# Switched Capacitors: a bridge between analog and digital SP

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**ABSTRACT**

Switched capacitor filter design is an hybrid technique. Occasional subject in both analog circuit and DSP courses, we made use of it in order to introduce digital filter concepts from an Analog Signal Processing perspective. Laboratory assignments take into practice sampling effects and IIR filter design issues, such as analog to digital transformations or cascade realization. Distortion and noise become allies to make the student perception sharper. A PCB design that meets some specifications given, turns into an accomplished challenge. That means self confidence to keep on track.

## 1. INTRODUCTION

Signal Processing is an amazing discipline. So prodigious that getting into its world is quite a challenge. So vast that almost all areas of the Electrical Engineer curricula play a role. Switched capacitor theory is one of them.

When EE students reach our course on Signal Processing, they have a solid background in Analog Circuit Theory as well as experience in discrete circuit implementation [1]. They are able to consider the problems that arise in a design, managing terms like dynamic range, component sensibility, PCB layout and noise. They had already built their active analog filter using operational amplifiers, resistances and capacitors, tested its performance using proper instruments, and compared it to self-created PSpice [2] simulations.

Having all this knowledge in perspective, the introduction to Signal Processing theory should take advantage of it. We understand that the first steps in SP are quite abstract, involving subjects such as discrete variables and systems, Z transforms and the sampling theorem. As we focus the course to put concepts into practice, they will be prepared, by the end of the semester, to calculate and test a digital filter.

We could find a way to ease the path. Filter theory, sampling effects, analog and digital mixed PCB design; all concepts are included in one subject: a switched capacitor filter.

## 2. THE TASK

### 2.1. Theoretical material

There are two digital principles that puts switched capacitor filter theory together. First, the way we model an analog integrator. Second, the need of sampling the analog signal.

A resistance can be integrated in a CMOS chip, but it requires a great surface. Instead, there is a way to simulate it, using a capacitor whose charge is injected through a switch. One possible implementation of such a resistance is shown in Figure 1.

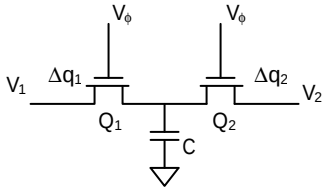


**Figure 1: SC Shunt Resistance**

In order to achieve high precision in values, CMOS technologies can implement ratio of small capacitors, obtaining results that can reach .1% error. The transfer function of an integrator whose resistance is replaced by the SC model is proportional to the ratio of the capacitors involved. This property entitles the integrator as the basic unit of the filter implementation.

It is clear now the signal becomes sampled, and an equivalent transfer function is required. Depending on the type of model (parallel or series resistance), a different "digital" integrator is obtained:

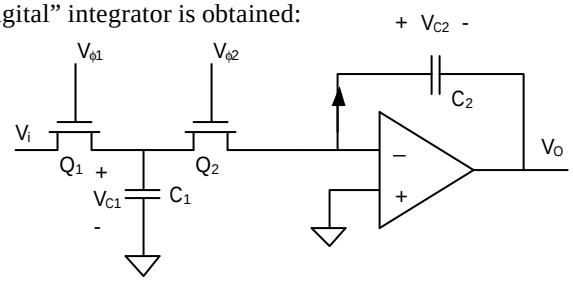


**Figure 2 (a): SC Shunt Integrator**

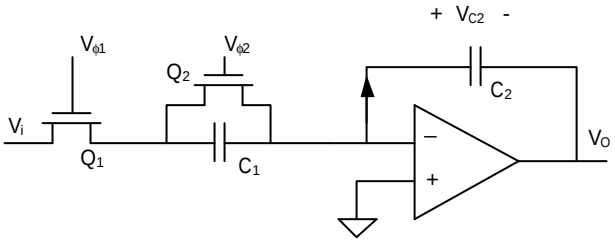


**Figure 2 (b): SC Series Integrator**

$$H(z) = -\frac{C_1}{C_2} \cdot \frac{z^{-1}}{1 - z^{-1}}$$ Forward integrator (Shunt)

$$H(z) = -\frac{C_1}{C_2} \cdot \frac{1}{1 - z^{-1}}$$ Backward integrator (Series)

