JOINT MAXIMUM-LIKELIHOOD CHANNEL ESTIMATION AND SIGNAL DETECTION FOR SIMO CHANNELS

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ABSTRACT

In wireless communication systems, channel state information is often assumed to be available at the receiver. Traditionally, a training sequence is used to obtain the estimate of the channel. Alternatively, the channel can be identified using known properties of the transmitted signal. However, the computational effort required to find the joint ML solution to the symbol detection and channel estimation problem increases exponentially with the dimension of the problem. To significantly reduce this computational effort, we formulate the aforementioned problem in a way that makes it possible to solve it via the use of sphere decoding, an algorithm that has polynomial expected complexity. We also provide simulation results and a complexity discussion.

1. INTRODUCTION

The pursuit for high-speed data services has resulted in a tremendous amount of research activity in the wireless communications community. To obtain high reliability of the transmission, particular attention has been payed to the design of receivers (see, e.g., [1] and references therein).

In the system design, one often assumes knowledge of the channel coefficients at the receiver. These are typically obtained by sending a training sequence, thus sacrificing a fraction of the transmission rate. However, in practical systems, due to rapid changes of the channel and/or limited resources, training and channel tracking may be infeasible. One possible remedy is to differentially encode the transmitted data and thus eliminate the need for the channel knowledge. Another one is to exploit known properties of the transmitted data to learn the channel blindly – for instance, one can exploit the fact that the transmitted data belongs to a finite alphabet.

We consider a problem of joint maximum likelihood (ML) channel estimation and signal detection in a communication system where the transmitter uses only one antenna but the receiver employs multiple antennas. Let N denotes the number of receive antennas and let T be the length of a data packet during the transmission of which the channel can be assumed to be constant. Then the channel output can be written as

$$X = \mathbf{h}\mathbf{s}^* + W, \tag{1}$$

where $\mathbf{h} \in \mathcal{C}^{N\times 1}$ is the single-input multi-output (SIMO) channel gain, $\mathbf{s} \in \mathcal{C}^{T\times 1}$ is the transmitted symbol sequence, and $W \in \mathcal{C}^{N\times T}$ is an additive noise matrix whose elements are assumed to be i.i.d. complex Gaussian random variables $\mathcal{C}(0, \sigma^2)$. Furthermore, the entries in $\mathbf{s}$ are assumed to be unitary, i.e.,

$$|s_k|^2 = 1, \quad k = 1, 2, \ldots, T. \tag{2}$$

[Note that the sphere decoding algorithm performs the closest point search in a rectangular lattice and the available expected complexity results assume the same. To make use of them, in this paper we shall assume that s_k belongs to a QPSK constellation. However, extension of sphere decoding to the detection of PSK-modulation schemes is readily available [5].]

The joint ML channel estimation and signal detection problem can be stated as follows:

$$\min_{\mathbf{h},\mathbf{s}\in\mathcal{S}^T} \|X - \mathbf{h}\mathbf{s}^*\|^2, \tag{3}$$

where $\mathcal{S}^T$ denotes the (finite) T-dimensional integer lattice. Problem (3) is a mixed optimization problem: it is a least-squares problem in $\mathbf{h}$ and an *integer least-squares* problem in $\mathbf{s}$. Traditionally, the solution to the integer least-squares problems is found by an exhaustive search over the entire symbol space. The complexity of exhaustive search is exponential in T and often infeasible in practice. Therefore, low-complexity heuristic techniques, usually iterating between the $\mathbf{s}$ and $\mathbf{h}$ estimation, are often employed (see, e.g., [2] and references therein). On the other hand, in communication applications, the sphere decoding [3] is recognized as a technique for solving integer least-squares problems at polynomial expected complexity [4]. In this paper, we show how to employ the sphere decoding algorithm to solve

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the mixed problem (3). The algorithm requires no channel knowledge at the receiver and no iterations. The expected complexity for large SNRs is found. Simulation results are also included.

2. SOLVING THE JOINT PROBLEM

For any given $\mathbf{s}$, the channel $\hat{\mathbf{h}}$ that minimizes (3) is given by

$$\hat{\mathbf{h}} = X\mathbf{s}/\|\mathbf{s}\|^2 = \frac{1}{T}X\mathbf{s}, \tag{4}$$

where we used the assumption that $|s_k|^2 = 1$, for any k. Substituting (4) in (3) gives

$$\|X \underbrace{(I - \frac{1}{T}\mathbf{s}\mathbf{s}^*)}_{=P_s} \|^2 = \text{tr}(XP_sX^*) = \text{const} - \frac{1}{T}\mathbf{s}^*X^*X\mathbf{s}$$

