IMAGE INTERPOLATION USING WAVELET-BASED CONTOUR ESTIMATION

Hasan F. Ates

Dept. of Electrical Engineering
Princeton University
Princeton, NJ 08544 USA
hasan@princeton.edu

Michael T. Orchard

Dept. of Electrical and Computer Engineering
Rice University
Houston, TX 77005 USA
orchard@ece.rice.edu

ABSTRACT

Successful image interpolation requires proper enhancement of high frequency content of image pixels around edges. In this paper, we introduce a simple edge model to estimate high resolution edge profiles from lower resolution values. Pixels around edges are viewed as samples taken from one dimensional (1-D) continuous edge profiles according to 1-D smooth edge contours defining the sampling instants. The image is highpass filtered by wavelets and subpixel edge locations are estimated by minimizing the modeling error in the wavelet domain. Interpolation is carried out by applying the model, wherever applicable, together with a baseline interpolator (here, bilinear) in order to make edges look sharper without introducing artifacts. The results are compared to bilinear interpolation, and significant improvement in terms of SNR, edge sharpness and contour smoothness is observed.

1. INTRODUCTION

Resolution enhancement of images becomes complicated due to the localized high-frequency nature of pixel values across edges. Standard methods such as bilinear or bicubic interpolation[1] fail to capture these sudden changes around edges and cause visually unacceptable artifacts such as blurring or ringing. Various techniques have been proposed to improve the visual quality of interpolated images. Unsharp masking[1] and nonlinear interpolation filters[2] help to sharpen edges, only to produce artifacts and increase noise in return. Adaptive interpolation schemes[3, 4, 5] use simple edge models to estimate edge locations and interpolate selectively among pixels around edges. These are typically *ad hoc* methods and the outcome is significantly affected by the mistakes made in edge locations. Wavelet[6] or fractal[7] based techniques enhance the high-frequency content of the interpolated image by predicting the high-frequency details in the wavelet or fractal domain. Since no explicit edge model is involved, these techniques usually end up increasing any noise present in the unmagnified image. Edge directed interpolation of Xin Li, *et al.*[8, 9], avoids using an edge-map by estimating local covariance characteristics at low resolution and using them to direct the interpolation at high resolution. Even though it achieves sharpness across the edge and smoothness along the edge contour, it is not clear whether the technique provides the level of adaptivity required for pixels around edges.

In order to enhance the sharpness of an edge without incurring any additional artifacts, an accurate model for the high-frequency content of the edge profile (i.e., pixel values across the edge) is needed. When a high resolution image is downsampled, the high frequency components get aliased to the low frequency terms. Aliasing can be avoided by pre-filtering before downsampling; yet, this might blur the appearance of the edge and high-frequency components are lost in any case. In this paper, we develop a simple yet powerful edge model to estimate the aliased high-frequency terms from downsampled pixel values around an edge contour. The model is based on the observation that the edge profile changes smoothly and slowly along an edge contour. Therefore, pixel values around the contour can be modelled as samples from the same 1-D edge profile, with sampling instants determined according to the location of the pixel with respect to the edge contour. The model is applied in the wavelet domain after the image is high-pass filtered using wavelets. For a given edge, 1-D wavelet coefficient profile and edge contour are chosen by minimizing the approximation error of the model. For the interpolation step, the model is used together with a baseline interpolator. Since the model is meant to capture the high-frequency deviations, the baseline interpolator is needed for stable interpolation. By properly combining the two interpolation schemes, we achieve sharper and more accurate edge profiles and smooth edge contours without introducing much noise or artifacts.

There are several interesting features of this “contour + profile” model. First of all, it provides accurate estimation of edge locations as long as the model is applicable. For such “well behaving edges”, the high-frequency enhancement is properly defined in terms of the original pixel values, in contrast to the *ad hoc* enhancement schemes used in former edge-based interpolation ideas. Most of the previous schemes tend to obscure the smoothness along the edge contour. On the other hand, “contour + profile” model improves

