EMPIRICAL CHOICE OF SMOOTHING PARAMETERS IN OPTICAL FLOW WITH CORRELATED ERRORS

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ABSTRACT

Optical flow estimation algorithms such as the Lukas-Kanade method and Horn and Schunk method require selection of a tuning parameter. In the former case a neighbourhood size, in the latter, a penalty parameter. Selection of these tuning parameters is difficult in general but has a profound effect on the results. So automatic methods of selection are of great interest. In previous work we have developed such methods based on white noise assumptions and here we show how to adjust for the effect of spatially correlated errors. These always occur in practice and can degrade the performance of white noise based procedures.

1. INTRODUCTION

There is by now, a considerable literature on optical flow estimation e.g. [1],[2],[3],[4]. The question is how to estimate motion (i.e. velocities) from a sequence of image intensities $I = I(t, x_1, x_2)$. A basic approach is based on the brightness constraint equation (BCE) which assumes brightness does not change with time so

$$\begin{aligned} \frac{dI}{dt} &= 0 = \frac{\partial I}{\partial t} + u\frac{\partial I}{\partial x_1} + v\frac{\partial I}{\partial x_2} \\ &= I_t + uI_{x_1} + vI_{x_2} \end{aligned}$$

where I_t, I_{x_1}, I_{x_2} are image intensity gradients; u, v are the x_1, x_2 components of optical flow. The image gradients can be estimated (as $\hat{I}_t, \hat{I}_{x_1}, \hat{I}_{x_2}$- from image intensities by temporal and spatial differencing) so the BCE provides one constraint on the two velocities u, v. To estimate the two velocities then, further information is needed and this is provided by assuming spatial continuity of u, v.

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Such constraint information can be manifested either as in [5](HS) by Tikhonov regularization or as in [6](LK) by local estimation. We deal only with the LK method here. Our approach can handle the HS approach but will be pursued elsewhere. More recent approaches to optical flow estimation have emphasized the necessity to take account of outliers due e.g. to occlusion; thus the above methods are modified [4],[2]. Since we are emphasizing automatic choice of neighbourhood size here we study initially the simpler algorithm of [6]. Our approach can be extended to handle the more recent more complex algorithms but again this will be pursued elsewhere.

Optical flow estimation is of course an ill-conditioned inverse problem [7] and the problem of estimating tuning parameters in ill-conditioned inverse problems has a large literature e.g.[8]. There are two approaches: deterministic, based on minimising a surrogate for a mean squared error quality measure e.g.[9]; stochastic based on a(n) (empirical) Bayesian approach e.g[10],[11].

As discussed in [12] the Bayesian approach is usually computationally very demanding and approximate solutions have to be sought. We pursue the deterministic approach here; its computational demands are usually modest by comparison. The Bayesian approach has been mentioned in the computer vision literature [13] but not applied to optical flow.

While there has been some work on tuning parameter selection in image processing e.g.[14] there has been only a little on optical flow [15],[16],[17]. In these works the error signal $\hat{I}_t + u\hat{I}_{x_1} + v\hat{I}_{x_2}$ is treated as white noise; but it is clearly correlated due to estimation of the image gradients by filtering.

In the much simpler problem of signal estimation it is well understood in the statistics literature that the correlation can severely affect the choice of tuning parameter and hence the quality of signal estimation

[18]. That is also true here and our aim is then to show how to adjust for correlation in the error signal.

In section 2 we formulate the BCE with noise. In section 3 we develop our new selection criterion. Section 4 contains results and conclusions in section 5.

2. NOISY BCE

The BCE is an ideal. Noise in the image capture process; noise in estimating the temporal and spatial gradients; violation of the assumptions behind the BCE (e.g. due to variations in ambient illumination) mean that the BCE does not hold exactly. This phenomenon can be modeled by introducing a noise term

$$-I_t = uI_{x_1} + vI_{x_2} + \epsilon$$

This view was discussed e.g. by [19] but he did not deal with spatial correlation in ϵ which is what we pursue here. Again [16] covered only the white noise case. Our approach follows the general method of [12]. That discussion covered only white noise but the development is easily modified to handle coloured noise c.f.[20].

