FIXED POINT ERROR ANALYSIS OF CORDIC PROCESSOR BASED ON THE VARIANCE PROPAGATION

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ABSTRACT

The effects of angle approximation and rounding in the CORDIC processor have been intensively studied for the determination of design parameters. However, the conventional analyses provide only the error bound which results in large discrepancy between the analysis and the actual implementation. Moreover, some of the signal processing architectures require the specification in terms of the mean squared error (MSE) as in the design specification of FFT processor for OFDM. This paper proposes a fixed point MSE analysis based on the variance propagation for more accurate error expression of CORDIC processor. It is shown that the proposed analysis can also be applied to the modified CORDIC algorithms. As an example of application, an FFT processor for OFDM using the CORDIC processor is presented. The results show close match between the analysis and simulation.

1. INTRODUCTION

The COordinate Rotation DIgital Computer (CORDIC) is an effective method for the calculation of trigonometric functions, multiplication, division, and conversion between binary and mixed radix number systems [1, 2]. There are a number of digital signal processing (DSP) applications using the CORDIC-based hardware in modern digital signal processing systems such as modulation, digital filtering, and fast Fourier transforms (FFT). For the optimal design of systems using the CORDIC processor, the analysis of various error sources is required as in other DSP problems. Since Walther [2] first briefly discussed the accuracy of CORDIC computation, there have been many papers on the analysis of error bound [4–7]. Specifically, Y. Hu [4] analyzed various error sources of angle approximation, round-off, and normalization error in greater detail for all modes of the CORDIC arithmetic, and X. Hu and Bass [5] reformed the model by adding neglected error source about the direction of rotation. Kota and Cavallaro [6] proposed a partial normalization scheme to reduce the numerical error in the computed inverse tangent of CORDIC backward rotation mode, and Antelo *et al.* [7] presented a prescaling algorithm to compensate for the disadvantage of [6]. However, the conventional analyses provide only the upper bound of the error, which is not sufficient for several reasons. More specifically, although the upper bound analysis helps the designer select the parameters for stable operation, the difference between the analysis and practical result is very large. Moreover, in the case of application such as FFT employing several CORDIC processors, the error is propagated and added to another errors. As a result, the error bound of the output provides very little information. Hence, the mean squared error (MSE) analysis as well as the upper bound error is needed for more accurate analysis of CORDIC processor.

The MSE analysis have already been developed for many DSP systems such as the quantization error analysis of digital filter, FFT, and DCT implementations. The MSE analysis can provide the designer with the average SNR of the system for the given parameters. Hence, we attempt to provide the MSE analysis of CORDIC processor, which can be applied to many DSP systems employing CORDIC. From the simulations, it is shown that the MSE analysis is much closer to the simulation than the upper bound error analyzed in the conventional literature. The proposed analysis can also be used for the analysis of modified CORDIC family [3]. To demonstrate how our analysis can be applied to a typical DSP system employing the CORDIC processor, simulation of FFT processor for OFDM using the CORDIC processor [8] is presented.

2. CONVENTIONAL CORDIC ALGORITHM

The CORDIC processor computes a set of trigonometric functions using vector rotation. These functions can be computed by a series of specific incremental rotation angles, where each rotation is performed by a shift/add operation. The rotation angle θ can be represented as [3]

$$\theta \simeq \sum_{i=0}^{N-1} \sigma(i)\alpha(i), \qquad (1)$$

where N is the number of rotations. The term $\sigma(i)$ is a sequence of ±1s which determines the direction of remaining angle. The $\alpha(i)$ is the elementary rotation angle of the i-th rotation. The CORDIC algorithm consists of two parts, namely iteration process and scaling process. The iteration process relates the output vector $\boldsymbol{v}(i+1)$ to its input vector $\boldsymbol{v}(i)$ as

$$\begin{aligned}\boldsymbol{v}(i+1) &= \boldsymbol{P}(i)\cdot\boldsymbol{v}(i),\\ \phi(i+1) &= \phi(i) - \sigma(i)\alpha(i) \quad \text{for } i = 0, 1, ..., N-1,\end{aligned}$$