In order to introduce all the alternatives, we should mention that there is also a bilinear integrator, which requires four switches and a double frequency clock.

$$H(z) = -\frac{C_1}{C_2}\frac{z+1}{z-1} = -\frac{C_1}{C_2}\frac{1+z^{-1}}{1-z^{-1}} \qquad \text{(Bilineal)}$$

Another double frequency clock circuit samples a forward or a backward integrator at half period. It is called midpoint integrator or LDI.

$$H(z) = -\frac{C_1}{C_2}\frac{z^{-\frac{1}{2}}}{1-z^{-1}} \qquad \text{(LDI)}$$

IC designers take into account that stray capacitances play an important part, so they modify the basic circuits. An example of a stray capacitance insensitive integrator is shown in figure 3.

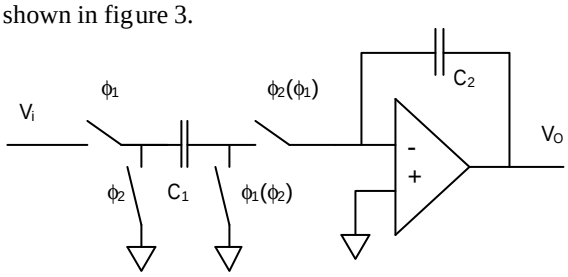


**Figure 3: Stray C insensitive SC Integrator**

Depending on the choice of switches, we may get both integrators, shunt or series.

There are several implementations issues that make designs quite interesting:

- Frequency Response Distortion

Each integrator described above has a different analog to digital mapping. That effect causes significant distortion of the frequency response, that has to be taken into count while calculating the components of a filter.

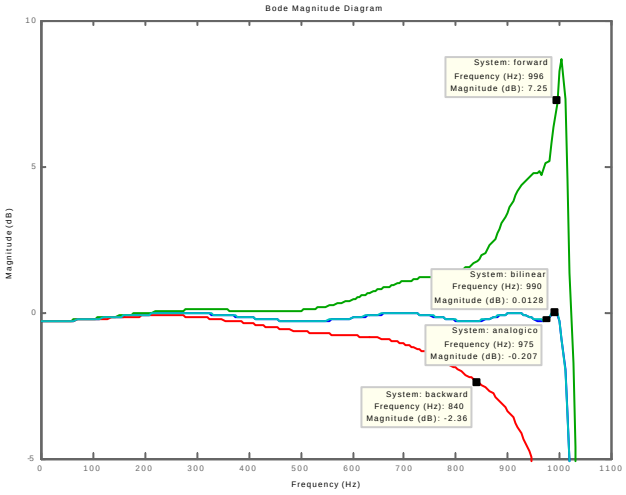


**Figure 4: Frequency Response Distortion of an 8th order Elliptic Filter using diverse transformations**

An example of such effect can be seen in Figure 4, where the magnitude of the frequency response of an elliptic filter (Cauer) is calculated with the aid of MATLAB [3] from three diverse cases: analog, backward integrator and bilineal.

- Stability

Although the sampling frequency is always at least 50 times the cut-off frequency of the filter, high order forward implementations have little gain margin.

Figure 5 displays the pole-zero location in the z plane, for the elliptic filter of Figure 4 using different integrators.