Hence solving (3) is achieved by solving

$$\max_{\mathbf{s}\in\mathcal{S}^T} [\mathbf{s}^*X^*X\mathbf{s}]. \tag{5}$$

Let $\hat{\lambda} = \lambda_{\max}(X^*X)$ be the maximum eigenvalue of X^*X, and let $\rho > \hat{\lambda}$ (e.g., $\rho = \text{tr}(X^*X)$). The problem (5) is equivalent to

$$\min_{\mathbf{s}\in\mathcal{S}^T} \mathbf{s}^*\underbrace{(\rho I - X^*X)}_{=\mathcal{H}}\mathbf{s}. \tag{6}$$

The optimization problem (6) is an integer least-squares problem and a straightforward way to solve it is via exhaustive search [2]. However, note that due to the choice of ρ, the matrix $\mathcal{H}$ is positive definite and therefore it allows for Cholesky factorization

$$\mathcal{H} = R^*R, \tag{7}$$

where R is an upper-triangular matrix. Thus the sphere decoding algorithm of Fincke and Pohst [3] can be applied to solve (6). Rather than exhaustively searching over the entire lattice $\mathcal{S}^T$, the sphere decoding algorithm performs a limited search inside a hypersphere of radius r, i.e., finds the point $\mathbf{s}$ that minimizes (6) among all lattice points that satisfy

$$\mathbf{s}^*X^*X\mathbf{s} \leq r^2. \tag{8}$$

[More about the choice of the radius r in the next section.] The closest lattice point to the origin inside the sphere is the solution to (6). The algorithm can be stated as follows:

Input: radius r, matrix $R = [r_{ij}],\ 1 \leq i, j \leq T$.

1. Set $k = T$, $r_k = r$.
2. (Bounds for s_k) Set $z = \frac{r_k}{r_{kk}}$, $UB(s_k) = \lfloor z \rfloor$, $s_k = \lceil -z \rceil - 1$
3. (Increase s_k) $s_k = s_k + 1$. If $s_k \leq UB(s_k)$ go to 5, else to 4.
4. (Increase k) $k = k + 1$; if $k = T + 1$, terminate algorithm, else go to 3.
5. (Decrease k) If $k = 1$ go to 6. Else $k = k - 1$, $\hat{s}_{k|k-1} = \sum_{j=k+1}^{T} \frac{r_{kj}}{r_{kk}} s_j$, $r'^2_k = r'^2_{k+1} - r^2_{k+1,k+1}(s_{k+1} - \hat{s}_{k+1|k+2})^2$, and go to 2.
6. Solution found. Save $\mathbf{s}$ and go to 3.

[Note that in order to directly employ sphere decoding, we need to write (1) in its "real-equivalent" form. For more details, see, e.g., [4].]

The sphere decoding algorithm can be interpreted as a generalized nulling and canceling (see, e.g., [6]) where, after a component of the vector $\mathbf{s}$ that satisfies (8) is found, its contribution to $\mathbf{s}^*X^*X\mathbf{s}$ is subtracted. However, unlike in nulling and canceling, components of $\mathbf{s}$ are never fixed until an entire vector $\mathbf{s}$ which satisfies (8) is found. Therefore, the algorithm essentially performs a search on the tree, as illustrated in Figure 1, where the nodes on the k^{th} level of the tree correspond to the vectors $[s_1 \ldots s_k]$. The complexity of

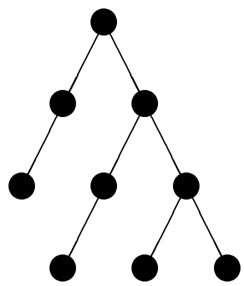

Fig. 1. *Tree search of the sphere decoding algorithm.*

the algorithm is proportional to the number of nodes visited. It depends on the choice of search radius. The choice of the radius and complexity of the algorithm are discussed in the next section.

3. CHOICE OF THE RADIUS AND COMPLEXITY OF THE ALGORITHM

A simple heuristic for solving (3) consists of finding the eigenvector corresponding to the maximum eigenvalue of X^*X (or, equivalently, the dominant right singular vector of X) and then projecting it onto $\mathcal{S}^T$ (i.e., rounding each entry). This heuristic can also be exploited as a starting point of the sphere decoding search – the norm of the heuristic solution can be used as the search radius. However, we cannot say much about the complexity of sphere decoding for this deterministic choice of the search radius.

Alternatively, when the distribution of the objective of the minimization is known, one can make a probabilistic choice of the search radius. In [4], the objective of the minimization is chi-square distributed, and the radius is chosen according to the variance of that distribution, scaled in such a way that the probability of finding a point inside the sphere is very high. Furthermore, the expected complexity of the algorithm is found and shown to be polynomial over a wide range of SNRs. We shall show next that the argument in (6) has a chi-square distribution at high SNRs. This suggests a probabilistic choice of the search radius. Furthermore, the expected complexity results of [4] extend to the current paper.