This research was supported in part by the NSF Grant DMS9872890

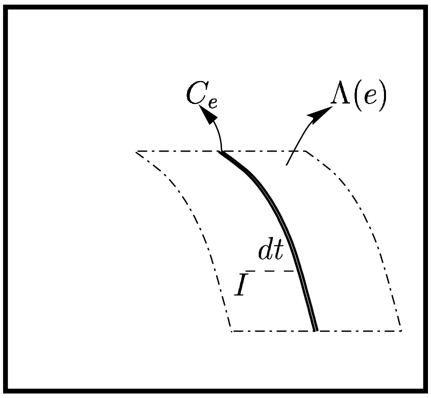

Fig. 1. Edge Contour and Its Neighborhood

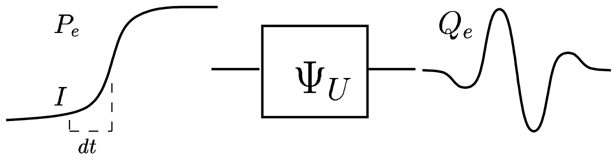

Fig. 2. Edge and Wavelet Coefficient Profiles

such smoothness by using a 1-D smooth contour description and low-pass filtering along the contour direction. Section 2 introduces the model and relevant notation and discusses when the model is applicable. Section 3 is about the contour estimation algorithm in the wavelet domain. In Section 4, we provide the details of the interpolation scheme, and discuss the pros and cons of various different approaches. Experiment results and discussions about the performance are included in Section 5. We conclude the paper by highlighting the future directions to be taken.

2. CONTOUR + PROFILE MODEL FOR EDGES

Typically, pixel values within a local neighborhood of an edge tend to change smoothly and slowly along the edge orientation. This observation forms the basis of our "contour + profile" model in images. A 1-D continous edge profile is sampled according to the subpixel edge locations provided by the 1-D smooth contour, and pixel values are modelled as these sampled values plus small additive noise (see Figure 1,2). In its most general form, the edge profile is defined along the direction orthogonal to the edge orientation at a certain point. We further simplify the model by using horizontal profiles for edges close to being vertical, i.e. "vertically oriented", and vertical profiles for "horizontally oriented" edges. The corresponding 1-D contour is defined accordingly as a function of vertical and horizontal axis respectively. For diagonal edges both profiles need to be used for accurate modelling. In the rest of the paper, we assume the edge to be vertically oriented. The same ideas apply trivially for the horizontal case.

Specifically, pixel values are given by the following formulation (see Figure 1,2); $\forall(m,n) \in \Lambda(e)$,

$$I(m,n) = P_e(m - C_e(n)) + \gamma(m,n), \qquad (1)$$

where P_e is the edge profile, C_e is the contour, $\Lambda(e)$ is the set of neighboring pixels for the edge e. γ is assumed to be a small independent additive white Gaussian noise (AWGN) process.

Since higher frequencies are more sensitive to changes in location, contour estimation is governed mainly by the high frequency components of the edge profile. In addition, edge spectrums decrease as $1/f$ and therefore aliasing is much more pronounced for high frequencies compared to low frequencies. Wavelet filters at different scales divide the edge spectrum into portions of dyadic proportion. This dyadic scaling matches the 1/f-like behavior of the edge spectrum. Hence, each portion of the spectrum can be weighted properly based on how much it contributes to location uncertainty and how much it is affected by aliasing. However, experience shows that performance loss is negligible when only the highest frequency band is used and other bands are discarded. In view of these facts, contour estimation is carried out after the image is high-pass filtered using wavelets in the horizontal direction. The edge model is still applicable in the wavelet domain: $\forall(m,n) \in \Lambda(e)$,

$$w(m,n) = Q_e(m - C_e(n)) + \gamma(m,n). \qquad (2)$$

Edge profile P_e is filtered by ψ_u to produce wavelet coefficient profile Q_e (see Figure 2), where ψ_u is the upsampled wavelet filter:

$$\psi_u(t) = \begin{cases} \psi(k), & \text{if} \quad t = k, \quad k \in \mathbb{Z} \\ 0, & \text{else} \end{cases} \qquad (3)$$

Note that wavelet coefficients are determined by the pixel values that lie within the support of the wavelet filter. Consequently, when there are multiple high frequency patterns within a local neighborhood (i.e., texture and/or multiple edges at different directions), the assumptions of the model fail. We will later comment on this issue.

