DESIGN OF SPACE-EFFICIENT, WIDE- AND NARROW TRANSITION-BAND, FIR FILTERS

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ABSTRACT

We propose a method for designing a filter to meet a set of specifications, which can be implemented with reduced area. Our approach combines a prefilter implementation structure whose function has been developed over the years with our previously reported variable precision technique for intelligently quantizing the filter coefficients. Our area-efficient structure uses frequency masking with a single filter model to give good performance with low order and low coefficient sensitivity. Our method does use a novel connection of a simple prefilter structure used in a frequency masking technique to give good designs for the previously unattainable wide-band filter designs. The technique also gives designs with superior (very sharp) transition regions. Both types produce filters that efficiently use digital circuitry, leading to space-efficient designs that are significantly smaller than would otherwise be the case. Some examples are given to demonstrate the effectiveness of our design.

1. INTRODUCTION

Recently, space-efficient FIR digital filter implementations have become more important. In particular, research has revolved around reducing the number of multiplications used. This often comes at the expense of more additions. Several alternatives have been proposed.

One way to decrease the arithmetic complexity of FIR filter realization focuses on the quantization of the signals and coefficients. Finding "good" quantized coefficients has attracted significant attention [1]-[3]. In this paper, we introduce a sensitivity-based algorithm to reduce the wordlength of the coefficients as much as possible, thus reducing the number of filter operations needed.

Another way to reduce the hardware cost is to reduce the order of the FIR filter by using a relatively simple structure. In [4]-[6], the scheme of using a prefilter-equalizer is explored, with good performance being achieved for the design of narrow-band filters. The realization is a cascade connection of a recursive prefilter with a conventionally designed equalizer. The biggest advantage of this method is that the prefilter only requires two adders and some delays regardless of the overall filter order. The prefilter transfer function can be written as:

$$H(z) = 1 + z^{-1} + \ldots + z^{-(L-1)} = \frac{1 - z^{-L}}{1 - z^{-1}}$$

Considering implementation (see Fig. 1), the simplicity of this prefilter is very attractive. However, the attenuation provided by the prefilter is insufficient for all but narrowband filter design. We concern ourselves in this paper with extending its application to the design of wide-band filters. Thus, we introduce a new prefilter structure in this paper. We use the masking filter and delay-complementary concepts introduced in [7], [8] and extend them to realize area-efficient wide-band and narrow transition-band symmetric FIR filters.

In Section 2, our variable precision coefficient algorithm that reduces the overall realization by using variable wordlength coefficients based on sensitivity analysis for FIR filter, will be explained. In Section 3, we provide a new prefilter structure to make it possible to extend the simple RRS (Recursive Running Sum) structure in [4] to wideband FIR filter design. Examples and concluding remarks are also given in Section 4 and Section 5, which show that the computational complexity of the filter realization can be greatly reduced.

2. VARIABLE PRECISION COEFFICIENTS

The proposed method is based on the fact that the frequency response of a filter has different sensitivities to different coefficients. It is wise to vary the wordlength so that more bits will be used for the highly sensitive coefficients while fewer bits are used for the less sensitive coefficients. As a result, a space-efficient filter implementation can be realized without introducing additional inaccuracy into the filter response.

Estimating the variable wordlength of each coefficient starts by evaluating the sensitivity $S_{en}(n)$ of the frequency response to each of the coefficients. This sensitivity reflects the degree of influence on the frequency response of a digital filter that any one of the coefficients exerts under small perturbations.

$$S_{en}(n) = \frac{1}{2p}\int_{-p}^{p} \left|H(w) - \tilde{H}(w)\right|^2 dw \qquad (1)$$

where $H(w)$ is the frequency response with accurate floating point coefficients and $\tilde{H}(w)$ is the frequency response with the n^{th} coefficient changed to its nearest power-of-two. The next step in determining the wordlength of each coefficient is to estimate the quatization error of the designed filter such that the implemented filter meets the specifications. We propose the following algorithm to estimate the roundoff error.

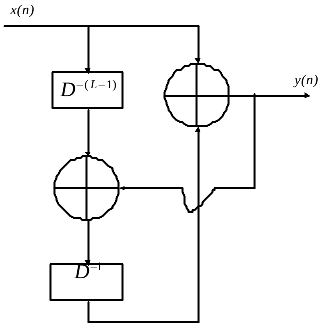

Figure 1. Implementation of the RRS Structure

The transfer function of an FIR filter is

$$H(w) = \sum_{n=0}^{N-1} h(n) e^{jwn} \qquad (2)$$

For convenience, we consider the design of an odd length-N, symmetric FIR filter, so that the magnitude response is

$$H(w) = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{(N-3)/2} h(n)\cos\left(\left(\frac{N-1}{2} - n\right)w\right) \qquad (3)$$