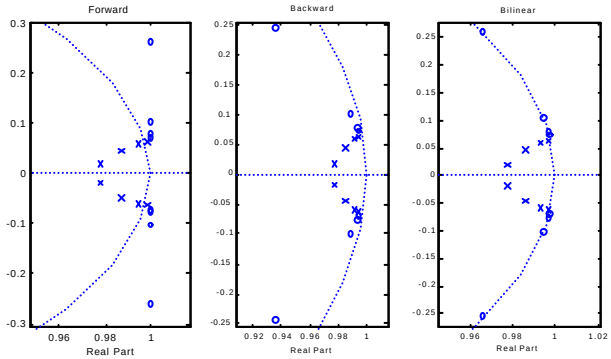


**Figure 5: Pole-Zero Location of an 8th order Elliptic Filter using diverse transformations**

In this case it is clear that the forward integrator cannot be implemented.

- Sampling effects

The switched capacitor circuit acts as a sampling system. Depending on the application, the consequences may be helpful or have to be corrected.

A switched capacitor filter that emulates an analog one, should band-limit the incoming signals and get rid of the staircase outputs. Part of the student's task appertain to the antialiasing and recover filter designs.

## 2.2. Specifications

From the several IC's that perform SC filters, there are two basic realizations. The ready-to-use kind, which allows just the cut-off frequency setting, such as the MAX294, from MAXIM [4]; and the state variable implementation, a cascade of second order stages to carry out almost any filter, like the classical MF-10 [5].

The student's assignment includes both designs, to be accomplished by a group of four or five of them. They have to start up a PCB of their own, showing that it meets the prescribed specifications; and prepare a report with all considerations they have taken, the limitations they have reached, and a set of conclusions.

Besides the frequency response constraints, the general purpose MF-10 filter should take into consideration:

- Maximum dynamic range achievable
- Maximum input signal
- Maximum frequency range
- PCB design with noise minimization
- Input and output signals should be continuous-time

Task goals are detailed in the following items:

a) Transfer function expressions, H(s) and H(z) to be able to compare them theoretically.
b) Fitting clock frequency and oscillator design.
c) Evaluation of offset and feasible corrections.
d) PSpice simulations.
e) Dynamic range measurement.
f) PCB Implementation.
g) Frequency response measurements, comparison with analytical responses.
h) Ready to cut-off frequency changes.
i) Antialiasing and reconstruction filters design.
j) Performance comparison with similar analog filters.

Similar demands are to be met for MAX294 SC filter, leaving out the design. In this case, it is an $8^{th}$ order lowpass elliptic SC filter, which cannot be changed.

## 3. SOLVING THE PROBLEM

The first part of the solution involves some theory of IIR digital filter design. They have to compare the transformations to figure out which one is implemented in the IC. After that, they have to consider a margin and redesign the filter, in order to achieve the original specifications. Figure 6 shows a MATLAB example with a

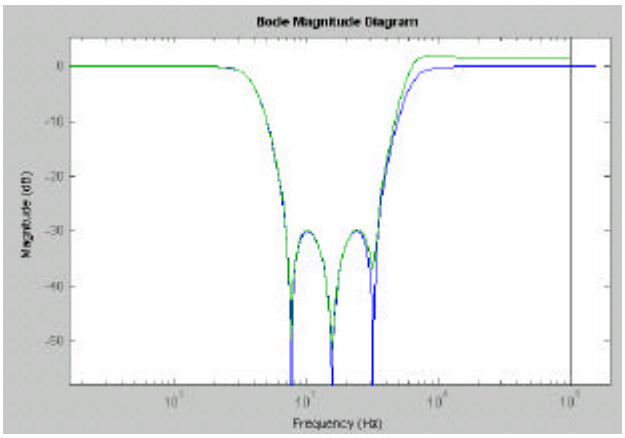


**Figure 6: Band-reject Chebyshev II Filter for 200 KHz clock frequency**

band-reject Chebyshev II filter, for a given clock frequency. Increasing it to 1 MHz, makes the curves identical. Due to similarities found, the students could conclude that the MF-10 uses the forward integrator.

Changing the clock frequency affects the response, but it also compromise dynamic range, thus its value is very important. It can be seen that concerning frequency response accuracy, the higher the better.