3. CONTOUR ESTIMATION

Wavelet coefficient profile Q_e and edge contour C_e are assumed to be smooth 1-D functions. Q_e is a compact signal with support determined by the length of the wavelet filter and shape of the edge profile (see Figure 2). Strictly speaking, it cannot be bandlimited. Yet, the effective bandwidth is typically small and since we would like to magnify the image by two, we assume Q_e to have bandwidth equal to 2π. In other words, the Nyquist sampling period is 0.5 units. For a given edge, profile and contour are chosen as two smooth functions that minimize the approximation error of the model:

$$D_{\Lambda(e)} = \sum_{(m,n)\in\Lambda(e)} (w(m,n) - Q_e(m - C_e(n)))^2,$$

$$[\tilde{Q}_e, \tilde{C}_e] = \arg\min_{\substack{[Q_e, C_e] \\ Q_e \in \mathcal{B}_{[2\pi]}}} \left(D_{\Lambda(e)} + \delta \| \mathbf{H} C_e \|_{L_2}^2 \right), \qquad (4)$$

where $\mathcal{B}_{[2\pi]}$ is the linear subspace for bandlimited functions with bandwidth 2π, and $\mathbf{H}$ is some highpass filter. Contour smoothness is controlled by penalizing the high-frequency deviations of the contour using an appropriate weight δ.

For a given contour, the solution for the optimal profile simply becomes the orthogonal projection of the wavelet coefficients onto a linear subspace. Contour estimate is updated using an iterative gradient-based approach so as to minimize the above error expression.

4. MODEL-BASED IMAGE INTERPOLATION

Estimated contours provide subpixel accurate edge orientations along which smooth interpolation is possible. Standard interpolation techniques could be modified for selective edge-directed interpolation using similar ideas as in [3]. Even though improved image quality is observed, these methods suffer from mistakes in edge locations. Furthermore, since such methods don't include explicit models for the edge profile, high frequency enhancement around the edge does not necessarily correspond to the aliased components of the true high resolution image. This leads to artifacts around the edge and failure to maintain a visually smooth edge contour.

We propose to use the model introduced in Section 2 for estimating the proper enhancement to sharpen edges without sacrificing contour smoothness. Wherever the model is applicable, missing pixels of the high resolution image could also be treated as samples from the 1-D continuous edge profile. Unfortunately, unlike wavelet coefficients, the edge profile is not a compact signal. This leads to ringing when bandlimited interpolation is used to define the continuous edge profile. The common solution to this problem is to use a localized kernel, such as linear interpolator. As mentioned before, aliasing at low frequencies is negligible and wavelet coefficient profile is capable of resolving aliasing at high frequencies. Therefore, we combine the two approaches such that bilinear interpolation is used as the baseline interpolator, and estimated wavelet coefficient profile provides the necessary adjustment for high frequencies.

The two schemes define a set of linear equations, and pixel values of the interpolated image are given by the least square solution of these equations. Specifically, define $\mathbf{F}_L$ as the bilinear interpolation filter; $\mathbf{\Psi}_U$ is the discrete upsampled (by 2) wavelet filter; $\mathbf{S}_{[dt]}$ is (FFT-based) shifting by dt units; $I_{(2)}$ is the interpolated image and I_U is the upsampled low resolution image. For a given edge e, define $\mathbf{m}_n = \{m;\ (\lfloor \frac{m}{2} \rfloor, \lfloor \frac{n}{2} \rfloor) \in \Lambda(e)\}$. Then, we would like the interpolated edge to satisfy, $\forall\, i, j$ such that $\mathbf{m}_i, \mathbf{m}_j \neq \emptyset$,

$$\mathbf{S}_{[C_e^2(i)]} \mathbf{\Psi}_U I_{(2)}(\mathbf{m}_i, i) = \mathbf{S}_{[C_e^2(j)]} \mathbf{\Psi}_U I_{(2)}(\mathbf{m}_j, j), \qquad (5)$$

