DETECTION FOR MIMO SYSTEMS WITH IMPRECISE CHANNEL KNOWLEDGE

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ABSTRACT

In this work, we investigate the signal detection for MIMO systems with imprecise channel knowledge. The optimal detector is one which best matches the "total" observation matrix and a "total" signal matrix which has a finite alphabet constraint and a Sylvester structure constraint. An iterative local optimization with interference cancellation (LOIC) algorithm is proposed to achieve low complexity and exploit the finite alphabet constraint. Simulation results show that our proposed algorithms can detect the signals with BER close to the case of perfect channel knowledge, if a rough channel estimate is available initially.

1. INTRODUCTION

In wireless MIMO communications, extensive researches have been conducted on the signal estimation and detection for completely known (perfectly trained) and completely unknown (blind) channel models. However, in practice, neither model is realistic because wireless channels are slowly time-varying, as a result of physical channel impairments and traffic dynamism.

In this paper, we consider the signal detection for MIMO systems with *imprecise* receiver channel knowledge:

$$\underline{y}(z) = \mathbf{H}(z)\underline{x}(z) + \underline{n}(z), \mathbf{H}(z) = \bar{\mathbf{H}}(z) + \Delta\mathbf{H}(z), \quad (1)$$

where the channel transfer function $\mathbf{H}(z)$ is assumed to comprise of a known part $\bar{\mathbf{H}}(z)$ and an unknown part $\Delta\mathbf{H}(z)$. The channel imprecision can be the channel estimation error as a result of imperfect training, or alternatively be the gradual change in the channel response.

Assuming each entry of $\mathbf{\Delta H}(z)$ and $\underline{n}(z)$ is i.i.d. Gaussian distributed, the generalized maximum likelihood detector (GMLD) is the proposed formulation for signal detection. Signal estimation techniques such as LS and TLS can provide initializations which hopefully should reside in a local vicinity of the true signal. In order to further improve the detection accuracy, the structural and finite alphabet constraints should be effectively exploited. In this paper, we aim at designing iterative signal detectors with low complexity which can utilize the constraints to gradually improve the match of the signal estimate and the channel estimate with the observation. The proposed method operates as a sequence of local optimizations with the interferences from out-of-window symbols cancelled.

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2. CHANNEL MODEL AND GMLD

2.1. Channel Model

Definition 1 (Sylvester Matrices)
The Row-Sylvester Matrix of $\mathbf{H}(z) = \mathbf{H}_0 + \cdots + \mathbf{H}_d z^{-d}$ *with* ξ *block rows is defined as:*

$$\mathbf{\Gamma_R}^{(\xi)}[\mathbf{H}(z)] \equiv \begin{pmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_d & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_d & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \cdots & \mathbf{H}_d \end{pmatrix}.$$

The Column-Sylvester Matrix of $\mathbf{H}(z)$ *with* ξ *block columns is defined as:*

$$\mathbf{\Gamma_C}^{(\xi)}[\mathbf{H}(z)] \equiv \left(\mathbf{\Gamma_R}^{(\xi)}[\mathbf{H}^H(z)]\right)^H. \quad (2)$$

Consider a MIMO-ISI channel with ISI-degree d: $\mathbf{H}(z) \equiv \mathbf{H}_0 + \mathbf{H}_1 z^{-1} + \cdots + \mathbf{H}_d z^{-d}$. Note that the case $d = 0$ particularizes to the frequency flat channel. Let M and N denote the number of transmitting and receiving antennas, respectively. We assume a block fading scenario where the channel remains constant for the duration of the coherence interval. Let T denote the block size (in symbols) which is less than the coherence interval length. Additionally, we assume a zero-padding of length d is appended at the end of a block transmission of T channel uses so that the inter-block interference is eliminated. With these assumptions, we can write the channel model as (1).

