ANGULAR MULTICHANNEL SIGMA FILTER

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ABSTRACT

Adaptive nonlinear filtering methods are preferred in situations when it is necessary to adapt a filter behavior for varying signal and noise statistics. In case of the impulsive noise corruption, the problem is stated often as searching for the switching function that allows reducing the filter effect only to noisy samples. Thus, the undesired smoothing of non-corrupted image areas, which results in a blurring, especially of small image structures and details, is reduced. We provide a new adaptive framework between basic vector directional filter and identity operation based on the angular multichannel definition of the Lee sigma filter [4].

1. INTRODUCTION

Multichannel signal processing has been the subject of extensive research during the last years, primarily due to its importance to color image processing. The most common image processing tasks are noise filtering and image enhancement. These tasks are an essential part of any image processing system whether the final image is utilized for visual interpretation or for automatic analysis.

It has been widely recognized that the processing of color image data as vector fields is desirable due to the correlation that exists between the image channels, and that the nonlinear vector processing of color images is the most effective way to filter out noise. For this reasons, the new filtering technique presented in this paper is also nonlinear and utilizes the correlation among the color image channels.

A number of nonlinear, multichannel filters, which utilize correlation among multivariate vectors using various distance measures, have been proposed [1],[3],[5-10]. The most popular nonlinear, multichannel filters are based on the ordering of vectors in a predefined sliding window. The output of these filters is defined as the lowest ranked vector according to a specific ordering technique.

2. VECTOR DIRECTIONAL PROCESSING

Let $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ be a set of multichannel vector-valued samples spawned by a filter window of a finite size N and $\mathbf{x}_{(N+1)/2}$ a central sample corresponding to the window reference position. Let us consider that each input vector $\mathbf{x}_i$ is associated with the angle measure α_i [7],[10] given by

$$\alpha_i = \sum_{j=1}^{N} \mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1, 2, ..., N \tag{1}$$

where

$$\mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) = \cos^{-1}\left(\frac{\mathbf{x}_i . \mathbf{x}_j^T}{|\mathbf{x}_i| . |\mathbf{x}_j|}\right) \tag{2}$$

$$= \cos^{-1}\left(\frac{x_{i1}x_{j1} + x_{i2}x_{j2} + ... + x_{im}x_{jm}}{\sqrt{x_{i1}^2 + x_{i2}^2 + ... + x_{im}^2}\sqrt{x_{j1}^2 + x_{j2}^2 + ... + x_{jm}^2}}\right) \tag{3}$$

represents the angle between two m-dimensional vectors $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{im})$ and $\mathbf{x}_j = (x_{j1}, x_{j2}, ..., x_{jm})$. If the ordering scheme of ordered angular measures

$$\alpha_{(1)} \le \alpha_{(2)} \le ... \le \alpha_{(r)} \le ... \le \alpha_{(N)} \tag{4}$$

is implied to the input vector-valued samples $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$, it results in the ordered input set

$$\mathbf{x}^{(1)}; \mathbf{x}^{(2)}; ...; \mathbf{x}^{(r)}; ...; \mathbf{x}^{(N)} \tag{5}$$

The sample $\mathbf{x}^{(1)}$ associated with the minimum angular distance $\alpha_{(1)}$, i.e. the sample $\mathbf{x}^{(1)}$, minimizing the sum of angles with other vectors, represents the output of the basic vector directional filter (BVDF) [10].

If the filter output is formed by the first r terms of the ordered set (5), these r order-statistics constitute the output of generalized vector directional filter (GVDF) [10]. Since the GVDF passes to the filter output the set of r vectors with the smallest angular measures, it eliminates the samples with atypical directions in the vector space. In order to estimate the noisy sample, the GVDF is closely connected with an additional filter such as α-trimmed average filter, multistage median filter or some morphological filters. This necessary connection significantly increases the filter complexity.

3. STANDARD SIGMA FILTER

In many filtering situations with a low degree of the observed impulses (outliers or bit errors) the problem is design the filter so that the desired noise-free image features will be invariant to a filtering operation [2]. Thus, the undesired amount of smoothing resulting often in a blurring is reduced. To take the advantage of the optimal filtering situation, some adaptive approaches for noisy color images were developed [5-8], where the smoothing filter estimates only the noisy central sample. Otherwise, the identity operation is performed and the central sample remains unchanged.

