

# SIMULTANEOUS GROUP AND PHASE CORRECTION FOR THE ESTIMATION OF DISPERSIVE PROPAGATING WAVES IN THE TIME-FREQUENCY PLANE

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## ABSTRACT

This paper deals with seismic signal processing. We propose a new algorithm which performs the dispersion filter estimation using a linear array of geophones. This estimation enables a robust characterization and extraction of the dispersive propagating waves from a seismic profile. The novelty of our method is the use of a simultaneous time delay and phase shift correction in order to estimate the dispersion filter. The resulting algorithm is semi-automatic and requires very few input parameters. The main advantage of our algorithm is that the signals are analysed in the time-frequency domain using all sensors simultaneously while classical approaches are either multi-sensor in the frequency domain or single sensor in the time-frequency domain. Validations on synthetic and real data show the reliability of the algorithm.

## 1. INTRODUCTION

Analysis of seismic surface waves applied to near surface structures is an increasingly important tool in civil and seismic engineering as the shear wave velocity variation with depth can be recovered. The surface waves propagate along the surface, are highly energetic, and have a depth penetration which depends on the wavelength (*i.e.* on the frequency). As the shear wave velocities generally vary with depth, the surface waves are dispersive. This explains that their duration increases with distance (see figure 1(a)). In this paper we focus on the estimation of the dispersion of the surface waves. This has two main interests. Firstly, the parameters of the dispersive waves make it possible to invert for the shear velocity as a function of depth. Secondly, once the dispersion is known, the extraction of the propagating waves becomes efficient. This is crucial for the petroleum industry where the energetic surface waves are considered as noise [1].

In this paper we propose a new approach for estimating the dispersion of a propagating wave. In the literature,

there are two kinds of methods. The first one is based on 2D Fourier transform (or FK representation). It gives a global (or mean) estimation because group and phase parameters are usually estimated at the maximum of the 2D spectrum [2]. In 1981 McMechan [3] proposed an efficient Fourier based technique, slant stack, for estimating the dispersive parameters at all frequencies. This method does not require as many traces as the classical FK but the lack of temporal resolution makes it difficult to apply. The second set of methods is based on an improved interpretation of Time-Frequency Representations (TFR) of each trace separately [4], [5], [6]. Their advantage is the time resolution induced from the TFR. These 1D methods lead to the estimation of the group velocity between the source and each sensor. We propose in our method a trade-off between the two approaches by designing a multi-sensor analysis (2D) which include TFR. Our method can also be applied on a small number of sensors. In addition, our algorithm estimates simultaneously the group and phase velocities between the sensors. Section 2 is devoted to the design of the method. In section 3, we show the efficiency of the method on field data. Finally, in section 4 the noise robustness and stability are estimated on heavily corrupted synthetic data.