With the definition of the following quantities: $\bar{\mathbf{H}} \equiv [\mathbf{H}_0\ \mathbf{H}_1\ \cdots\ \mathbf{H}_d]$, $\Delta\mathbf{H} \equiv [\Delta\mathbf{H}_0\ \Delta\mathbf{H}_1\ \cdots\ \Delta\mathbf{H}_d]$, $\mathcal{X} \equiv \mathbf{\Gamma_R}^{(d+1)}[\underline{x}(z)]$, $\mathbf{Y} \equiv [\underline{y}_0\ \underline{y}_1\ \cdots\ \underline{y}_{T+d-1}]$, the imprecise channel model considered in this paper is:

$$\mathbf{Y} = \mathbf{H}\mathcal{X} + \mathbf{N} = (\bar{\mathbf{H}} + \Delta\mathbf{H})\mathcal{X} + \mathbf{N} \quad (3)$$

$$\Delta\mathbf{H}[i,j] \sim i.i.d.\ N(0,1) \quad (4)$$

$$\mathbf{N}[i,j] \sim i.i.d.\ N(0,1), \text{ independent of } \Delta\mathbf{H} \quad (5)$$

where $[i, j]$ denotes the (i, j)th entry of a matrix.

Normalization: Note that the channel model (3)-(5) has already been normalized without loss of generality. Suppose initially the unnormalized channel model is given as:

$$\mathbf{Y}_o = \sqrt{SNR \cdot \sigma_N^2 / M}(\bar{\mathbf{H}}_o + \sigma_H \Delta\mathbf{H})\mathcal{X}_o + \sigma_N \mathbf{N}, \quad (6)$$

where SNR is the total SNR at all transmit antennas and each element of $\mathcal{X}_o$ belongs to the normalized finite constellation set, e.g. BPSK. After normalization we would have:

$$\frac{1}{\sigma_N}\mathbf{Y}_o = \left(\frac{1}{\sigma_H}\bar{\mathbf{H}}_o + \Delta\mathbf{H}\right)\left(\sqrt{\frac{SNR \cdot \sigma_H^2}{M}}\mathcal{X}_o\right) + \mathbf{N}.$$

Training Form: Consider the following channel model:

$$\mathbf{Y}_t = \mathbf{H}_o(\sigma_N/\sigma_H \mathbf{I}) + \sigma_N \mathbf{N}_t, \quad (7)$$

$$\mathbf{Y}_o = \mathbf{H}_o(\sqrt{SNR \cdot \sigma_N^2 / M}\mathcal{X}_o) + \sigma_N \mathbf{N}. \quad (8)$$

This can be explained as a communication transaction where pilot symbols are transmitted to learn the channel before the data transmission: (1) start with a *diffuse prior* assumption on $\mathbf{H}_o$, that is, the inverse of the covariance matrix of each row of $\mathbf{H}_o$ is set to be $\mathbf{0}$, reflecting the lack of channel knowledge *a priori*, (2) transmit a training block of $\sigma_N/\sigma_H \mathbf{I}_M$ followed by d padding zeros to learn the channel, after which $\mathbf{H}_o[i,j] | \mathbf{Y}_t \sim N(\sigma_H/\sigma_N \mathbf{Y}_t[i,j], \sigma_H^2)$. With $\bar{\mathbf{H}}_o \equiv \sigma_H/\sigma_N \mathbf{Y}_t$ and $\Delta\mathbf{H} \equiv \mathbf{N}_t$, we can arrive at (6) and consequently (3). Hence we are motivated to rewrite (3) as (9) and call it the *training form* of the channel model:

$$[\bar{\mathbf{H}}\ \mathbf{Y}] = \mathbf{H}[\mathbf{I}\ \mathcal{X}] + [-\mathbf{\Delta H}\ \mathbf{N}]. \quad (9)$$

The discussion above demonstrates how uncertainties due to imperfect training can be incorporated into (3). More generally, a Bayesian statistics framework is proposed in [1] as a unifying model for MIMO-ISI channel which can characterize diverse assumptions on the channel knowledge.

2.2. Generalized Maximum Likelihood Detector

For the channel model in training form (9), consider the generalized maximum likelihood detection (GMLD) with $\mathbf{H}$ and $\mathcal{X}$ being the unknown parameters:

$$[\mathbf{H}\ \mathcal{X}]_{GMLD} = \arg\max_{\mathbf{H}, \mathcal{X} \in \mathcal{SF}} p([\bar{\mathbf{H}}\ \mathbf{Y}] | \mathcal{X}, \mathbf{H}) \quad (10)$$

$$= \arg\min_{\mathbf{H}, \mathcal{X} \in \mathcal{SF}} \|[\bar{\mathbf{H}}\ \mathbf{Y}] - \mathbf{H}[\mathbf{I}\ \mathcal{X}]\|_F^2. \quad (11)$$

where $\mathcal{S}$ and $\mathcal{F}$ denote structurally and finite alphabet constrained sets, respectively[1]. It can be shown [1] that for wide-sense stationary and ergodic inputs, GMLD is a good approximation of the optimal maximum likelihood detector which minimizes the block error probability.

