CONTEXT-BASED GRAPHICAL MODELING FOR WAVELET DOMAIN SIGNAL PROCESSING

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ABSTRACT

Wavelet-domain hidden Markov tree (HMT) modeling provides a powerful approach to capture the underlying statistics of the wavelet coefficients. We develop a mutual information-based information-theoretic approach to quantify the interactions between the wavelet coefficients within a wavelet tree. This graphical method enables the design of a context-specific hidden Markov tree (HMT) by adding or deleting links from the traditional tree structure. The performance of the model is demonstrated on segmenting two-dimensional synthetic textures having intricate substructures, although the method can be used for signals of arbitrary dimensions.

1. INTRODUCTION

A multi-level wavelet decomposition splits the original signal spectrum into a set of frequency subbands. Crouse et al. [1] have modeled each wavelet node as a two-state Gaussian mixture, and the states sampled by a *sequence* of wavelet coefficients (connected across the scales forming a wavelet tree) are modeled as a Markov process. This results in a *hidden* Markov tree (HMT) model, since the state being sampled by a given wavelet coefficient is assumed unknown, or "hidden". The HMT structure [1] connects each (parent) node in scale j with 2^M (children) nodes at the next finer scale $j+1$, M being the dimensionality of the signal space. This approach ignores the scaling coefficients at the coarsest level from any parametric statistical modeling. Moreover, the links between the wavelet coefficients are treated as fixed rather than being adaptive to the statistics of the signal under consideration.

Recently, there has been increasing interest in a more general class of probabilistic models of which the HMT is a special case. The approach, known as graphical models [2], tries to "learn" the complex dependencies directly *from data* rather than making a-priori assumptions on interactions between the *hidden* and the *observed* variables. In our approach, we explore possible existence of interactions between the wavelet coefficients not directly connected by the traditional HMT structure. The importance of including additional links beyond those in [1] is quantified in terms of the mutual information. The links between the non-adjacent wavelet node pairs are then added to the modified HMT structure and the Expectation-Maximization (EM) algorithm is modified to include the changes in the structure. This approach allows us to design a wavelet tree structure in an adaptive signal-dependent fashion that tries to emulate the "true" underlying dependencies between wavelet coefficients.

In the multi-dimensional case, the discrete wavelet transform decomposes the original signal into a set of oriented frequency bands. For example, a two-dimensional image under discrete wavelet transform decomposes into four subbands: *HH*, *HL*, *LH* and *LL*, where *LL* represents the coarsest representation of the original image. In our approach, we include the coarsest *LL* subband as the common root node to the *HH*, *HL* and *LH* trees and modify each subband tree structure through addition of potentially significant links as dictated by mutual-information calculations. Each modified subband tree was trained separately via a modified EM algorithm.

The remainder of the text is organized as follows. In Sec. 2, we discuss the adaptations of the traditional HMT scheme. Although our proposed strategy is independent of the dimensionality of the signal space, we have chosen to use two-dimensional signals (digital "Brodatz" images [3]) to illustrate the efficacy of our algorithm. The performance of the proposed graphical model in segmenting two-dimensional textures is compared in Sec 3 against the traditional HMT quadtree structure. The work is summarized and conclusions are discussed in Sec. 4.

2. SIGNAL-SPECIFIC TREE DESIGN

A. Wavelet Decomposition and HMT model

The main drawback of the HMT-based stochastic multi-scale modeling for multi-dimensional ($M > 1$) signals is the fixed tree structure for all the subbands, independent of the signal under investigation. The HMT modeling also ignores the coarsest *LL* band from parametric optimization via the EM algorithm. However, it has been shown [4] that the wavelet coefficients of most "natural" images possess certain dependencies across scales and even the coarsest *LL* subband contains significant information about the original two-dimensional signal.

In our modification of the HMT scheme, we place the scaling coefficient from the coarsest *LL* subband as the root node to each of the three subband wavelet trees. The solid lines in Fig. 1 represent the default interconnections present in the HMT for a two-level wavelet tree coupled with a root scaling node, whereas the dotted lines represent the potential additional links considered here. Our objective is to quantify the "importance" of all the potential dashed links in Fig. 1. We add to the traditional HMT structure those links having importance higher than a predefined threshold. The EM algorithm [1] is modified appropriately for the adapted HMT structure.