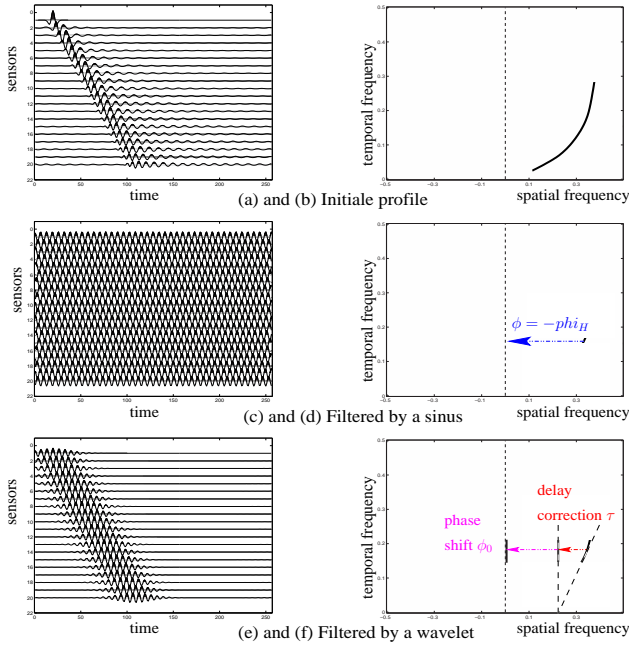
## 2. ESTIMATION OF THE DISPERSION

### 2.1. Notations

In multi-sensor analysis, dispersion is characterized by the transfer function  $h$  between sensors. Noting  $w_n$  the dispersive wave at sensor  $n$ , we have  $w_{n+1} = (w_n * h_n)$ . We note  $H_n(f) = \|H_n(f)\|e^{j\phi_{nH}(f)}$  the Fourier Transform (FT) of  $h_n$ , where  $\phi_{nH}(f)$  is the phase of  $H_n(f)$ . We assume standard assumptions which are that the dispersion is spatially stationary (*i.e.*  $h_n = h$ ), and that the module  $\|H(f)\|$  is equal to one. This simplifies  $H(f)$  to  $1 \cdot e^{j\phi_H(f)}$ . Note that in a non dispersive case,  $h$  is only a delay  $\tau$ , and  $\phi_H(f) = 2\pi f\tau$ .

## 2.2. Estimation at one frequency

To explain the algorithm we focus on the dispersion estimation on a synthetic dispersive wave (figure 1(a) in time and 1(b) in frequency) at one frequency  $f$ . To estimate  $\phi_H(f)$ , Fourier based methods correspond to filtering the traces with a sinusoid. This leads to the profile figure 1(c) in time and 1(d) in frequency. The phase shift  $\phi$  which lines up this profile is  $-\phi_H(f)$ . FK [2] methods find  $\phi$  by computing the 2D Fourier transform along the time and spatial axis, and this technique therefore requires many traces. Slant stack [3] is based on the Fourier Transform (FT) of the Radon Transform, so  $\phi$  is estimated by finding the velocity correction which maximizes the energy of the summation of all traces. However the slant stack does not separate waves which have the same dispersion curve, even if such waves arrive at the sensor at different times.



**Fig. 1.** Filtering with different time-frequency resolutions

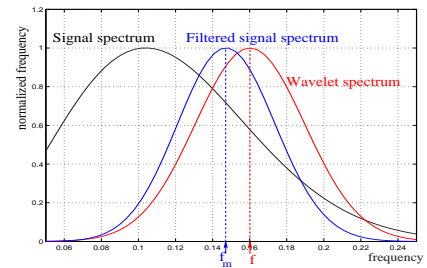
Our purpose here is to use a trade-off between time and frequency resolution to decrease the interference between the different waves. Instead of convolving our profile with a sine wave, we convolve with a short time duration sine wave. This wavelet is a bandpass filter. The output of this filtering is presented on figure 1(e) in time and 1(f) in frequency. The dispersion correction is done here by applying successively a delay correction followed by a phase shift correction (see figure 1(f)). This means that  $\phi_H(f)$  is approximated at the order 1 by  $2\pi f\tau + \phi_0$ , where  $\tau$  and  $\phi_0$  are respectively a delay and a phase shift. As in the case of the slant stack [3]  $\phi_0$  and  $\tau$  can be estimated as the value of  $\phi_0$  and  $\tau$  for which the energy after stack is maximized.

This leads to  $\phi_H(f) = -(2\pi f\tau + \phi_0)$ . The benefit here is the improved time resolution on the profile 1(e) as compared to the simple frequency analysis figure 1(c). In presence of several waves, we can therefore estimate the time location of each one. This simplifies the interpretation and decreases the interferences between different waves.

## 2.3. Estimation at all frequencies

We need to estimate  $\phi_H(f)$  at all the frequencies. So a set of bandpass filters is required. We choose to use the Continuous Wavelet Transform (CWT) because its time-frequency resolution is adapted to seismic waves [7]. Rather than filtering the profile at one frequency  $f$ , and then look for  $\tau$ , and  $\phi_0$ , the two operations can be inverted to accelerate the algorithm. This is possible because the CWT is linear. We consequently filter at each frequency the stack of all traces issued from the different delay and phase shift corrections. This means that the wavelet transform is applied to the stacked signal to visualize where in the time-frequency plane the stacking operation magnifies the energy.