[1]In the sequel, the simplified notations $\mathcal{S}$ and $\mathcal{F}$ are used to represent sets of matrices of different sizes. The specific constraints should be obvious from the context.

3. ITERATIVE SIGNAL DETECTION

3.1. Initialization with TLS or LS

Formulation (11) can be rewritten as:

$$\mathcal{X} = \arg\min_{\mathcal{X} \in \mathcal{SF}} \left\| [\bar{\mathbf{H}}\ \mathbf{Y}] - [\bar{\mathbf{H}}\ \mathbf{Y}][\mathbf{I}\ \mathcal{X}]^\dagger [\mathbf{I}\ \mathcal{X}] \right\|_F^2, \quad (12)$$

where $\dagger$ is the Moore-Penrose pseudo inverse. Formulation (12) bears a pleasing geometric explanation: find $\mathcal{X} \in \mathcal{SF}$ to maximize the projection of the "total" observation matrix $[\bar{\mathbf{H}}\ \mathbf{Y}]$ onto the row span of the "total" transmitted signal matrix $[\mathbf{I}\ \mathcal{X}]$.

If the constraints are dropped and $N > M(d+1)$, the solution is given by TLS, which can be obtained in two steps: (1) set the row span of $[\mathbf{I}\ \mathcal{X}]$ to the subspace spanned by the first $M(d+1)$ principal components of $[\bar{\mathbf{H}}\ \mathbf{Y}]$. This step performs dimension reduction. (2) given the row span of $[\mathbf{I}\ \mathcal{X}]$, use the known part $\mathbf{I}$ as a signature to resolve the ambiguity in $\mathcal{X}$. After these, the estimate $\mathcal{X}_{TLS}$ should be quantized to a valid initialization $\mathcal{X}_{TLS}^{(0)} \in \mathcal{SF}$:

$$\mathcal{X}_{TLS}^{(0)} = \arg\min_{\mathcal{X} \in \mathcal{SF}} \|\mathcal{X} - \mathcal{X}_{TLS}\|_F^2. \quad (13)$$

For frequency flat MIMO channels, the condition $N > M(d+1)$ is satisfied if the channel is PR (prefect recoverable) by linear filtering in the ideal noise-free case.

For MIMO-ISI channels with $N \le M(d+1)$, TLS is not applicable. An alternative initialization can be obtained by least squares constructed from the nominal channel $\bar{\mathbf{H}}(z)$:

$$\underline{\mathbf{x}}_{LS}^{(0)} = Q\left[\left(\mathbf{\Gamma_C}^{(T)}[\bar{\mathbf{H}}(z)]\right)^\dagger \underline{\mathbf{y}}\right], \quad (14)$$

where $Q[\cdot]$ is the quantization operation, $\underline{\mathbf{x}} \equiv [\underline{x}_0^H\ \cdots\ \underline{x}_{T-1}^H]^H$, and $\underline{\mathbf{y}} \equiv [\underline{y}_0^H\ \underline{y}_1^H\ \cdots\ \underline{y}_{T+d-1}^H]^H$.

3.2. Iterative Projection

Formulation (11) leads to an iterative optimization procedure which alternates between optimizing over $\underline{\mathbf{x}}$ and optimizing over $\mathbf{H}$, given in Algorithm 1. To simply notation, we introduce $\mathcal{H} \equiv \mathbf{\Gamma_C}^{(T)}[\mathbf{H}(z)]$. It can be shown that such an procedure is a particularization of the EM (expectation maximization) algorithm [2] with $\mathbf{H}$ being the *missing data*. Since each step cannot increase the cost function (11), Algorithm 1 will converge to a local minimum.