Oscillator design is an easy task for them because of their experience. What makes this job interesting is the PCB layout, where they have to manage analog and digital signals, in order to minimize the noise. Concepts like clock feedthrough, bypass capacitors, single point grounding are essential for a successful project. Students

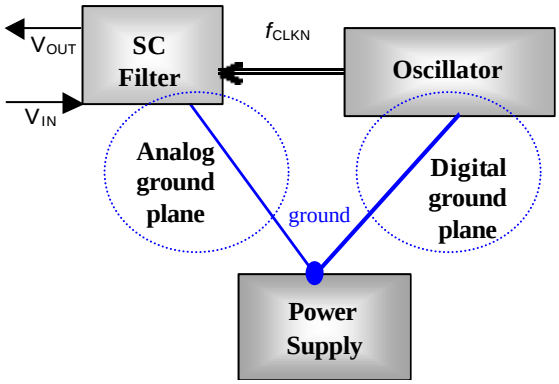


**Figure 7: Single Point Grounding PCB Layout**

must realize that any SC filter can be a part of a digitalization process, acting as the antialiasing stage. In that case, they should provide clock synchronization between the SC device and A to D converter.

Offset is an issue in some realizations, but no important conclusions can be derived from it.

Dynamic range (DR), as in all system designs, is a relevant matter. DR Optimization is a hard task, and it involves almost every step of the process. It plays an important role in the power design, it depends on the PCB design, it doesn't change linearly with the clock frequency and it is a direct result of the chosen order of the state variable stages. Performance comparison is particularly pertinent to analog and digital filter cascade implementation design. This hands-on experience helps enhance their next assignment, a digital filter implementation.

DR measurement can be performed in many ways, but our goal is to gain ground in signal processing. That is why students analyze harmonic distortion, and find the upper DR limit from the spectrum of the signal. Given a desired distortion level, they amplify the input until they reach to that point. Figure 8 displays the spectrum of the sinusoidal signal at the output of a $7^{th}$ order Legendre lowpass filter, and a 46 dB level of distortion.

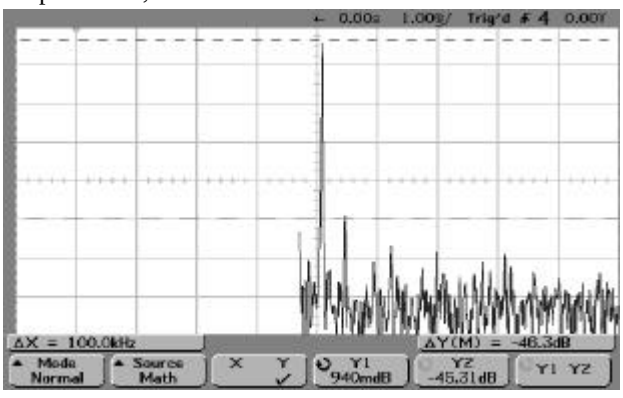


**Figure 8: SC Filter Output Signal Spectrum**

Measurement results are encouraging, considering all the variables involved. A MAX294 frequency response provided by the manufacturer and what the students got from their board is shown in Figure 9.