We rewrite the model as

$$\begin{aligned} y &= \mu + \epsilon \\ y &= -I_t \\ \mu &= uI_{x_1} + vI_{x_2} = g^T f \\ f, g &= (u,v)^T, (I_{x_1}, I_{x_2})^T \end{aligned}$$

Here y represents a lexicograhically ordered image of $M \times M$ pixels as do $u, v, I_{x_1}, I_{x_2}, \mu$ (so e.g. y is an $M^2 \times 1$ vector). For the moment we allow an arbitary noise covariance $cov(\epsilon_P, \epsilon_Q) = \Gamma_{P,Q}$ (where e.g. $P = (x_1, x_2)$ is a point on the plane). We can thus write $var(\epsilon) = \Gamma$ where Γ is $M^2 \times M^2$.

3. AUTOMATIC NEIGHBOURHOOD SELECTION

We need to measure the quality of the optical flow estimator. In [21],[22] is proposed a criterion which measures angle between velocity vectors. Unfortunately this measure does not account for magnitude differences and partly for that reason is not a metric. If estimating speed matters as it often does then this measure is inadequate.

Mean squared error (mse) for flow velocities is a simple measure that is a metric and measures both speed and direction. However the development of a tuning paramter selector based on such a measure is not straight forward and will be pursued elsewhere. Here we use mse based on the BCE. So our quality measure or statistical 'risk' is

$$\begin{aligned} R &= E \parallel \hat{\mu} - \mu \parallel^2 \\ &= E \parallel g^T(\hat{f} - f) \parallel^2 \end{aligned}$$

where $\parallel z \parallel^2 = z^T z$ and $\hat{f}$ is the optical flow estimator from the algorithm of interest; in our case an overlaped version of [6]. Ideally we would choose the neighbourhood size to minimize R. Now R cannot be calculated since f is unknown so the idea is to find an empirically computable surrogate for R and minimize that instead. It can be shown (modifying [12]) that an unbiassed estimator of R (known as Stein's unbiassed risk estimator SURE) is (with $e = y - \hat{\mu}$ = residual)

$$\hat{R} = \parallel e \parallel^2 - 2trace(\Gamma \frac{\partial e^T}{\partial y}) + trace(\Gamma)$$

Dropping terms not dependent on the neighbourhood size leaves

$$\hat{R} = -2\hat{\mu}^T y + \parallel \hat{\mu} \parallel^2 + 2trace(\Gamma \frac{\partial \hat{\mu}^T}{\partial y})$$

Now substitute the (overlapped) LK estimator

$$\begin{aligned} \hat{f}_P &= M_P^{-1} \Sigma y_Q g_Q w_{P-Q} \\ M_P &= \Sigma g_Q g_Q^T w_{P-Q} \end{aligned}$$

Here w_P is a hump shaped kernel (e.g. a Gaussian or a triangle) whose region of support defines the neighbourhood of pixels used to calculate $\hat{f}_P$. As long as the kernel is continuous its use eliminates Gibbs ringing and 'zigzagging' in $\hat{R}$. Continuing, we obtain after some algebra, and now assuming the noise is spatially stationary

$$\begin{aligned} \hat{R} &= \Sigma(-2\hat{\mu}_P y_P + \hat{\mu}_P^2 + 2\gamma_0 g_P^T M_P^{-1} h_P) \\ h_P &= \Sigma \Gamma_{P-Q} w_{P-Q} g_Q \end{aligned}$$

where Γ_P is the spatial autocorrelation function (ACF) and γ_0 the variance. These are estimated from the residuals $e_P = y_P - \hat{\mu}_P$. We call our criterion SURE$_{CN}$. For a white noise process $\Gamma_P = 0, P \neq 0$ then $h_P = g_P w_0$ and the criterion collapses to that of [16] (which we call SURE$_{WN}$).

4. RESULTS

We illustrate our new results with a well known example; the rotating Rubik cube sequence used in [22]. The ACF (and variance) is estimated from residuals from a preliminary optical flow estimation (which could use the neighbourhood size that minimizes the SURE$_{WN}$

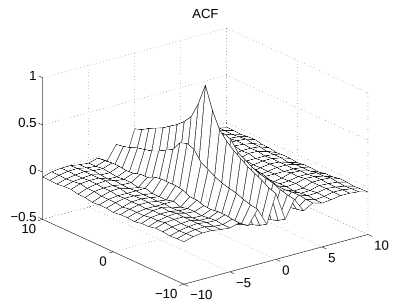

Figure 1: ACF of Rubik Cube Sequence.