Algorithm 1 (Iterative Projection (IP))

1. *Initialize* $\mathcal{X}$ *as* $\mathcal{X}_{TLS}^{(0)}$ *or* $\mathcal{X}_{LS}^{(0)}$.
2. *Fix* $\mathcal{X}$, *and estimate* $\mathbf{H}$ *as* $\mathbf{H} = [\bar{\mathbf{H}}\ \mathbf{Y}][\mathbf{I}\ \mathcal{X}]^\dagger$.
3. *Fix* $\mathbf{H}$, *and optimize over* $\mathcal{X}$.

$$\mathcal{X} = \arg\min_{\mathcal{X} \in \mathcal{SF}} \|\mathbf{Y} - \mathbf{H}\mathcal{X}\|_F^2 \quad (15)$$

$$= \arg\min_{\mathcal{X} \in \mathcal{SF}} \|\underline{\mathbf{y}} - \mathcal{H}\underline{\mathbf{x}}\|^2 \quad (16)$$

4. *The above procedure may be repeated.*

3.3. Local Optimization with Interference Cancellation

The bottleneck of Algorithm 1 is on (16), which is a quadratic minimization with finite-alphabet constraints. Finding the exact minimization for (16) is equivalent to maximum likelihood sequence detection for a known channel. With the technique of dynamic programming, it has been shown in [3] that the detection complexity is $O(2^{M(d+1)})$ per user per symbol for BPSK.

One way to reduce the complexity is to use quantized linear estimators such as LS or MMSE. For quantized linear estimators, the finite alphabet constraint is only utilized on a coarse granularity, i.e., as a final quantization step for the sequence estimate. Another disadvantage for quantized LS is that the complexity is still high. If the quantized LS solution is used for Algorithm 1, $\mathcal{H}^\dagger \underline{\mathbf{y}}$ needs to be calculated in each iteration where $\mathcal{H}$ is of size $N(T+d) \times TM$.

It can be verified that $\|\underline{\mathbf{y}} - \mathcal{H}\underline{\mathbf{x}}\|^2$ is convex in $\underline{\mathbf{x}} \in \mathcal{R}^{MT}$. The convexity assures that the unconstrained global optimal $\underline{\mathbf{x}}_{LS}$ can be reached by a series of local search. However, the finite alphabet constrained set $\mathcal{F}$ is not a convex set. Nevertheless, the convexity argument motivates us to design low complexity (and suboptimal than MLD) algorithms based on greedy search to solve (16).

Rewrite (16) as:

$$\min_{\underline{\mathbf{x}} \in \mathcal{F}} \|\underline{\mathbf{y}} - \mathcal{H}[(\mathbf{I} - \mathbf{W}_i) + \mathbf{W}_i]\underline{\mathbf{x}}\|^2 \qquad (17)$$

where $\mathbf{W}_i$ is a series of "windowing" matrices, i.e., the j-th element of $\underline{\mathbf{x}}_i \equiv \mathbf{W}_i\underline{\mathbf{x}}$ is either the j-th element of $\underline{\mathbf{x}}$ (in window) or 0 (out of window). We propose to approximate (17) with iterative optimizations over each window. Given each window $\mathbf{W}_i$, we restrict the minimization (17) to that over the in-window symbols, leaving out-of-window symbols fixed or equivalently having their interferences cancelled. Introduce $\underline{\mathbf{r}}(\underline{\mathbf{x}}) \equiv \underline{\mathbf{y}} - \mathcal{H}\underline{\mathbf{x}}$. The details are given in Algorithm 2, which is named local optimization with interference cancellation (LOIC).

The potential advantages of Algorithm 2 are two fold: (1) it operates on a local window and thus further reduces the complexity, (2) the finite alphabet constraints are exploited in a finer granularity than the quantization of LS.

Algorithm 2 (LOIC)

1. *Initialize* $\underline{\mathbf{x}}^{old}$ *from the last iteration in Algorithm 1. Compute* $\underline{\mathbf{r}} = \underline{\mathbf{r}}(\underline{\mathbf{x}}^{old})$.