Originally, Lee [4] introduced the standard deviation in the filter design for noisy gray-scale images. Mathematically, the standard deviation is given by

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \qquad (6)$$

where μ is a mean of observed (input) data $x_1, x_2, ..., x_N$. The behavior of the developed sigma filter is described as

$$\begin{array}{ll} \text{IF } \left|x_{(N+1)/2} - \mu\right| \geq k\sigma & \text{THEN } y = f(x_1, x_2, ..., x_N) \\ & \text{ELSE } y = x_{(N+1)/2} \end{array} \qquad (7)$$

where y is the sigma filter output, $f(\cdot)$ characterizes the smoothing function, σ is the standard deviation (6), k is the smoothing factor, μ is the mean of the input set and $x_{(N+1)/2}$ is the central sample. If $\left|x_{(N+1)/2} - \mu\right|$ is greater than or equal to the product $k\sigma$, the central sample $x_{(N+1)/2}$ is most probably noisy and it will be estimated by the smoothing function $f(\cdot)$, usually defined as the median of the input set. The condition $\left|x_{(N+1)/2} - \mu\right| < k\sigma$ means that the central sample is noise-free and the sigma filter will perform the identity operation. For the smoothing factor $k = 0$ sigma filter performs the smoothing operation, whereas for $k \to \infty$ sigma filter reduces to identity operation. In the rest of this paper we will use the standard choice $k = 1$.

4. ANGULAR MULTICHANNEL SIGMA FILTER

The main problems related to the transformation of scalar filtering methods to the multichannel case, is the sample ordering [7],[9] and the quantification of distances between vector-valued samples. The distances of vector samples can be computed in many ways [7] such as Minkowski metric, absolute distance, Euclidean distance, Chess-board distance, Canberra distance, Czekanowski coefficient and various similarity measures given by angles and correlation that exist between two multichannel samples.

In case of multichannel images, the directional processing [3],[5],[6],[9],[10] preserves well the color chromaticity. For that reason, we provide the vector directional sigma filter based on the angular distances between two vectors. If the distance measures $x_i - \mu$, for $i = 1, 2, ..., N$, is rewritten in term of vector angles $A(\mathbf{x}_i, \boldsymbol{\mu})$, the standard definition (6) is defined as follows

$$\sigma_A = \sqrt{\frac{1}{N}\sum_{i=1}^{N} A^2(\mathbf{x}_i, \boldsymbol{\mu})} \qquad (8)$$

where N is the window size and $\boldsymbol{\mu} = (\mu_1, \mu_2, ..., \mu_m)$ is the mean of the input set $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$, i.e.

$$\boldsymbol{\mu} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i \qquad (9)$$

Thus, the corruption of the central sample $\mathbf{x}_{(N+1)/2}$ is determined by a simple comparison of the product $k\sigma_A$ with the angle between the central sample $\mathbf{x}_{(N+1)/2}$ and the sample mean $\boldsymbol{\mu}$. It forms the following inequality

$$A(\mathbf{x}_{(N+1)/2}, \boldsymbol{\mu}) \geq \sigma_A \qquad (10)$$

If the angle $A(\mathbf{x}_{(N+1)/2}, \boldsymbol{\mu})$ is greater than or equal to the angular standard deviation σ_A, i.e. $A(\mathbf{x}_{(N+1)/2}, \boldsymbol{\mu}) \geq \sigma_A$, then the central sample $\mathbf{x}_{(N+1)/2}$ is probably noisy because the corresponding angle $A(\mathbf{x}_{(N+1)/2}, \boldsymbol{\mu})$ reflects the significant difference between $\mathbf{x}_{(N+1)/2}$ and its neighborhoods. The high similarity between $\mathbf{x}_{(N+1)/2}$ and the input set $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ reflected by the angle $A(\mathbf{x}_{(N+1)/2}, \boldsymbol{\mu})$ smaller than the standard deviation σ_A.

When the detection rule (10) is combined with the BVDF performing a pure directional operation, the output $\mathbf{y}$ of the proposed angular sigma filter is given by

$$\begin{array}{ll} \text{IF } A(\mathbf{x}_{(N+1)/2}, \boldsymbol{\mu}) \geq \sigma_A & \text{THEN } \mathbf{y} = \mathbf{x}^{(1)} \\ & \text{ELSE } \mathbf{y} = \mathbf{x}_{(N+1)/2} \end{array} \qquad (11)$$

where $\mathbf{x}^{(1)}$ characterizes the BVDF output (5) and $\mathbf{x}_{(N+1)/2}$ is the input central sample. The noisy $\mathbf{x}_{(N+1)/2}$ is estimated by the BVDF, whereas the identity operation is related to the uncorrupted central sample $\mathbf{x}_{(N+1)/2}$.