Substituting $m = (N-1)/2 - n$, yields

$$H(w) = h(0) + 2\sum_{m=1}^{(N-1)/2} h(m)\cos(mw) \qquad (4)$$

Quantizing the coefficients of this filter leads to the new frequency response:

$$\tilde{H}(w) = \tilde{h}(0) + 2\sum_{m=1}^{(N-1)/2} \tilde{h}(m)\cos(mw) \qquad (5)$$

From Eqs. (4) and (5), the quantization error can be derived as

$$\begin{aligned} Q(w) &= H(w) - \tilde{H}(w) \\ &= q(0) + 2\sum_{m=1}^{(N-1)/2} q(m)\cos(mw), \end{aligned} \qquad (6)$$

where $q(m) = h(m) - \tilde{h}(m), \quad m = 0, \ldots, (N-1)/2$.

To obtain an expression for the variance $S_q(w)$ of $Q(w)$, according to [1], we make the assumption that $q(m)$ is modeled by a zero mean random variable. Then

$$\begin{aligned} S_q(w) &= E\{Q(w)Q(-w)\} \\ &= E\left\{\left(q(0) + 2\sum_{m=1}^{(N-1)/2} q(m)\cos(mw)\right)^2\right\} \\ &= E\left\{q^2(0) + 4\sum_{m=1}^{(N-1)/2} q(0)q(m)\cos(mw)\right\} \\ &\quad + E\left\{4\sum_{m=1}^{(N-1)/2} q^2(m)\cos^2(mw)\right\} \\ &\quad + E\left\{8\sum_{n=1}^{(N-3)/2}\sum_{m=n+1}^{(N-1)/2} q(n)q(m)\cos(nw)\cos(mw)\right\} \end{aligned} \qquad (7)$$

Now, we modify the equations given in [1] to reflect our use of variable precision coefficients. First, define

$$r_q(m,k) = E\{q(m)q(m+k)\}, \; 0 \le m \le (N-1)/2 - k \qquad (8)$$

Eq. (7) is then rewritten as

$$\begin{aligned} S_q(w) &= r_q(0,0) \\ &\quad + 4\sum_{m=1}^{(N-1)/2} r_q(m,0)\cos^2(mw) \\ &\quad + 4\sum_{m=1}^{(N-1)/2} r_q(0,m)\cos(mw) \\ &\quad + 8\sum_{n=1}^{(N-3)/2}\sum_{m=n+1}^{(N-1)/2} r_q(n,m)\cos(nw)\cos(mw) \end{aligned} \qquad (9)$$

As is standard practice, we assume that the error due to the quantization of different coefficients is independent and uniformly distributed within the quantization bins, that is, we assume that the correlation function $r_q(m,k)$ is

$$r_q(m,k) = \begin{cases} \dfrac{2^{-2d_m}}{12}, & m-k=0 \\ 0, & m-k \neq 0 \end{cases} \qquad (10)$$

where d_m is the least important nonzero bit in the canonical signed digit (CSD) presentation of the m^{th} coefficient. Thus, we can rewrite Eq. (9) to yield

$$S_q(w) = \frac{2^{-2d_0}}{12} + \sum_{m=1}^{(N-1)/2} \frac{2^{-2d_m}}{3}\cos^2(mw) \qquad (11)$$

Eq. (11) is used to evaluate the quantization error. The algorithm for determining the optimal set of variable length coefficients is discussed in more detail in [9].

3. PROPOSED PREFILTER STRUCTURE

The prefilter proposed in [4], [5], with 2 adders, N delays and no multipliers, is a very simple realization. To use this idea to synthesize a wide-band filter, we use a frequency-response masking technique. The prefilter leaves the task of correcting the passband to the equalizer, and only addresses the attenuation and transition bandwidth. The masking prefilter consists of a model filter and a masking filter, whose transfer function can be given as:

$$H(z) = G(z)F(z),$$

where $G(z)$ is the model filter and $F(z)$ is the masking filter. In this paper, we propose to use the masking filter, $F(z)$, implemented by instances of the model filter $G(z)$ with the use of the folding technique described by Parhi in [10]. It is possible to map all the sub-filters in this structure to a single hardware unit [7], which results in efficient hardware with fewer adders at the cost of more delays.

To make use of the simple realization, the RRS prefilter proposed in [4] is used to generate a narrow-band low-pass filter, $G(z)$, which can be used to synthesize a wide-band high-pass filter by subtracting the output of $G(z)$ from the delayed version of the input. The proposed filter structure is given in Fig. 2.