2. *FOR* $i = 0, \cdots, I$,

$$\underline{\mathbf{r}} = \underline{\mathbf{r}} + \mathcal{H}\underline{\mathbf{x}}_i^{old}, \qquad (18)$$

$$\underline{\mathbf{x}}_i^{new} = \arg\min_{\underline{\mathbf{x}}_i \in \mathcal{F}} \|\underline{\mathbf{r}} - \mathcal{H}\underline{\mathbf{x}}_i\|^2, \qquad (19)$$

$$\underline{\mathbf{r}} = \underline{\mathbf{r}} - \mathcal{H}\underline{\mathbf{x}}_i^{new}, \qquad (20)$$

3. *Step 2 can be repeated until it converges in the sense the step 2 can not reduce the cost function* $\|\underline{\mathbf{r}}(\underline{\mathbf{x}})\|^2$.

Note that since the channel is FIR, the contributions of a small windowed signal $\underline{\mathbf{x}}_i$ is limited. Thus (19) can be effectively reduced to a LS problem with a small dimension.

In Algorithm 2, the windows $\mathbf{W}_i, i = 1, \cdots, I$ are left unspecified. Because of the shift-invariant structure, we can restrict our attention to sliding windows. Different choices of $\mathbf{W}_i, i = 1, \cdots, I$ will lead to different implementation complexity. In the simulations of this paper, for frequency flat channels, we choose MT non-overlapping windows $\mathbf{W}_i$, each having only a single 1. This corresponds to the case that each user's signals are optimized with signals from all other users cancelled. In this case, after the interference cancellations, (19) is optimized with matched filtering (for BPSK). This special case for flat channels has been proposed in [4]. Compared with [4], Algorithm 2 allows more general interference cancellation structures, which are particularly useful for frequency selective channels. For MIMO-ISI channels, we choose T overlapping sliding windows, one for each time index. $\mathbf{W}_0$ is chosen to be $diag[\mathbf{I}_{wM}\ \mathbf{0}]$. Then (19) becomes:

$$\underline{\tilde{\mathbf{x}}} = \arg\min_{\underline{\tilde{\mathbf{x}}} \in \mathcal{F}} \|\underline{\tilde{\mathbf{r}}} - \mathbf{\Gamma_C}^{(w)}[\mathbf{H}(z)]\underline{\tilde{\mathbf{x}}}\|^2, \qquad (21)$$

$$\approx Q\left[\left(\mathbf{\Gamma_C}^{(w)}[\mathbf{H}(z)]\right)^\dagger \underline{\tilde{\mathbf{r}}}\right], \qquad (22)$$

where $\underline{\tilde{\mathbf{r}}}$ and $\underline{\tilde{\mathbf{x}}}$ are sub-vectors selected accordingly. To reduce the complexity of (21), the quantized LS (22) is used as an suboptimal approximation. Note that (21) is in a similar form with (16) but the dimension is greatly reduced.

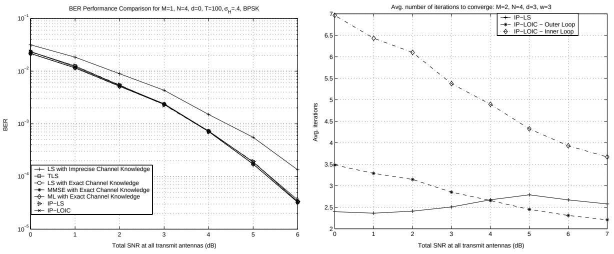
One might wonder why not simply extending the setup for flat channels and perform *per-user per-symbol interference cancellation* and how the window size affects the performance. This is briefly explained as follows. Since the search space is a discrete set of grid points, greedy search can only be assured to converge to local optimum. A larger local window would have better chance to escape from local optimum. However, a larger window requires more local checks for potential updates, unless a suboptimal approximation such as (22) is used. The complexity and performance tradeoff for a general class of iterative greedy search algorithms has been investigated in greater details in [1].

4. SIMULATIONS

The simulation model is generated as:

$$[\bar{\mathbf{H}}\ \mathbf{Y}] = \mathbf{H}[\mathbf{I}\ \sqrt{SNR \cdot \sigma_H^2/M}\mathcal{X}_o] + [-\mathbf{\Delta H}\ \mathbf{N}] \qquad (23)$$

where $\mathbf{H}$ is fixed and normalized $\mathbf{tr}\{\mathbf{H}\mathbf{H}^H\} = MN(d+1)/\sigma_H^2$. We set $T = 100$ and $\sigma_H = 0.4$ in all simulations. In other words, $E(\|\mathbf{\Delta H}\|_F^2) = 16\%\|\mathbf{H}\|_F^2$.