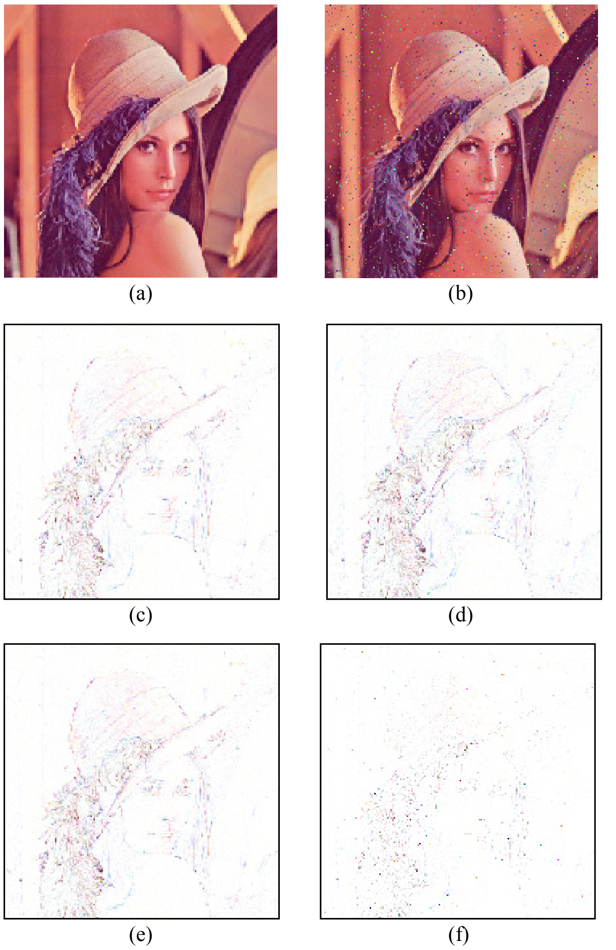

Fig. 1 Achieved results related to the test image Lena. (a) test image Lena, (b) 4% impulsive noise, (c-f) estimation error of (c) VMF, (d) BVDF, (e) DDF, (f) proposed method

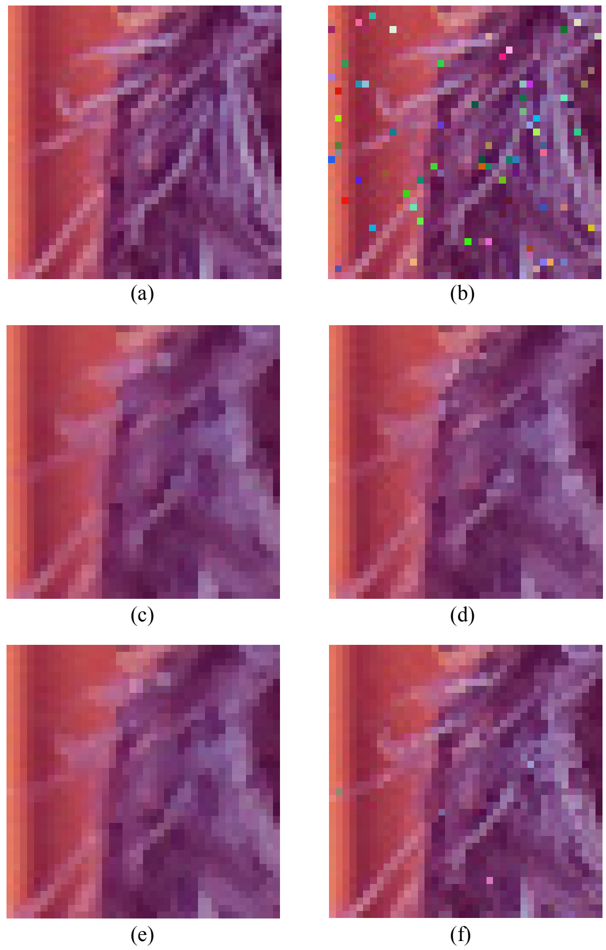

Fig. 2 Zoomed parts of results related to the test image Lena (a) test image Lena, (b) 4% impulsive noise, (c) VMF output, (d) BVDF output, (e) DDF output, (f) proposed method

5. EXPERIMENTAL RESULTS

The efficiency of the new method was tested for color test images Lena (Fig. 1a) and Peppers corrupted by impulsive noise [2],[7]. The impulse probability p_v was ranged from no corruption to 10% impulsive noise with the stepsize 1%. The filtering results (Tab.1-3, Fig.3) were evaluated through mean absolute error (MAE), mean square error (MSE) and normalized color difference (NCD) [7].

