Automatic Image Registration by Stochastic Optimization of Mutual Information

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ABSTRACT

Image registration is the process by which we determine a transformation that provides the most accurate match between two images. The search for the matching transformation can be automated with a suitable choice of metric, but can be very time-consuming and tedious. In this paper, we consider a registration algorithm that combines a simple yet powerful search strategy based on optimization of mutual information using a stochastic gradient, together with a wavelet-based multi-resolution pyramid. It is tested using a pair of fundus eye images, which is matched using a six-parameter affine transformation. This extends previous work based on the three-parameter transformation [7]. The registration algorithm is implemented in a multi-resolution manner using a wavelet pyramid.

1. INTRODUCTION

Consider a pair of images $F_I(x,y)$ and $F_R(x,y)$ with coordinates (x,y). These are *registered* by finding a transformation $T_\rho(.)$ of a certain class such that $F_R(T_\rho(x,y))$ best matches $F_I(x,y)$, where ρ is a vector of transform parameters. In this paper, we consider $T_\rho(.)$ to be the class of six-parameter transforms specified by a 2D affine transformation in the plane. These can be represented by the 3-by-3 matrix shown below:

$$T_P(x,y) = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and we can write ρ as the vector: $\rho=[a\ b\ c\ d\ e\ f]$. Note that these six parameters specify translation, scaling and skewing (or rotation) in the plane.

Prior work on optimization techniques for image registration can be found in references [2-4]. Maes et. al. [2] use the Marquardt-Levenberg search technique to optimize mutual information (MI). The required derivatives are explicitly calculated based on a partial volume interpolation of the criterion, and the search is implemented in a multi-resolution framework. Thevenaz et. al. [4] develop a scheme to maximize mutual information in a multi-resolution manner, which is applied to medical imagery. Parzen windows are used to compute the histogram, it smooths the MI surface and thus achieves the differentiability required to compute the gradient and the Hessian matrix. Their optimizer is designed specifically for the MI criterion. In [8] Can and Stewart derive a 12-parameter transformation model for the curved retina, and develop an automatic algorithm for registration, which is based on feature extraction of some vascular landmarks.

Unlike the methods discussed above, the optimization method described in this work does not require an explicit derivation of the required gradient vector. Section 2 describes the registration algorithm and optimization search technique, associated results are presented in section 3.

2. THE REGISTRATION ALGORITHM

2.1. A Multi-Resolution Search Technique

Mutual information (MI) has been extensively studied for the registration of medical imagery [2-4], it measures redundancy between two images by looking at their intensity distributions and represents a measure of the relative entropy between two sets. Thus in the context of image registration mutual information has been used as a similarity measure, which indicates through its maximum the best match between a reference image and an input image. The simplest search strategy for determining the transformation that optimizes mutual information is the exhaustive search, but this is computationally expensive with the computational cost increasing exponentially with the number of transformation

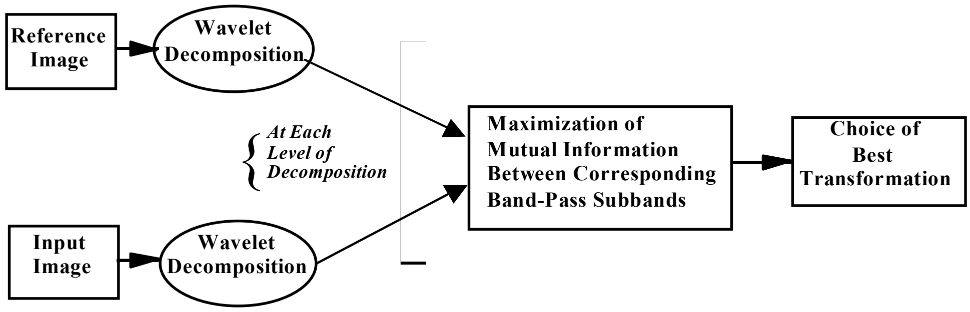

Figure 1
Summary of the Wavelet and Mutual Information Image Registration Method

parameters and the size of the dataset. In this work, mutual information is optimized using a stochastic gradient search technique, at each decomposition level of a multi-resolution wavelet-like pyramid. We use a multi-resolution framework based on the Simoncelli steerable pyramid [5].