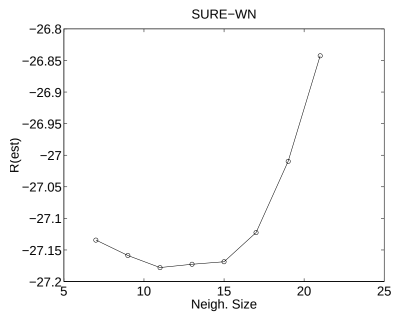

Figure 2: SURE$_{WN}$ for Rubik Cube Sequence

criterion). We have found that, over a number of different image sequences ,the neighbourhood size used to estimate the ACF is not critical (i.e. does not affect the shape of the resulting SURE$_{CN}$) unless it is very small (e.g. 3×3) or very large (e.g. 21 × 21). Next the SURE$_{CN}$ criterion is calculated according to the expression at the end of section 3. Results are shown in Figs.1-5. The ACF is shown is Fig.1. The unusual structure is clearly bias due to the edges in the image sequence. But the ACF does not need to be well estimated for the criterion to function successfully. In Figs.2,3 we see the minimizing neighbourhood with SURE$_{CN}$ is smaller than for SURE$_{WN}$. This phenomenon seems to be typical over several image sequences. It is a positive feature of the new criterion since it ensures the optical flow estimator is more sensitive to local features. Also the SURE$_{WN}$ criterion is relatively flat near its minimum and so points to a range of reasonable neighbourhood sizes, say 9-15. The SURE$_{CN}$ criterion has a more pronounced minimum (at 7). Again this phenomenon seems to be typical.

In Figs.4,5 the optical flow corresponding to the minimizing neighbourhood sizes are shown. The optical flow based on the SURE$_{WN}$ neighbourhood has

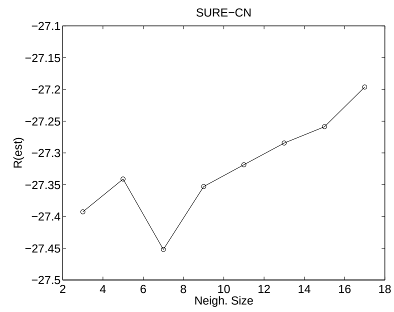

Figure 3: SURE$_{CN}$ for Rubik Cube Sequence

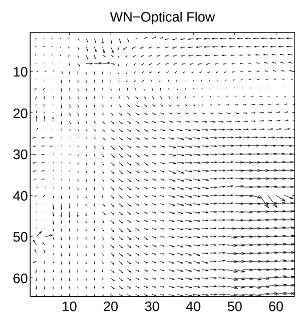

Figure 4: Optical Flow for Rubik Cube Sequence - White Noise based Neighbourhood Size

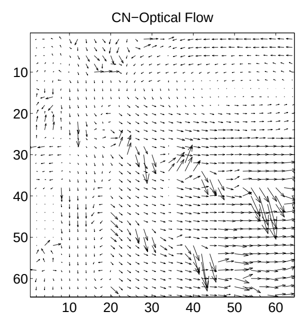

Figure 5: Optical Flow for Rubik Cube Sequence - Coloured Noise based Neighbourhood Size

smoothed out the velocity a little too much; the optical flow based on SURE_{CN} shows more structure but is perhaps just a little too noisy. Indeed we found the result with neighbourhood size of 9 (not shown), visually more pleasing. However as we have pointed out elsewhere tuning parameter values near the minimizing one should in general also be looked at.

5. CONCLUSION

We have shown how to adjust, for correlated noise, a previously developed criterion for automatic selection of neighbourhood size for the LK method of optical flow estimation. The new criterion tends to yield smaller neighbourhood sizes, an encouraging result which allows the optical flow estimator to be more sensitive to local variation. In function estimation [18],[20] WN criteria sometimes fail altogether in the presence of correlation. We have so far not seen that phenomenon here. In future work we will modify our method to apply to robust optical flow estimators.

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