The new filter provides excellent improvement in comparison with standard filters such as VMF, BVDF and DDF. This is also confirmed by the results depicting the estimation error (Fig. 1c-f) and also by the zoomed parts of images (Fig. 2). It can be seen that the estimation error of the proposed method has a random structure (Fig. 1f), whereas standard filters (Fig. 1c-f) fail at image edges. The proposed method achieves excellent results especially in terms of MAE and NCD criteria (Figs. 3a,c) that reflect the capability of the filter to preserve the image-details and color chromaticity. Concerning the impulsive noise attenuation expressed through MSE (Fig. 3b), the proposed method was superior to standard filters for noise intensity smaller than 8%. Fig. 4 shows that the small amount of additive Gaussian noise σ_G does not influence the impulse detection capability of the proposed method.

Table 1 Performance of methods related to the noise-free image.

Image	Lena			Peppers		
Method	MAE	MSE	NCD	MAE	MSE	NCD
VMF	3.190	45.4	0.038	2.885	36.7	0.041
BVDF	3.605	52.7	0.039	3.458	50.2	0.040
DDF	3.288	46.8	0.038	2.907	37.5	0.040
Proposed	1.320	22.8	0.017	1.034	17.6	0.019

Table 2 Performance of methods related to 4% impulsive noise.

Image	Lena			Peppers		
Method	MAE	MSE	NCD	MAE	MSE	NCD
Noisy	3.103	356.2	0.036	3.176	389.4	0.036
VMF	3.418	50.9	0.040	3.130	43.3	0.044
BVDF	3.813	59.6	0.041	3.726	61.5	0.043
DDF	3.475	51.2	0.040	3.138	43.7	0.042
Proposed	1.301	31.5	0.016	1.308	57.2	0.021

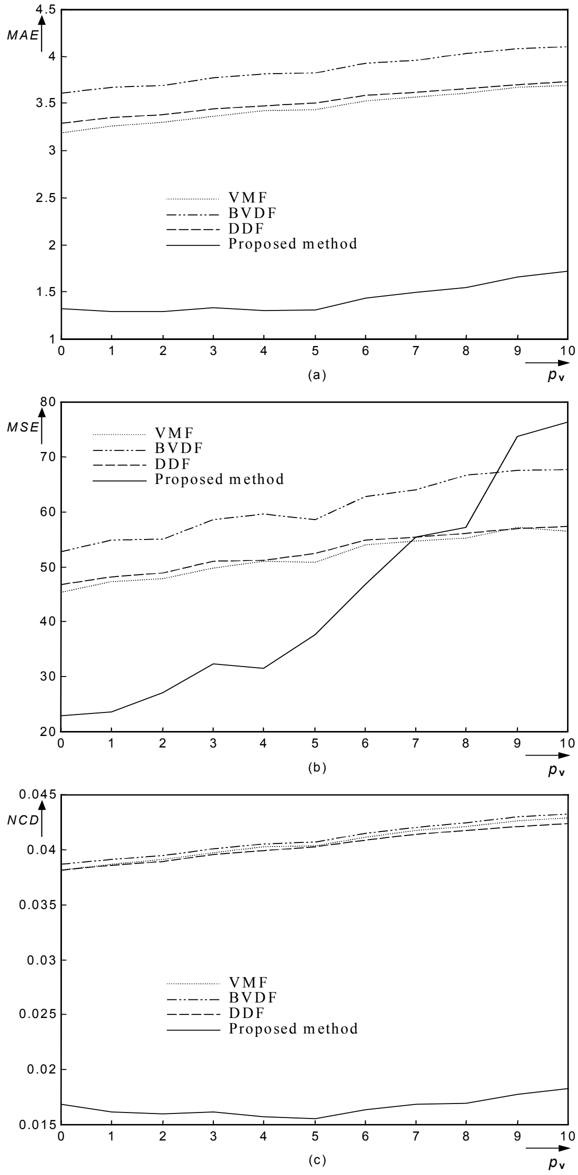

Fig. 3 Objective criteria in dependence on impulse probability. (a) MAE, (b) MSE, (c) NCD

Table 3 Performance of methods related to 8% impulsive noise.

