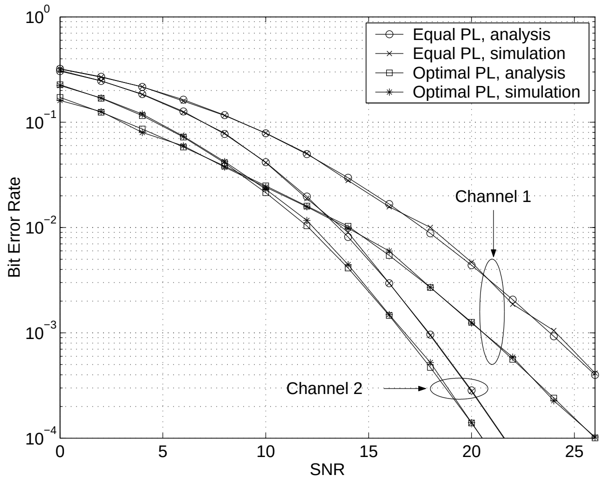

Fig. 1. BER performance, $\mathbf{R}_h$ known.

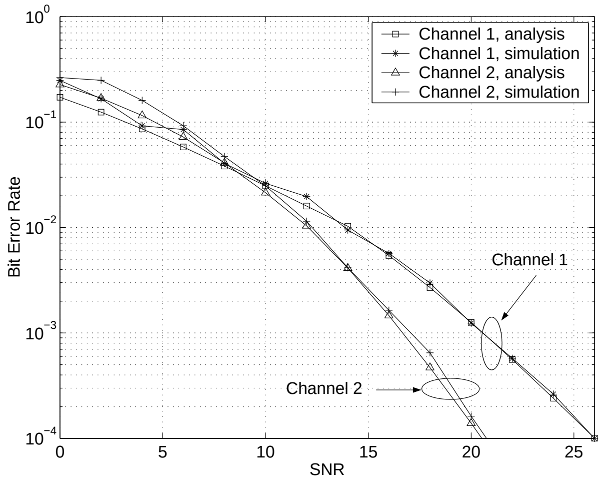

Fig. 2. BER performance, $\mathbf{R}_h$ estimated.

equal power loading is the same as that of differential modulation without beamforming. Hence, these BER curves demonstrate clearly the advantage of combining differential modulation with optimally loaded beamforming. Fig. 2 depicts the the error probability when $\mathbf{R}_h$ is estimated using the detected symbols [1]. We see that when the SNR is reasonably high, using estimated $\mathbf{R}_h$ instead of $\mathbf{R}_h$ does not degrade the performance.

5. CONCLUSIONS

We have analyzed the error probability performance of differential space-time modulation that relies on orthogonal space-time block coding. Based on this performance analysis, combining differential space-time modulation with transmit-beamforming and power loading was shown to enhance the error probability performance in correlated channels. Both analytical and simulation results confirmed that considerable performance gain can be achieved.

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