Assume that $SNR \gg 1$. Consider the eigenvalue decomposition of X^*X,

$$X^*X = \begin{bmatrix} \hat{u} & \hat{G} \end{bmatrix} \begin{bmatrix} \hat{\lambda} & 0 \\ 0 & \hat{\Lambda} \end{bmatrix} \begin{bmatrix} \hat{u}^* \\ \hat{G}^* \end{bmatrix}$$

Now, taking $\rho = \hat{\lambda}$, we can write the objective of minimization in (6) as

$$\begin{aligned} \mathbf{s}^*(\rho I - X^*X)\mathbf{s} &= \mathbf{s}^*(\hat{\lambda} I - \hat{u}\hat{u}^*\hat{\lambda} - \hat{G}\hat{\Lambda}\hat{G}^*)\mathbf{s} \\ &= \mathbf{s}^*[\hat{\lambda}(\underbrace{I - \hat{u}\hat{u}^*}_{=\hat{G}\hat{G}^*}) - \hat{G}\hat{\Lambda}\hat{G}^*]\mathbf{s} \\ &= \mathbf{s}^*\hat{G}(\hat{\lambda} I - \hat{\Lambda})\hat{G}^*\mathbf{s} \qquad (9) \end{aligned}$$

Furthermore,

$$\begin{aligned} X^*X &= \|\mathbf{h}\|^2\mathbf{s}\mathbf{s}^* + \mathbf{s}\mathbf{h}^*W + W^*\mathbf{h}\mathbf{s}^* + W^*W \qquad (10) \\ &= \hat{\lambda}\hat{u}\hat{u}^* + \hat{G}\hat{\Lambda}\hat{G}^* \qquad (11) \end{aligned}$$

Note that for $SNR \gg 1$,

$$X^*X = \|\mathbf{h}\|^2 T \left(\frac{\mathbf{s}}{\sqrt{T}}\right)\left(\frac{\mathbf{s}^*}{\sqrt{T}}\right),$$

and hence, at high SNRs, $\hat{\lambda}$ and $\hat{\Lambda}$ become

$$\lambda = T\|\mathbf{h}\|^2, \quad \Lambda = 0. \qquad (12)$$

Combining (10), (11), and (12), we obtain

$$\begin{aligned} \hat{G}^*(X^*X)\mathbf{s} &= \lambda\hat{G}^*\mathbf{s} + (\hat{G}^*\mathbf{s})(\mathbf{h}^*W\mathbf{s}) + \\ &+ \hat{G}^*W^*\mathbf{h}T + \hat{G}^*W^*W\mathbf{s} \\ &= \hat{\Lambda}(\hat{G}^*s) \end{aligned}$$

Neglecting the higher order terms,

$$\lambda(\hat{G}^*\mathbf{s}) \approx -\hat{G}^*W^*\mathbf{h}T$$

Therefore, for high SNRs, $(\hat{G}^*\mathbf{s})$ is circular Gaussian with zero mean. To find its variance, note that

$$T\hat{G}^*W^*\mathbf{h} = T\hat{G}^* \begin{bmatrix} W_1^*\mathbf{h} \\ \vdots \\ W_N^*\mathbf{h} \end{bmatrix},$$

where W_k is the k^{th} column of W. Also, note that

$$E\left[W_k^*\mathbf{h}\mathbf{h}^*W_l\right] = E\left[\mathbf{h}^*W_lW_k^*\mathbf{h}\right] = \sigma^2\|\mathbf{h}\|^2\delta_{k,l}$$

Therefore,

$$\begin{aligned} \text{cov}(\hat{G}^*\mathbf{s}) &= \frac{T^2}{\lambda^2}\text{cov}(G^*W^*\mathbf{h}) \\ &= \frac{T^2}{\lambda^2}\|\mathbf{h}\|^2\sigma^2 I = \frac{T}{\lambda}\sigma^2 I \\ &= T\sigma^2(\lambda I - \Lambda)^{-1}, \end{aligned}$$

where $\Lambda = 0$ was inserted for convenience. Therefore,

$$(\hat{G}^*\mathbf{s}) \sim \mathcal{N}\left[0, T\sigma^2(\lambda I - \Lambda)^{-1}\right].$$

At high SNRs,

$$\lambda \approx \hat{\lambda}, \quad \Lambda \approx \hat{\Lambda},$$

and, therefore,

$$(\hat{G}^*\mathbf{s}) \sim \mathcal{N}\left[0, T\hat{\sigma}^2(\hat{\lambda} I - \hat{\Lambda})^{-1}\right],$$

where $\hat{\sigma}^2$ is an estimate of σ^2. This estimate can be obtained (see, e.g., [7]) as the mean of the $(N-1)$ smallest eigenvalues of XX^* (or, alternatively, the smallest $(N-1)$ non-zero eigenvalues of X^*X).