2.2. Mutual Information

The concept of mutual information (MI) represents a measure of relative entropy between two sets, which can also be described as a measure of information redundancy. If A and B are two images to register, with $P_A(a)$ and $P_B(b)$ defined as the marginal probability distributions, and $P_{AB}(a,b)$ defined as the joint probability distribution of A and B, then mutual information is defined as :

$$I(A,B)=\sum_{a,b} p_{AB}(a,b).\log\frac{p_{AB}(a,b)}{p_A(a).p_B(b)}$$

These probabilities are computed using the histograms of the two images A and B, $h_A(a)$ and $h_B(b)$, and their joint histogram $h_{AB}(a,b)$. The MI surface is smooth when the transformed image is obtained using spline interpolation [4].

2.3. SPSA Optimization Technique

The optimization technique, which is implemented in this work is the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm. It was first introduced by Spall [6], where a detailed description can be found. It has recently attracted attention for solving challenging optimization problems for which it is difficult or impossible to obtain an analytic expression for the gradient of the objective function. SPSA is based on an easily implemented and highly efficient gradient approximation that relies only on measurements of the objective function to be optimized. It does not rely on explicit knowledge of the gradient of the objective function, or on measurements of this gradient.

Let L represent mutual information, the objective function to be optimized. We consider a parameter search space of affine transformations, consisting of six parameters. Starting with an initial guess ρ_0, at each iteration the gradient approximation is calculated based on only two function measurements. At iteration k, the update law for the parameters is steepest ascent:

$$\rho_{k+1} = \rho_k + a_k\, g_k$$

where the gradient vector $g_k = [(g_k)^1\ (g_k)^2\ \ldots\ (g_k)^m]$ for the m-dimensional parameter space is determined by

$$(g_k)^i = \{L(\rho_k + c_k\,\Delta_k) - L(\rho_k - c_k\,\Delta_k)\}/\{2\,c_k\,(\Delta_k)^i\},\quad \text{for } i=1,2\ldots m$$

In this study, six parameters are to be updated at each iteration, so m = 6. Each element $(\Delta_k)^i$ of the vector, Δ_k takes on a value of +1 or –1, as generated by a Bernouilli distribution, and a_k and c_k are positive sequences of the form:

$$a_k = a/(k+A)^\alpha \quad \text{and} \quad c_k = c/(k+1)^\gamma,\quad \text{with } 0<\gamma<\alpha<1.$$

Note that a_k and c_k decrease to zero. The constants a, c, A, α and γ are optimized and fixed within the range of values suggested by Spall [6], which would ensure convergence.

2.4. The Registration Algorithm

In this section, multi-resolution registration combining wavelet features, mutual information and the SPSA optimization scheme is tested on a pair of biomedical fundus images of a curved retina.

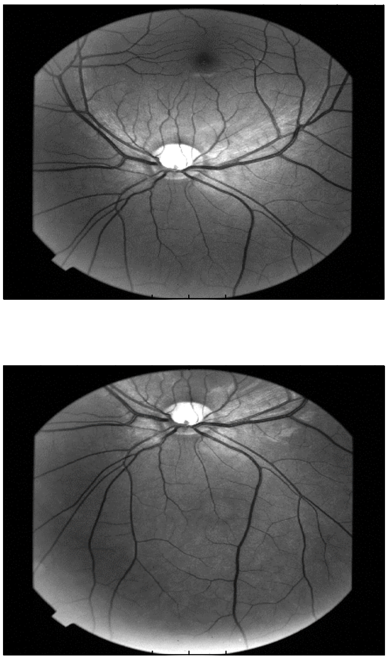

Figure 2
A pair of fundus images to be registered

A pair of fundus images shown in figure 2 is used to test the algorithm, and the results are illustrated visually by the mosaic of figure 3. The registration process is outlined as follows:

Step 0: Three levels of decomposition are computed using Simoncelli filters. These correspond to decimations of 8, 4 and 2 of the original image, respectively. Level 3 represents the coarsest image with a decimation of 8, and so on. At the top level of the pyramid we use the original gray level image in the process.

The actual optimization is done in two-steps:

Step 1: Starting with an initial guess of the correct x and y translations. Only the three parameters, which correspond to rotation and shift in the x and y directions, are optimized for the coarsest image at level 3.

Step 2: The results obtained in step 1 are converted into entries for the six-parameter matrix with the translation values doubled, and this is used as a starting point for the next level. The full six-parameter optimization is done on the higher resolution images of remaining levels.

For each of the two steps (1 and 2), the constants a, c, A, α and γ for the SPSA algorithm must be chosen and optimized. These values are then fixed and used to produce the results provided in the next section.