A two-dimensional image, subjected to wavelet decomposition, produces four decomposed signal subbands HH^1, HL^1, LH^1 and LL^1 (superscript represents the level of decomposition). The HH^1 subband corresponds to the original signal highpass filtered in both dimensions. The filtering process is repeated sequentially, with LL^k, HH^k, LH^k and HL^k representing the filtered and downsampled output from LL^{k-1}. Assume that we perform K levels of wavelet decomposition. Each point in HH^K corresponds to four (2x2) points in the $HH^{K-1}$and (2^{K-1}x2^{K-1}) points at the finest resolution HH^1 signal. Suppose $c_{i,j}$ corresponds to the *LL* subband coefficient at position (i,j). In our structure $c_{i,j}$ will be connected to the root wavelet nodes of the three subbands *HH*, *HL* and *LH* at the same spatial location (i,j).

At this point, the three subbands (*HH*, *HL* and *LH*) that resulted from a two-dimensional wavelet decomposition are connected via a common root node provided by the coarsest *LL* band. We implement a signal-specific tree modification for each of these three subbands. Given a traditional HMT quadtree structure [1], we quantify the interaction between the non-neighboring wavelet nodes based on mutual information. We "add" direct links between the tree nodes that possess mutual information higher than a predefined threshold. It is to be noted that the choice of threshold is critical to adaptive tree modification. A high threshold leads no modification of the original HMT structure, whereas a very low threshold leads to a complicated HMT structure, yielding potential overfitting.

B. Mutual Information-based Link importance quantification

The Mutual Information (MI) quantifies the reduction of uncertainty in one variable due to the knowledge about the other. Suppose x and y are two random variables represented by probability distribution $p(x)$ and $p(y)$ respectively. The mutual information, $I(x,y)$ between the two variables x and y is defined as

$$I(x;y) = H(x) - H(x \mid y) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)} \qquad (1)$$

Where $p(x,y)$ represents the joint probability distribution of x and y.

In order to explore the potential of additional links beyond those in the traditional HMT [1], we first train the EM algorithm [1] on the traditional HMT (excluding the *LL* root node). This results in a Gaussian mixture model (GMM) for each of the five wavelet nodes (nodes 2-6 in Fig. 1). The root scaling node is assumed to be realized from a single underlying Gaussian source with its parameters directly estimated from the coarsest representation of the training samples. As explained above, one needs to quantify the individual marginal and the joint probability distribution of two random variables x and y in order to quantify the mutual information between them. The marginal distribution of the random variables corresponding to each node in the HMT is represented by Gaussian source(s) with trained parameters (means and variances are trained using the traditional EM algorithm, except for the root scaling node which is trained empirically using a *best-fit* Gaussian distribution). The joint probability distribution between any two nodes within a tree is estimated using a two-dimensional Gaussian distribution having a full covariance matrix. The covariance matrix is estimated empirically from the training samples of the image. Given the parametric representation of the marginal and joint probability distributions, we calculate the mutual information between a node pair (x,y) using equation (1).

Given a tree structure with six nodes (Fig. 1), one could possibly have 15 (${}^6C_2$) links between the node pairs, of which, the traditional HMT imposes four links (shown as solid lines in Fig. 1). Hence, for a two-level wavelet decomposition of an image, we focus on quantifying the significance of the remaining eleven possible links. It should be noted that the same procedure is applied for all three subbands individually. The traditional HMT structure is appended with the links that correspond to mutual information higher than the threshold value (Fig. 2).

The details of the traditional EM algorithm are presented in [1]. Hence, we shall only illustrate the modifications needed for the adapted tree structure. As discussed in [1], each link between a wavelet coefficient pair (indexed by i and its parent $p(i)$) is represented by a

2x2 transition matrix ($\varepsilon_{m,n}^{i,p(i)}$ where *m* and *n* represent the states of the wavelet nodes *i* and *p*(*i*) respectively, $m, n \in \{0,1\}$). In our approach, since the root node corresponds to a single Gaussian density, any link connecting the root node with one of its children is represented by a 2x1 matrix. As shown in Fig. 2, each leaf node (nodes 3-6) in each of the three subbands (*HH*, *HL* and *LH*) is connected to the top root node (node 1) directly and via node 2. As explained in [1], the EM algorithm for the hidden Markov tree structure revolves around estimating the intermediate model parameters (for example, α_i and β_i) defined as

$$\alpha_i = p(s_i = m, \tau_{1\backslash i} \mid \theta) \text{ and } \beta_i = f(\tau_i \mid s_i = m, \theta) \quad (2)$$