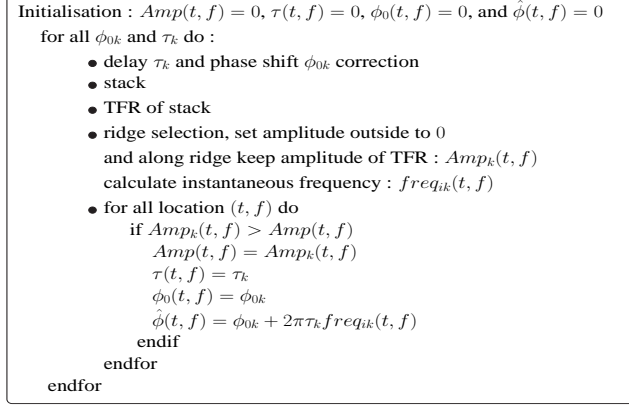
Compared with other representations [7], [8], [9], CWT has one advantage and two main drawbacks. The advantage is that there are no cross-terms. The first drawback is that when filtering our signal with a wavelet the main frequency  $f_m$  of the filtered signal is not exactly  $f$  (see figure 2). However, we can correct this error by estimating  $f_m$  with the instantaneous frequency of the filtered signal [10]. After this correction, we get  $\phi_H(f_m) = -(2\pi f_m\tau + \phi_0)$ . The second issue is that the CWT leads to a non concentrated representation of the signal. The representation has to be concentrated enough to present the different waves with no overlap of their associated pattern. To overcome this problem, we extract only the ridge of the module image for each of the  $\phi_0$  and  $\tau$  corrections. The image of the maxima (figure 5) is the output of this procedure.



**Fig. 2.** Frequency error

After the calculation of the different TFR has been done, for each time-frequency location  $(t, f)$ , we store the maximum modulus value among all the TFR generated for each correction  $\tau$  and  $\phi_0$  and its corresponding argument :  $\tau$ ,  $\phi_0$  and  $f_m$ . This enables the estimation of the phase filter

$\hat{\phi}_H(f_m(t, f)) = -(2\pi f_m(t, f)\tau(t, f) + \phi_0(t, f))$  at each coordinate  $(t, f)$ . The algorithm can be summarized by the flow chart figure 3.

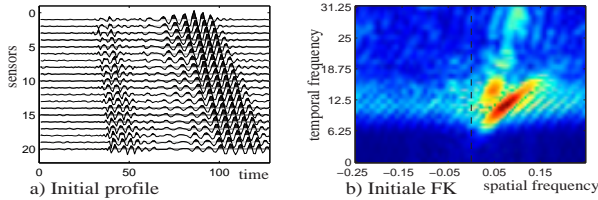


**Fig. 3.** Algorithm flow chart

The algorithm is related to 2D Fourier methods, but is more robust because we take advantage of the good time-frequency resolution of TFR. This simplifies the algorithm and improves its performance. An illustration is given on an example of field data in the next section.

### 3. DISCUSSION ON RESULTS

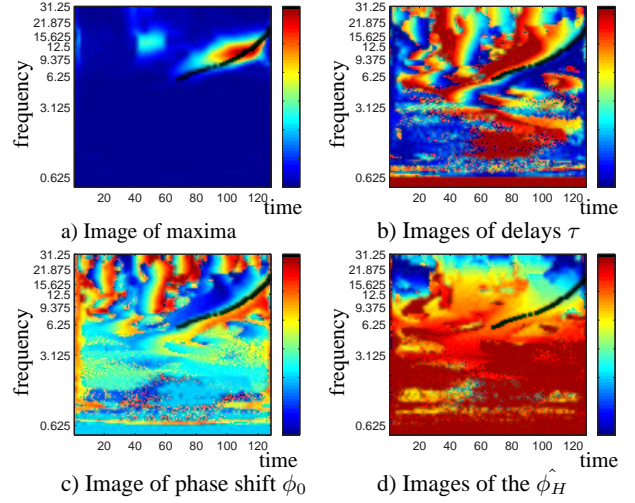
#### 3.1. Real data



**Fig. 4.** Initial Real data

Figure 4(a) and (b) present the real data respectively in the time domain and in the FK domain. The nonlinearity of the pattern on the FK reveals the dispersiveness.

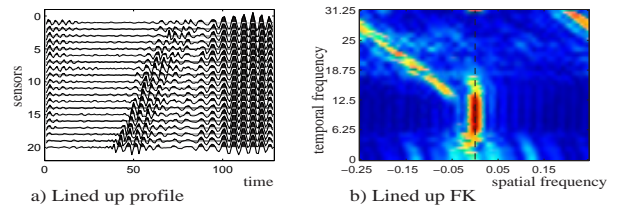
- figure 5(a) presents, in the time frequency domain the image of the maximum magnitude of each pixel in time-frequency plane corresponding to the energy maxima along the  $(\tau, \phi_0)$  axes. It reveals the presence of an energetic wave around coordinate (110,12).
- figure 5(b) and (c) present the delay  $\tau$  and phase corresponding arguments  $\phi_0$  for each location  $(t, f)$ .
- figure 5(d) presents the image of the estimated filter  $\hat{\phi}_H$  induced from image (b) and (c).