where $\boldsymbol{P}(i)$ represents the micro rotation in the i-th iteration, and $\phi(i)$ is the remaining angle after the i-th iteration. The rotational result is not restricted on the circle of fixed radius. Hence, the magnitude of the result needs to be normalized by the scaling process. That is, the output of the iteration process $\boldsymbol{v}(N)$ is divided by the scaling factor K where $K = \prod_{i=0}^{N-1} k(i)$. Hence, the final result of CORDIC processor $\boldsymbol{v}_o(N)$ can be represented as

$$\boldsymbol{v}_o(N) = \frac{1}{K}\cdot\boldsymbol{v}(N) = \left(\prod_{i=0}^{N-1} k(i)\right)^{-1}\cdot\boldsymbol{v}(N).$$

In case of circular rotation mode, $\boldsymbol{P}(i)$, $k(i)$, and $\alpha(i)$ are represented as

$$\boldsymbol{P}(i) = \begin{bmatrix} 1 & \sigma(i)2^{-i} \\ -\sigma(i)2^{-i} & 1 \end{bmatrix},$$
$$k(i) = \sqrt{1+2^{-2i}}, \quad \alpha(i) = \tan^{-1}(2^{-i}).$$

3. ACCURACY OF CORDIC ALGORITHM

3.1. MSE of Angle Approximation Error

One of the error sources of CORDIC algorithm is the angle approximation. If we wish to rotate the input vector by θ using the CORDIC algorithm, θ should be smaller than $|\theta| \le \sum_{i=0}^{N-1} \alpha(i) + \alpha(N-1)$ for the convergence [4]. In this case, the θ can be represented as $\theta = \sum_{i=0}^{N-1} \sigma(i)\alpha(i) + \delta$, where δ is the residual rotation angle, which is caused by finite combination of the elementary angles. Assuming there is no round-off error, the ideal CORDIC output $\boldsymbol{v}_o(\infty)$ is compensated by the compensation matrix $\boldsymbol{C}$ [4], i.e.,

$$\boldsymbol{v}_o(\infty) \;=\; \boldsymbol{C}\cdot\boldsymbol{v}_o(N) \;=\; \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \cdot \boldsymbol{v}_o(N).$$

Therefore, the angle approximation error $\boldsymbol{e}_a(N)$ of the $\boldsymbol{v}_o(N)$ is given by

$$\boldsymbol{e}_a(N) \;\triangleq\; \boldsymbol{v}_o(N) \;-\; \boldsymbol{v}_o(\infty) \;=\; (\boldsymbol{I} \;-\; \boldsymbol{C})\cdot\boldsymbol{v}_o(N), \tag{4}$$

where $\boldsymbol{I}$ is the 2×2 identity matrix. Now, the MSE of the angle approximation error is given by following theorem.

Theorem 2.1:
For the case that the input angle is given and thus the δ is deterministic, the MSE of the angle approximation error can be expressed as

$$E\,|\boldsymbol{e}_a(N)|^2 \;=\; 4\sin^2\frac{\delta}{2}\cdot E\,|\boldsymbol{v}(0)|^2 \;\simeq\; \delta^2\cdot E\,|\boldsymbol{v}(0)|^2, \tag{5}$$

where $E(\;\cdot\;)$ is the statistical expectation.

Proof:
Since the CORDIC output $\boldsymbol{v}_o(N)$ is independent of $\boldsymbol{C}$, the MSE of $\boldsymbol{e}_a(N)$ in eq. (4) can be expressed as

$$E\,|\boldsymbol{e}_a(N)|^2 \;=\; E\{\|\boldsymbol{I}-\boldsymbol{C}\|^2\cdot|\boldsymbol{v}_o(N)|^2\}$$

where $\|\;\cdot\;\|$ is the l_2-norm of a matrix. Assuming the infinite precision arithmetic, there is no round-off error, and then $\boldsymbol{v}_o(N)$ has the same energy as the CORDIC input $\boldsymbol{v}(0)$. Also, since $\|\boldsymbol{I}-\boldsymbol{C}\|^2 = 4\sin^2\frac{\delta}{2} \simeq \delta^2$, this proves eq. (5).