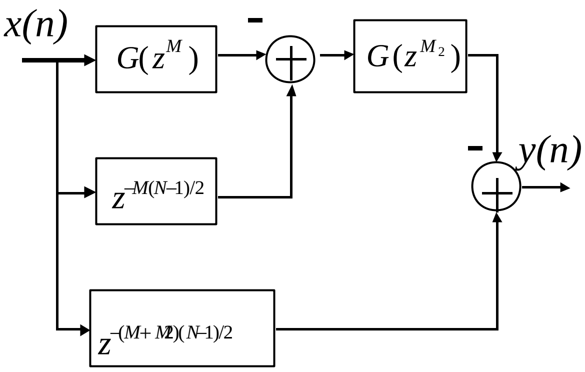

Figure 2. The proposed filter structure

Here, $G(z)$, whose frequency response is shown in Fig. 3(a), is the prototype narrow-band lowpass filter generated by the RRS method. For a linear phase FIR filter with *N-1*-order, the complementary filter $G_c(z)$ of this prototype filter is a wide-band highpass filter, which can be given by:

$$G_c(z) = z^{-(N-1)/2} - G(z) \qquad (12)$$

The magnitude response of $G_c(z)$ is also shown in Fig. 3(a). The filters $G(z^M)$ and $G_c(z^M)$ are derived by replacing each delay element of the prototype filters, $G(z)$ and $G_c(z)$ respectively, with M delay elements as shown in Fig. 3(b). The masking filter, $G_c(z^{M_2})$, which is used to cancel the undesired band in the filter, $G_c(z^M)$, is restricted to be identical to the prototype filter except for the periodicity as introduced in [5]. Then a narrow high-pass band filter can be created by equation (13):

$$H_{nh}(z) = G_c(z^M) \bullet G_c(z^{M_2}) \qquad (13)$$

If $G_c(z^{M_2})$ will not give a valid solution as desired, additional sub-filters with different periodicities should be added by cascading them together to remove the undesired band edges to obtain a valid masking filter. The wideband output can be obtained by subtracting the output of the masked filter again from the delayed version of the input, as shown in Fig. 2. The frequency response of the resulting wide-band lowpass filter $P(e^{jw})$ is shown in Fig. 3(c).

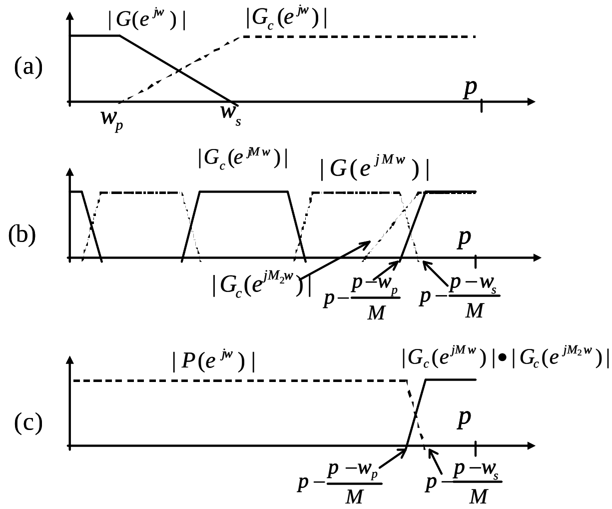

Figure 3. The frequency response of each subfilter in the proposed masking prefilter structure

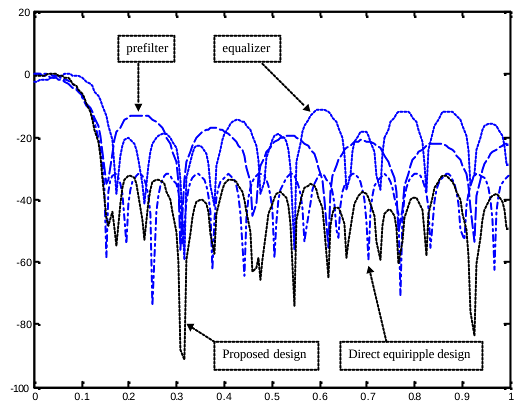

Figure 4. Proposed narrow-band low-pass filter design compared to the conventional Parks-McClellan design

4. EXAMPLES

Example 4.1

Consider an example filter design provided by Adams [4] with the following low-pass filter specification: the pass-band edge at $0.042p$, and the stop-band edge at $0.146p$, pass-band ripple, $DB_p = 0.2db$, stop-band ripple, $DB_s = -35db$. Fig. 4 shows the frequency responses of the quantized digital filters produced using our proposed algorithm, as well as the frequency response of the filter with the conventional equiripple (Parks-McClellan) design.

The prefilter is generated using the RRS structure introduced by Adams [4]. The next step is to design the equalizer with optimal fixed coefficients using our proposed variable precision algorithm. The reduced hardware result is shown in the Table 1. Note that D is for delays, A is for adds, and M is for multiplies. Since our proposed algorithm quantized the coefficients to either one or two nonzero bit CSD numbers, our design can be implemented by using a few shifts that substitute for the multiplications, thus reducing the required area greatly.