(a) (b)

Fig. 1. (a) BER for $M = 1, N = 4, d = 0$ (b) Average number of iterations for $M = 2, N = 4, d = 3$.

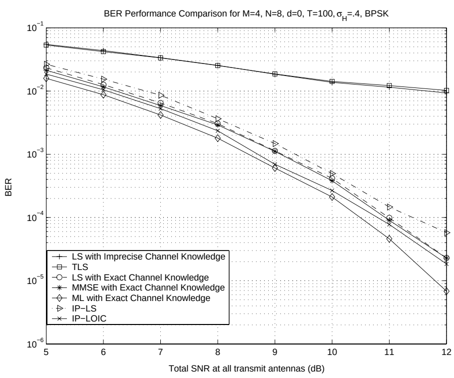


Fig. 2. BER performance comparisons for flat 4I8O.

Figure 1(a) and Figure 2 give the BER performance for a 1-in-4-out flat channel and a 4-in-8-out flat channel, respectively. In both figures, the performance of LS constructed with $\bar{\mathbf{H}}$ is much worse than the LS or MMSE with exact channel knowledge, confirming the need for further processing. While the performance of TLS in Figure 1(a) is close to optimal, its performance in Figure 2 is unsatisfactory. Recall that TLS consists of two steps: dimension reduction and ambiguity resolution. In the second step, TLS relies on the signature of $\mathbf{I}$ to resolve the ambiguity in possible left-multiplications of the much longer signal matrix $\mathcal{X}$ given the row span of $[\mathbf{I}\ \mathcal{X}]$. Our explanation is that this resolution is not robust. Designing more robust resolutions using signaling constraints is not discussed here due to space constraint and since the main focus is on iterative local optimization algorithms. For the 1-in-4-out channel (Figure 1(a)), all the detectors except the LS with imprecise channel knowledge have near optimal performance. We attribute this observation to the rich diversity offered by the SIMO channel which is effective in recovering the signals. For the MIMO channel (Figure 2), it can be seen that IP-LS (Algorithm 1 using LS approximation as step 3) achieves a performance close to the LS with exact channel knowledge. The performance of IP-LOIC (Algorithm 1 using Algorithm 2 as step 3) is even close to the optimal ML detector with perfect channel knowledge.

The BER performance for a 2-in-4-out ISI channel with $d = 3$ is shown in Figure 3. The window size w is chosen to be 3. It is observed that IP-LS is about 1-2dB worse than the LS with perfect channel knowledge. The proposed IP-LOIC is about 1dB better than MMSE with perfect channel knowledge. The performance of IP-LOIC with perfect channel knowledge is much better than MMSE, demonstrating its advantage in known channel detection problems. The performance gap between the perfect and imprecise channel knowledge detection with IP-LOIC suggests the existence of local minima in (11).

Figure 1(b) illustrates the complexity or the convergence speed for IP-LS and IP-LOIC. Take $SNR = 7$dB for example, the loop of IP-LS are executed 2.5 times on the average, and for iterative projection with LOIC, the loop of Algorithm 1 is executed 2.2 times on the average, and the inner loop of Algorithm 2 is executed 3.5 times on the average. Since Algorithm 2 involves inversion of a matrix of size 20×6(see (22)) while the LS algorithm for (16) involves solving the over-determined linear system with dimension 412×200, we can conclude that the proposed LOIC method has a lower complexity.

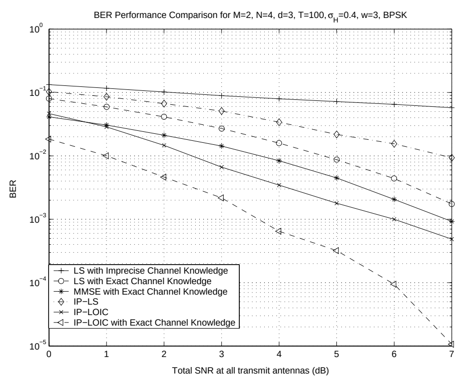


Fig. 3. BER performance comparisons for 2I4O, $d = 3$.

5. REFERENCES

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