**Fig. 5.** Four inputs for the user

From figure 5(a), we select the ridge of the pattern corresponding to the dispersive wave.  $\tau$  and  $\hat{\phi}_H$  (related respectively to group and phase velocity) can be extracted as the value at this location of figure 5(b), and (d). The correction of the dispersion leads to figure 6(a) and (b) where the wave is perfectly lined up.

Once the dispersion is corrected, subspace methods (SVD) can be applied for wave-field separation [11], [1]. This leads to the extraction of the first mode of the dispersive wave. Figure 7(a) and (b) present the extracted profile re-projected in its original configuration, and (c) and (d) show the difference (*i.e.* 7(c)= 6(a) - 7(a)).



**Fig. 6.** Correction of the dispersion on real data

This algorithm has been validated on sets of field data. We show in this example that the representation of the data in the time-frequency plane enables a simple interpretation of the data. In presence of several waves, with different velocities or different time-frequency locations, our algorithm is able to present them separated, and therefore to characterize and extract each individual one.

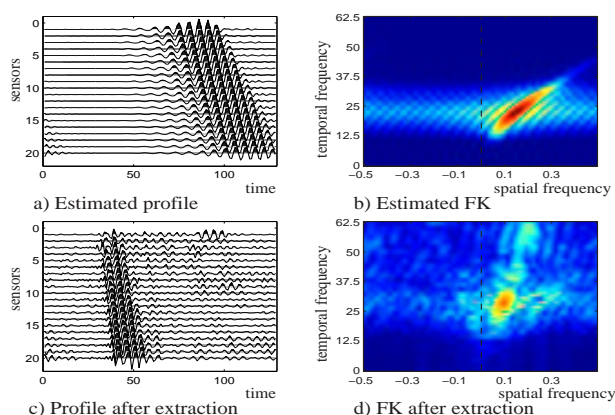


Fig. 7. Extraction of the dispersive wave

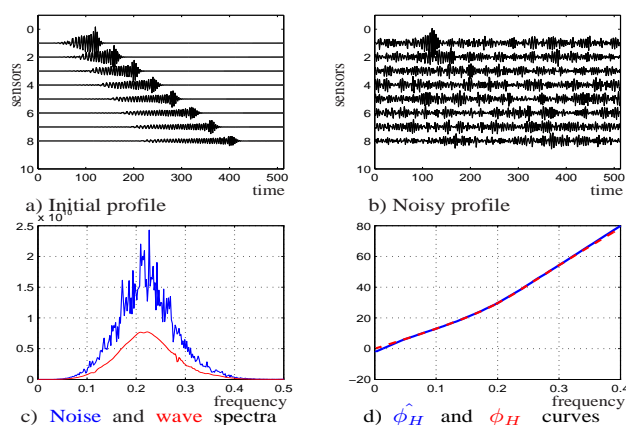


Fig. 8. Synthetic data result

### 3.2. Synthetic data

A synthetic data set is now considered. The initial profile is presented on figure 8(a). We want to show in this example the robustness of the dispersion estimation facing non coherent colored noise. Adding the colored noise leads to the profile presented on figure 8(b) where the wave is not even visible anymore. Figure 8(c) presents the spectral densities of the wave in red and of the noise in blue. Note that the noise has twice as high amplitude as the signal. Figure 8(d) presents the result of the estimation  $\hat{\phi}_H(f)$  superposed to the solution  $\phi_H(f)$ . In this case, 1D methods [6] cannot yield to a reasonable result because the amount of noise is too big.

## 4. CONCLUSION

We present a new algorithm for estimating the propagating filter of dispersive waves. This estimation has many applications in petroleum and engineering industry. We use the advantage of time-frequency representation to increase the

resolution in time and present the data in an image where the pattern of different waves are separated and easily recognized. In addition we keep the advantage of 2D analysis which is that it is noise robust and that it estimates the dispersion between sensors. The validation on real data and on highly noisy synthetic data show the good performances of this method. Our algorithm is stable, simple, and semi-automatic.

## 5. REFERENCES

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