Since the root node is modeled as a single Gaussian distribution, the corresponding α parameter, $\alpha_1 = p(s_1=1)$ is fixed at one. In the modified HMT structure, each leaf node has two parents. While calculating the parameter α for each one of the leaf nodes, one may choose either of the two links from the parent-pair. The iterative upward-downward optimization of α's and β's result in parametric optimization of the modified tree structure. For the root scaling node, $\beta_1 = \left[\prod_{i=2}^{6}\sum_{m=1}^{2}\beta_i(m)\varepsilon_{m,1}^{i,1}\right] \bullet g(x_1, \mu_1, \sigma_1^2)$ (μ_1 and σ_1 are best-fit Gaussian parameter approximation of the root node having five children, refer Fig. 2) signifies that the likelihood of the modified tree $(=\alpha_1\beta_1)$ [1] depends jointly on the scaling node parameters and its interaction with the wavelet nodes (nodes 2-6 in Fig. 2).

3. TEXTURE SEGMENTATION PERFORMANCE

We illustrate the efficacy of the proposed signal-specific hidden Markov tree design based on its performance on segmenting different textures. We have employed the Haar wavelet [5] although one might use any problem-specific wavelet. The proposed algorithm modifies the tree structure within each subband by adding the potential links beyond those in [1]. Although one might investigate the combined HMT structure and try to obtain the interactions between any two nodes (not necessarily in the same subband), we have only focused on intra-subband interactions in our modified tree structures since no significant performance gain was observed from inter-subband wavelet links. In other words, the potential of additional links was examined separately for the *HH*, *HL* and *LH* coefficients, with no links between these bands. Mutual-information calculations indicate that there exist minimal inter-subband dependencies between the wavelet coefficients for the synthetic textures presented here. The performance of the proposed algorithm is compared against the traditional HMT approach [1]. The comparison of the two algorithms yields insight into the benefit of including the statistics from the *LL* subband and the utility of modifying the HMT structure based on the signal statistics.

We have considered synthetic textures [3] for which "truth" is known. Consider a two-texture image (Fig. 4a) generated from two textures of the image database [3]. Each training sample is an image block of size 4x4 pixels. The training samples were subjected to a two-level wavelet decomposition.

Given a set of *N*=500 training trees for each of the three subbands (*HH*, *HL* and *LH*), we modify the HMT structure (Fig. 2) by adding links based on mutual information. Fig. 3 represents the distribution of the mutual information for different node pairs within the wavelet tree. The figure shows a strong correlation between the root scaling node with the wavelet nodes across the scales whereas mutual information between the wavelet nodes within the same scale are relatively week. As pointed earlier, we have also examined inter-subband wavelet interactions (e.g. between *HH* and *HL* coefficients) and found them to be very weak compared to intra-subabnd interactions.

The "ground truth" for the texture mixture is shown in Fig. 4(b) where black and white represent two component textures. Fig. 4(c) corresponds to segmentation performance of the traditional HMT whereas the performance of the proposed algorithm is shown in Fig. 4(d). The proposed algorithm produces better segmentation performance (91% *vis-à-vis* 85%) when compared against the traditional HMT structure.

Most "natural" images possess strong spatial correlation. In [6], a spatial HMM was used to capture the interactions between the eight adjacent image blocks. We have shown here (Fig. 6) that a simple averaging of the log likelihoods of nine surrounding blocks (including the central block) based on our model achieves comparable performance in texture segmentation vis-a-vis HMT-HMM [6] scheme (Fig. 5). We infer that since the proposed algorithm relaxes the stringent assumptions imposed by the wavelet-domain HMT modeling, we observe superior segmentation performance as compared to the combined HMT-HMM scheme. Our model uses significantly less number of parameters since the entire set of spatial HMM parameters are absent.

4. CONCLUSIONS

We have proposed a new graphical multi-scale stochastic modeling scheme for wavelet coefficients in a multi-dimensional signal space. The new algorithm allows the

HMT structure to adaptively add links between the wavelet coefficients within and across the scales. The proposed scheme also models the coarsest scaling coefficients in conjunction with the wavelet coefficients. The algorithm is shown to produce better segmentation and classification for two-dimensional "Brodatz" images.