The contour is linearly interpolated; i.e. $C_e^2(2n+1) = (C_e(n) + C_e(n+1))/2$. The bilinear interpolation gives,

$$I_{(2)} = \mathbf{F}_L I_U. \qquad (6)$$

The pixel values due to the low resolution image are kept the same using the constraint,

$$I_{(2)}(2m, 2n) = I_U(2m, 2n), \quad \forall (m, n).$$

The model equations (5) can be weighted by some constant $\lambda(e)$ depending on how reliable the model is. A measure of reliability is the percentage error of approximation for the wavelet coefficients of low resolution image:

$$\rho(e) = \frac{D_{\Lambda(e)}}{\sum_{(m,n) \in \Lambda(e)} w(m, n)^2}.$$

Typically, if $\rho(e) > 0.2$, then the model is assumed to fail and $\lambda(e)$ is set to zero. When $\rho(e) \leq 0.2$, $\lambda(e) = 0.5$ seems to yield a good tradeoff between enhancing edge profile and avoiding artifacts.

It needs to be mentioned that using bilinear interpolation in this fashion is not necessarily the best approach. The image could first be low-pass filtered by the scaling filter, and equation (6) could be modified accordingly in order to separate low frequency interpolation from high frequency interpolation. Bilinear interpolation could be replaced by a weighted linear interpolation scheme, where the weights would reflect the edge orientation. However, the performance gains are usually small; and since the goal of this paper is to model the high frequency behavior around edges, we decided to work with the simplest strategy.

For natural images, there exist many different types of localized high frequency patterns, which could come in the form of multiple intersecting or neighboring edges, texture and other complicated shapes and figures. In most of these cases, our model is not going to be applicable, and the performance will be limited by the choice of the baseline interpolation scheme. Our future research is geared towards generalizing this basic model to include these different types of patterns.

5. SIMULATIONS AND RESULTS

The scheme is tested around the hat of Lena image (see Figure 3). Since the model performs best around isolated and smooth edges, this image is suitable to see the application of the idea. We used length 9 wavelet filter from linear phase 9/7 filter pair. Contour length is chosen to be between 4 and 8. This defines a window of size $(9 \times (4-8))$ (for vertical edges) around the edge locations. The idea is applied separately in vertical and horizontal directions. If a pixel happens to belong to some $\Lambda(e)$ in both directions (which happens for diagonally oriented edges), then the final result is a weighted sum of the two outcomes, with weights given by the wavelet coefficient energy in a (3×3) window around the pixel.

The image is magnified by a factor of 4 in both dimensions. Figure 3 compares the scheme against using bilinear interpolation only. The preserved smoothness along major (strong) edges in the image shows the stength of our algorithm. The ragged contours of the bilinear interpolated image degrades the perceptive image quality. Using a contour-based enhancement scheme, we manage to introduce the necessary high frequency details to make up for the lost resolution. The SNR gain relative to the original high resolution image is more than 2 dB. Improvement in SNR goes up as high as 6 dB (in other parts of Lena), depending on the sharpness of the edge profile and the smoothness of pixel values along the edge contour. The method gives a similar performance when cubic B-spline interpolation is used instead. This is expected, since the success of the method depends on the accuracy of the model for high frequency details and not on the baseline interpolation scheme. Compared to using spline interpolation only, the SNR gain for the same region is about 1.4 dB.

As mentioned before, using bilinear interpolation together with bandlimited interpolation of wavelet profile is probably not the best choice for images. The slight increase in noise around the right edge of the hat can be attributed to the undesirable properties of these two methods. We expect to achieve a better image quality when a more localized description is used for the wavelet coefficient profile.

(a) Bilinear Interpolation

(b) Model-based Interpolation

Fig. 3. Comparison for $4\times$ scaling

6. CONCLUSION AND FUTURE WORK

Edge directed interpolation doesn't live up to its promise unless enhancing the edge sharpness is consistent with the smoothness of the high resolution edge contour. Our contour + profile model provides a simple framework, where interpolation of the edge profile is controlled by the subpixel accurate contour description. This leads not only to sharper edges but visually smooth and pleasing contours.

Future research will focus on extending this basic idea to more complicated situations such as multiple edges. Once this is achieved, support length of the wavelet filter becomes less of an issue and wavelet decomposition at different scales could be used in addition to the highest frequency scale to develop better interpolation schemes. The algorithm in its current form is computationally expensive, mainly because of iterative contour estimation. We need to develop simpler strategies for estimating edge locations.

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