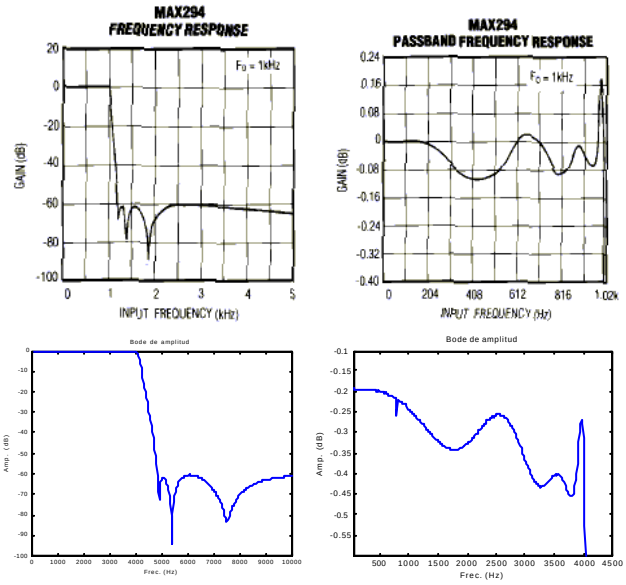


**Figure 9: MAX294 Magnitude Frequency Response**

Reconstruction filter design should take into account the "*sinc effect*" .It is not relevant in the band pass, as it is for a digital filter, but it assists the band stop attenuation with a factor of: $\left|\frac{\sin\left(p\frac{f}{f_s}\right)}{p\frac{f}{f_s}}\right| \cong \frac{1}{100} \Rightarrow 40\,dB$

As a consequence the reconstruction filter for this application can be as simple as a 1st order type. Choosing a convenient cut-off frequency, the zero-order-hold kind of output is eliminated.

Figure 10 displays a 2.2 KHz output signal from the MAX294 SC Filter in 4 KHz, before and after an analog RC reconstruction filter. In this particular design, the switching noise was not optimized.

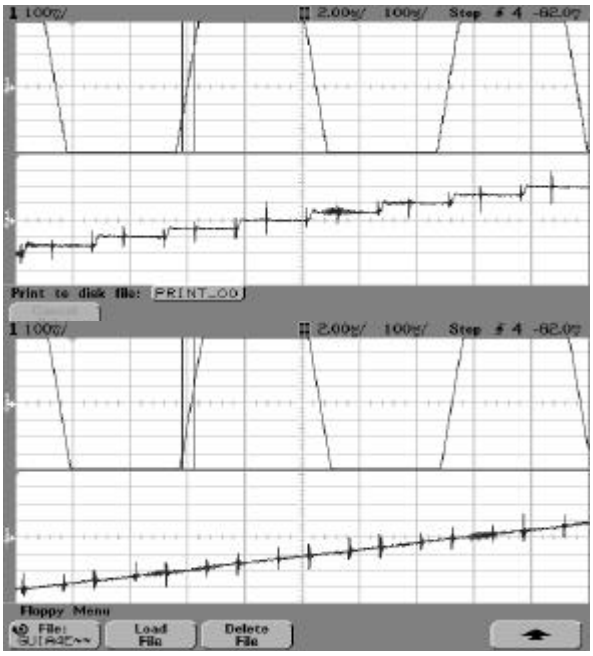


**Figure 10: MAX294 Output, before and after the reconstruction filter**

The final task, a technology comparison, gets students ready for digital signal processing. They carry out an analysis of the following parameters:

- Design complexity
- Cost
- Sensitivity
- Dynamic Range
- Bandwidth
- Repetitiveness
- Sampling and Reconstruction
- Trimming
- Accuracy
- Offset adjustment
- PCB area
- Number of components
- Oscillator issues

## 4. RESULTS

SC Filter design is seldom studied in depth. It is beyond an operational amplifier laboratory course, because of the digital concepts it includes. It is furthermore apart a DSP course, given the continuous-time nature of the signals processed. We introduced the subject to SP students 6 years ago. Ever since students have gone through DSP assignments more quickly: they have learned to plan ahead. We are confident that the knowledge SC can promote worth considering.

## 5. CONCLUSIONS

We have sketched a laboratory experiment about switched capacitor filter design. The assignment as well as the outcome have been explained. We have focused on the issues that concerns SP the most.

Provided a laboratory experience in both, active analog and SC filter design, students are able to manage and appreciate pros and cons of the SP world. We believe that every step they take in learning, should be supported by previous knowledge. In that path, they feel confident, and although they work hard, they realize they are not wasting their time.

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