The primary disadvantage of the new algorithm is that the adaptive structure might become complicated when three or more levels of wavelet decomposition is performed on the original data due to a large set of potential additional links. However, we expect the efficiency and flexibility of the approach to model all potentially important intra- and inter-subband interactions to outweigh the potential limitations.

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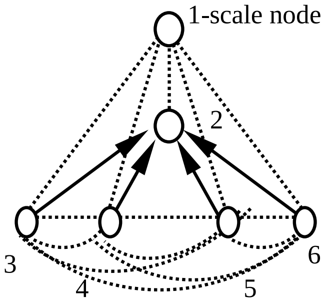


Fig. 1

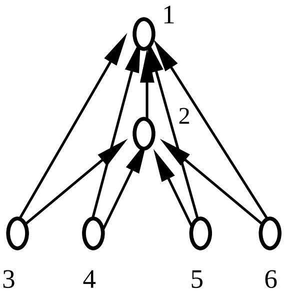


Fig. 2

Fig. 1: two-level HMT structure with "direct" and potential links"
Fig. 2: Modified HMT structure based on mutual information

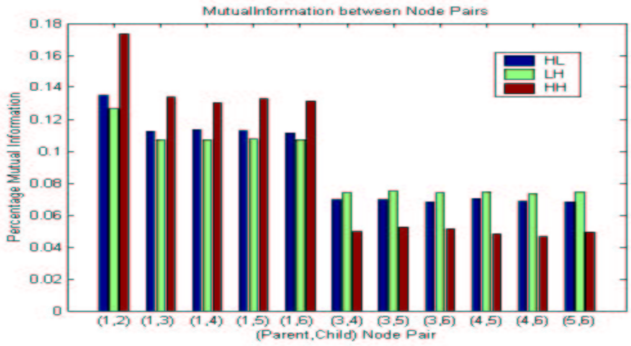
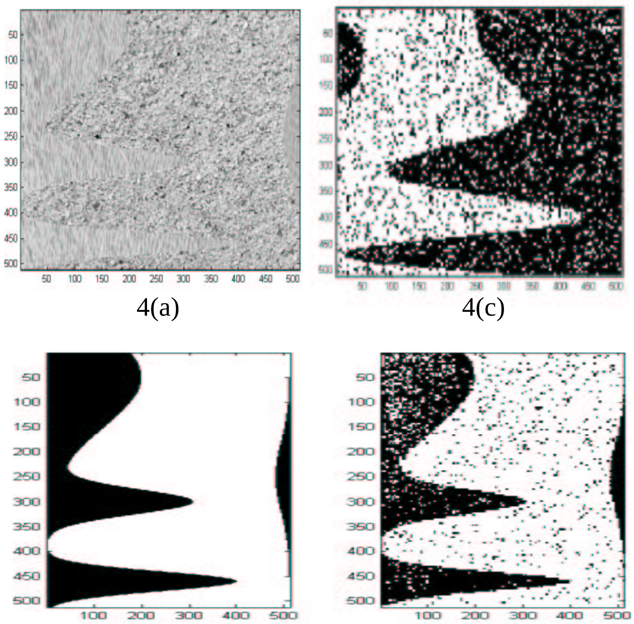


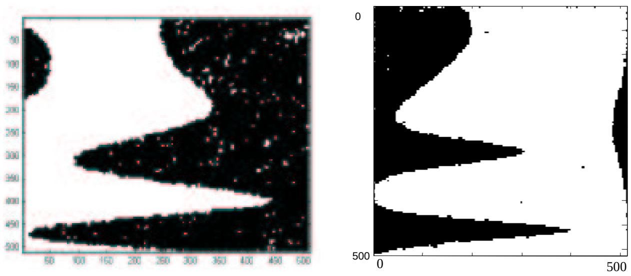
Fig. 3: Distribution of Mutual Information between node pairs



4(a) 4(c)

4(b) 4(d)

Fig. 4a: Mixture of two textures used for segmentation.
Fig. 4b: "True" labels corresponding to texture mixture.
Fig. 4c: 85% correct classification using traditional HMT.
Fig. 4d: 91% correct classification using modified HMT structure



5(a) 5(b)

Fig. 5a: 96% correct classification using HMT-HMM scheme
Fig. 5b: 99% correct classification using simple averaging of nine surrounding blocks of modified HMT outputs.