In summary, we have shown that the scaled term in (9),

$$\frac{1}{T\hat{\sigma}^2}(\mathbf{s}^*\hat{G})(\hat{\lambda} I - \hat{\Lambda})(\hat{G}^*\mathbf{s})$$

is chi-square distributed with $2(T-1)$ degrees of freedom. Thus we can choose the search radius $r^2 = \alpha T\sigma^2$ so that

$$\int_0^{\alpha T} \gamma(\lambda, T-1)d\lambda = p_{\text{fp}}, \qquad (13)$$

where $\gamma(\cdot,\cdot)$ denotes an incomplete gamma function and p_{fp} is set close to 1, say, 0.99. Furthermore, the expected complexity results derived in [4] hold.

Remark: Note that $|R| = 0$ for $\rho = \hat{\lambda}$. Therefore, to prevent any possible numerical problems we prefer to choose $\rho > \hat{\lambda}$, as stated in Section 2. Then the search radius may be chosen as

$$r' = (\rho - \hat{\lambda}) + r,$$

where r is chosen as implied by (13).

4. SIMULATION RESULTS

We consider a SIMO system employing $n = 4$ receive antennas and transmitting data in blocks of length $T = 20$. We compare the BER performance of the sphere decoding algorithm and the iterative least squares with projections (ILSP) algorithm (see, e.g., [2], [8]). [Simulation results are obtained by performing Monte Carlo runs in which $\mathbf{h}$ and W are varied.] The ILSP essentially finds the least squares estimate of the symbols and projects it onto the space $\mathcal{S}^T$ to obtain the symbol estimate $\hat{\mathbf{s}}$. Then $\hat{\mathbf{s}}$ is used to update the estimate of the channel and the two aforementioned steps repeated until convergence. As shown in Figure 2, the sphere decoding algorithm significantly outperforms the heuristic one. Figure 3 shows the complexity exponent, defined as $e = \log_T F$, where F denotes the total number of operations required to detect a vector $\mathbf{s}$. [For the sphere decoding, the total complexity is a sum of the operations for the QR factorization and the sphere decoding search]. It is evident that the sphere decoding has roughly cubic complexity over the range of SNRs of interest.

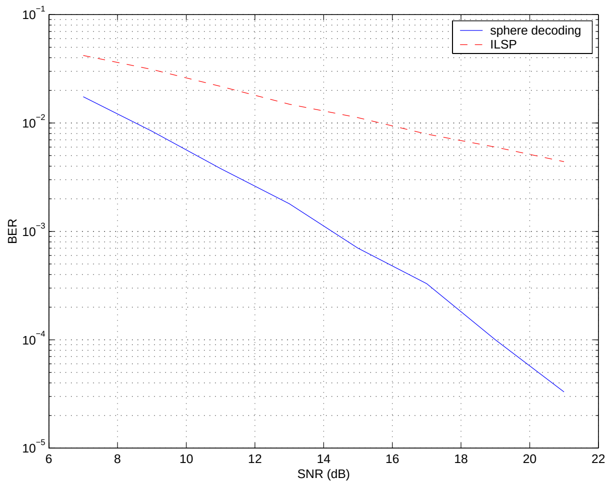

Fig. 2. BER comparison of sphere decoding and ILSP algorithms, $n = 4$, $T = 20$.

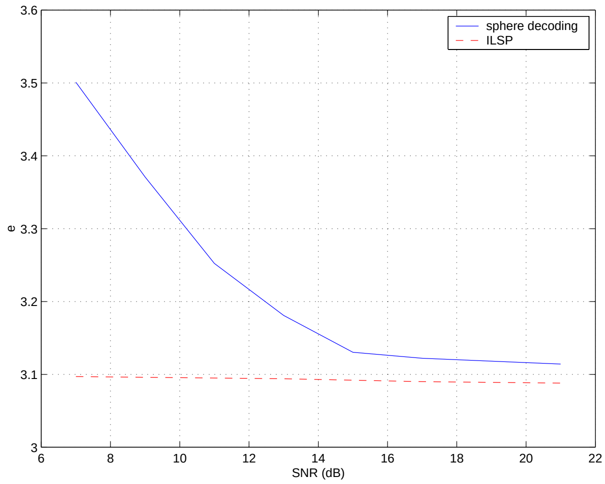

Fig. 3. Expected complexity exponent, $n = 4$, $T = 20$.

5. SUMMARY AND CONCLUSION

We considered the joint ML channel estimation and signal detection problem for single-input multiple-output wireless channels. To reduce the computational effort, we formulated the design problem so that it can be solved via the use of sphere decoding. It was shown that the algorithm, when applied to the problem herein, has polynomial expected complexity. The performance of the algorithm and its complexity were illustrated with an example. The extension to the multiple-input multiple-output scenario will be presented in a future paper.

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