■

When the input angle is not given, we just have the information that the residual angle $|\delta|$ is bounded by $\alpha(N-1)$ [4]. If we assume that the error is uniformly distributed, the MSE can be evaluated as follows.

$$\begin{aligned} E\,|\boldsymbol{e}_a(N)|^2 \;&=\; \int_{-\alpha(n-1)}^{\alpha(n-1)} 4\sin^2\frac{\delta}{2}\cdot\frac{1}{2\alpha(n-1)}\,d\delta\cdot E\,|\boldsymbol{v}(0)|^2 \\ &= \left(2 - 2\cdot\frac{\sin\alpha(N-1)}{\alpha(N-1)}\right)\cdot E\,|\boldsymbol{v}(0)|^2. \end{aligned} \tag{6}$$

For using variance propagation formula, variance and mean of the angle approximation error are required to be separated from its MSE. Variance of the angle approximation error is approximated as the MSE by the following theorem.

Theorem 2.2:
Let $\boldsymbol{e}_a(N) \triangleq [e_{ax}(N), e_{ay}(N)]^T$, where $[\;\cdot\;]^T$ represents vector transpose. Then

$$\mathit{Var}\{e_{ax}(N)\} \;=\; \mathit{Var}\{e_{ay}(N)\} \;\simeq\; \frac{E\,|\boldsymbol{e}_a(N)|^2}{2}, \tag{7}$$

where $\mathit{Var}(\;\cdot\;)$ is the statistical variance.

Proof:
The MSE of the angle approximation error can be described as

$$E\,|\boldsymbol{e}_a(N)|^2 \;=\; \mathit{Var}\{e_{ax}(N)\} + \mathit{Var}\{e_{ay}(N)\} + |E\boldsymbol{e}_a(N)|^2.$$

If δ is deterministic, since $|E\{\boldsymbol{v}(0)\}|^2 \ll E\,|\boldsymbol{v}(0)|^2$ in most case, $|E\{\boldsymbol{e}_a(N)\}|^2 \ll E\,|\boldsymbol{e}_a(N)|^2$. Otherwise,

$$|E\{\boldsymbol{e}_a(N)\}|^2 = \left(1-\frac{\sin\alpha(N-1)}{\alpha(N-1)}\right)^2\cdot|E\{\boldsymbol{v}(0)\}|^2. \tag{8}$$

From eqs. (6) and (8), it can be observed that $|E\{\boldsymbol{e}_a(N)\}|^2 \ll E\,|\boldsymbol{e}_a(N)|^2$. Hence,

$$E\,|\boldsymbol{e}_a(N)|^2 \;\simeq\; \mathit{Var}\{e_{ax}(N)\} \;+\; \mathit{Var}\{e_{ay}(N)\}. \tag{9}$$

■

3.2. MSE of Round-off Error

In the previous subsection, the analysis for the angle approximation error is presented. Now, we derive the errors caused by finite precision arithmetic in the micro rotations and multiplication of the scaling factor. For the analysis of these errors, let $[\;\cdot\;]_{\boldsymbol{Q}}$ denote quantization operator. Then the round-off errors are given by

$$\begin{aligned} \boldsymbol{e}_r(i) \;&\triangleq\; [\boldsymbol{v}(i)]_{\boldsymbol{Q}} - \boldsymbol{v}(i) \qquad \text{for}\quad i = 0,1,...,N-1, \\ \boldsymbol{e}_s \;&\triangleq\; [\boldsymbol{v}_o(N)]_{\boldsymbol{Q}} - \boldsymbol{v}_o(N), \end{aligned}$$

where $\boldsymbol{e}_r(i)$ is the round-off error introduced by $(i-1)$-th iteration process. Specifically, $\boldsymbol{e}_r(0)$ is the round-off error of the input vector. Also, $\boldsymbol{e}_s$ is the scaling error.