Table 1. Complexity comparison for Example 4.1

	D	A	M
Parks-McClellan	31	31	16
Adams (RRS)	37	25	12
Our Method	37	26	0

Example 4.2

A wide-band low-pass filter is designed using the proposed structure in combination with our variable precision algorithm. The designed filter has the following specifications: pass-band edge at $0.96p$, stop-band edge at $0.99p$, and the ripple at the pass-band and stop-band are $0.2db$ and $-35db$ respectively. The frequency responses of the quantized digital filters produced using our proposed algorithm and the conventional direct equiripple design are shown in Fig. 5.

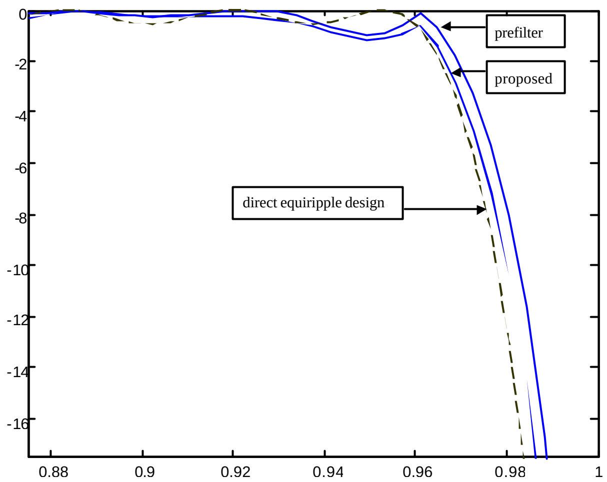

Figure 5. The response of the proposed wide-band filter with that of the direct equiripple design

The required order for meeting the specification using the conventional direct equiripple method is 103. However, the order of the equalizer using the proposed structure is only 61, and each coefficient can be quantized so that the CSD representation has only one nonzero digit. Therefore the required hardware is reduced greatly. Table 2 compares the results.

Table 2. Complexity comparison for Example 4.2

	D	A	M
Parks-McClellan	104	104	57
Our Method	167	66	0

5. CONCLUSIONS

In this paper, a variable precision algorithm is used to produce quantized coefficients that reduce the complexity required in implementing an FIR digital filter. When this method is combined with the novel use of a prefilter structure, which combines our masking technique and the RRS structure defined by other researchers (see [4] and [5]), the resulting filter implementations are very space efficient. Additionally, our design improves the transition band sharpness of the FIR filter design at a cost of only one delay. Moreover, our examples show that space-efficient wide-band filters can be obtained.

REFERENCES

[1] R. E. Crochiere, “A new statistical approach to the coefficient wordlength problem for digital filters,” *IEEE Trans. Cir & Sys.*, vol. 22, no.3, pp.190-196, 1975.

[2] A. G. Dempster, M. D. Macleod, “Variable statistical wordlength in digital filters,” *IEE Pro.-Vis. Image Signal Process*, vol.143, no.1, pp.62-65, Feb1996.

[3] L. S. DeBrunner, V. DeBrunner, P. Pinault, “Variable wordlength IIR filter implementations for reduced space designs,” *2000 IEEE SPS Workshop*, vol.1, pp.326-335, Oct 2000.

[4] J. W. Adams and A. N. Wilson, “A new approach to FIR digital filters with fewer multipliers and reduced sensitivity,” *IEEE Trans. Cir. & Sys.*, vol. CAS-30, no. 5, pp. 277-283, May 1983.

[5] J. W. Adams and A. N. Wilson, “Some efficient digital prefilter structures,” *IEEE Trans. Cir. & Sys.*, vol. CAS-31, no. 3, pp.260-266, Mar 1984.

[6] P.P. Vaidyanathan and G. Beitman, “On prefilters for digital FIR filter design,” *IEEE Trans. Cir & Sys.*, vol. CAS-32, no. 5, pp.494-499, May 1985.

[7] Y.C. Lim, “Frequency-response masking approach for the synthesis of sharp linear phase digital filters,” *IEEE Trans. Cir. & Sys.*, vol. CAS-33, pp. 357-364, Apr1986.

[8] O. Gustafsson, H. Johansson, and L. Wanhammar, “Design and efficient implementation of single filter frequency-response masking FIR filters,” *European Signal Processing Conference*, Tampere, Finland, Sep 2000.

[9] X. Hu, L. S. DeBrunner, and V. DeBrunner “An Efficient Design for FIR Filter with Variable Precision,” *ISCAS’02*, Phoenix, AZ, May 2002.

[10] K. K. Pahrhi, *VLSI digital signal processing systems*, Wiley, 2000.