Image	Lena			Peppers		
Method	MAE	MSE	NCD	MAE	MSE	NCD
Noisy	5.975	690.6	0.068	6.225	756.0	0.070
VMF	3.611	55.2	0.042	3.351	49.3	0.048
BVDF	4.026	66.7	0.042	3.989	74.6	0.046
DDF	3.659	56.0	0.042	3.370	51.3	0.046
Proposed	1.542	57.1	0.017	1.906	102.8	0.028

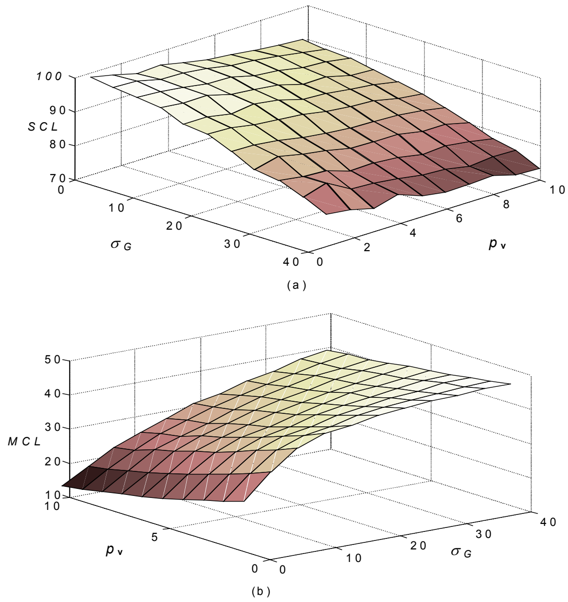

Fig. 4 Detection characteristics of the proposed method vs. mixed noise (additive Gaussian noise mixed with impulsive noise). (a) successful detection rate, (b) failed detection rate

6. CONCLUSION

The new angular sigma filter for the impulsive noise detection and its suppression in multichannel images was presented. The achieved results show that the new filter has excellent preservation capabilities and provides significant improvement in comparison with well-know vector filters such as VMF, BVDF and DDF.

7. REFERENCES

[1] J. Astola, P. Haavisto, and Y. Neuvo, "Vector Median Filters," Proc. IEEE, Vol.78, No.4, pp.678-689, April 1990.

[2] J. Astola, and P. Kuosmanen, *Fundamentals of Nonlinear Digital Filtering*, CRC Press, 1997.

[3] D.G. Karakos, and P.E. Trahanias, "Generalized Multichannel Image-Filtering Structure," IEEE Transactions on Image Processing, Vol.6, No.7, pp.1038-1045, July 1997.

[4] J.S. Lee, "Digital Image Smoothing and the Sigma Filter," Comp Vision, Graphics, Image Proc., Vol.24, No.2, pp.255-269, November 1983.

[5] R. Lukac, "Color Image Filtering by Vector Directional Order-Statistics," Pattern Recognition and Image Analysis, Vol.12, No.3, pp.279-285, August 2002.

[6] K.N. Plataniotis, D. Androutsos, and A.N. Venetsanopoulos, "Color Image Processing Using Adaptive Vector Directional Filters," IEEE Transactions on Circuits and Systems II, Vol.45, No.10, pp.1414-1419, October 1998.

[7] K.N. Plataniotis, and A.N. Venetsanopoulos, *Color Image Processing and Applications*, Springer Verlag, 2000.

[8] M. Szczepanski, B. Smolka, K.N. Plataniotis, and A.N. Venetsanopoulos, "Robust Filter for Noise Reduction in Color Images," Proc. CGIV'02 in Poitiers, pp.517-522, April 2002.

[9] K. Tang, J. Astola, and Y. Neuvo, "Nonlinear Multivariate Image Filtering Techniques," IEEE Transactions on Image Processing, Vol.4, No.6, pp.788-798, June 1995.

[10]P.E. Trahanias, D. Karakos, and A.N. Venetsanopoulos, "Directional Processing of Color Images: Theory and Experimental Results," IEEE Transactions on Image Processing, Vol.5, No.6, pp.868-881, June1996.