3. EXPERIMENTS AND RESULTS

Table 1 shows the optimization parameters for this pair of images after 100 iterations. Intermediate results are provided at the three levels of the Simoncelli decomposition and for the original image. At level 3 the parameters shown are translation and rotation, i.e. tx, ty and θ. The ending value provides the starting point for the subsequent six parameter optimization to be done for levels 2 and up. Note that parameters in the last column for the other levels, correspond to the x-translation and y-translation. We note that the 3-parameter optimization is more efficient than the 6-parameter update, this is affected by the choice of step-size parameters in the SPSA algorithm and this will be explored further. We also note that when the optimization works at the coarsest levels, it provides near optimal starting points for the higher levels of the wavelet pyramid.

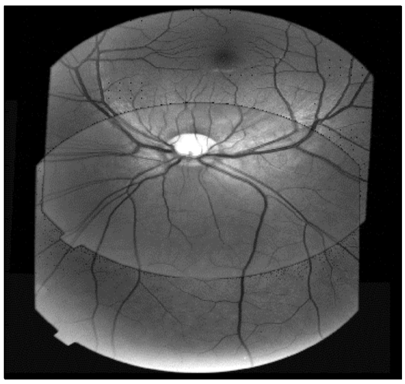

Figure 3
Mosaic from SPSA 6-parameter registration

Decomp Level	Starting Pt	Starting MI value	Ending Pt	Final MI value
3	-2/44/0	0.29685	-2.118/44.221/1.8078	0.41143
2	0.9995 -0.0315 -4.236 0.0315 0.9995 88.442	0.27993	0.9995 -0.0340 -4.2362 0.03341 0.9993 88.4415	0.28637
1	-0.9995 -0.0340 -8.4724 0.03341 0.9993 176.883	0.1692	0.9980 -0.0343 -8.4726 0.03448 0.9996 176.883	0.1717
original	0.9980 -0.0343 -16.9452 0.03448 0.9996 353.766	0.5873	0.9979 -0.0346 -16.9454 0.03532 0.9984 353.766	0.5886

Table 1: *Multi-Resolution Registration Parameters*

We evaluate these results visually, by obtaining the mosaic shown in figure 3 of the SPSA registration values shown in table 1. It is clear from figure 3, that the quadriatic parameters involving curvature are required in order to produce the best fit. This problem is currently under development to be included as the third step of this optimization process.

4. CONCLUSION

In this paper we have investigated a new optimization technique, and successfully applied it to produce affine registration parameters for a pair of biomedical images, in a multi-resolution framework. Current work involves testing this algorithm on a larger number of datasets, in order to compare its performance to the registration scheme of [8].

5. ACKNOWLEDGMENTS

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6. REFERENCES

1. L. Brown, “A Survey of Image Registration Techniques”, ACM Computing Surveys, vol.24, no.4, 1992.
2. F. Maes, A. Collignon, D. Vandermeulen, G. Marchal, P. Suetens, “Multimodality Image Registration by Maximization of Mutual Information,” *IEEE Trans. on Medical Imaging*, Vol.16, No.2, April 1997.
3. W.M. Wells, III, P. Viola, H. Atsumi, S. Nakajima, and R. Kikinis, “Multi-Modal Volume Registration by Maximization of Mutual Information,” *Medical Image Analysis*, Vol. 1, pp.35-51, 1996.
4. P. Thevenaz, and M. Unser, “Optimization of Mutual Information for Multiresolution Image Registration,” *IEEE Transactions on Image Processing*, Vol. 9, No. 12, December 2000.
5. A. Karasaridis, and E. Simoncelli, “A Filter Design Technique for Steerable Pyramid Image Transforms,” *Proceedings ICASSP-96*, May 7-10, Atlanta, GA, 1996.
6. J.C. Spall, “Multivariate Stochastic Approximation using a Simultaneous Perturbation Gradient Approximation,” *IEEE Transactions on Automatic Control,* Vol. 37, 332-341 (1992).
7. A. Cole-Rhodes, K. Johnson and J. Le Moigne, “Multi-Resolution Registration of Remote Sensing Images using Stochastic Gradient,” *SPIE Aerosense 2002, Wavelet Applications IX,* Orlando, FL, April 2002.
8. A. Can, C.V. Stewart, B. Roysam and H. Tanenbaum,”A Feature-Based, Robust Hierarchical Algorithm for Registering Pairs of Images of the Curved Human Eye”, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 24, No. 3, March 2002.