Theorem 2.3:
Total round-off error $\boldsymbol{e}_{or}$ is expressed as

$$\boldsymbol{e}_{or} \;=\; \frac{1}{K}\left(\boldsymbol{e}_r(N) \;+\; \sum_{i=0}^{N-1}\prod_{j=i}^{N-1}\boldsymbol{P}(j)\boldsymbol{e}_r(i)\right) \;+\; \boldsymbol{e}_s. \tag{10}$$

Proof:
This can be easily proved from the well-known error propagation formula [4]. Since the CORDIC operation is a linear transformation, the total round-off error is also the sum of the linear transformation of $\boldsymbol{e}_r(i)$ up to the N-th iteration with the scaling, which is the first term in the eq. (10), plus the final scaling error $\boldsymbol{e}_s$.

■

Since CORDIC is a linear transform, the covariance matrix of the output error can be determined from that of input only:

Theorem 2.4:
Since two elements of the vector $\boldsymbol{e}_r(i)$ are uncorrelated, its covariance matrix is represented as

$$\mathit{Cov}\{\boldsymbol{e}_r(i)\} \;=\; w_v(i-1)\cdot\boldsymbol{I} \qquad \text{for}\quad i = 0,1,...,N,$$

where $w_v(i-1)$ is the variance of each element of vector $\boldsymbol{e}_r(i)$. If $\boldsymbol{e}_r(0) = \boldsymbol{0}$, the covariance matrix of $\boldsymbol{e}_{or}$ can be described as follows.

$$Cov\{\boldsymbol{e}_{or}\} = \frac{1}{K^2}\Bigg(w_v(N-1) + \sum_{j=1}^{N-1}\prod_{i=j}^{N-1} w_v(j-1)k(i)^2\Bigg)\cdot \boldsymbol{I} + Cov\{\boldsymbol{e}_s\}. \quad (12)$$

Proof:
Let $\boldsymbol{e}_{or}(i)$ be the output error propagated from $\boldsymbol{e}_r(i)$. Then, from the variance propagation formula,

$$Cov\{\boldsymbol{e}_{or}(i)\} = \begin{cases} \frac{1}{K^2}\cdot \widetilde{\boldsymbol{P}}(i)Cov\{\boldsymbol{e}_r(i)\}\widetilde{\boldsymbol{P}}(i)^T & \text{for } 1 \le i < N, \\ \frac{1}{K^2}\cdot Cov\{\boldsymbol{e}_r(i)\} & \text{for } i = N, \end{cases}$$

where $\widetilde{\boldsymbol{P}}(i) \triangleq \prod_{j=i}^{N-1}\boldsymbol{P}(j)$. Hence, the covariance matrix of $\boldsymbol{e}_{or}$ can be described as

$$Cov\{\boldsymbol{e}_{or}\} = \sum_{j=1}^{N} Cov\{\boldsymbol{e}_{or}(j)\} + Cov\{\boldsymbol{e}_s\}, \text{ where}$$

$$\sum_{j=1}^{N} Cov\{\boldsymbol{e}_{or}(j)\} = \frac{1}{K^2}\Bigg(w_v(N-1)\cdot \boldsymbol{I} + \sum_{j=1}^{N-1} w_v(j-1)\cdot \widetilde{\boldsymbol{P}}(j)\widetilde{\boldsymbol{P}}(j)^T\Bigg) = \frac{1}{K^2}\Bigg(w_v(N-1) + \sum_{j=1}^{N-1}\prod_{i=j}^{N-1} w_v(j-1)k(i)^2\Bigg)\cdot \boldsymbol{I}.$$

■

Now that we have the variance, the mean of round-off error is needed for the derivation of MSE:
Theorem 2.5:

$$E\{\boldsymbol{e}_{or}\} = \frac{1}{K}\Bigg(E\{\boldsymbol{e}_r(N)\} + \sum_{j=1}^{N-1}\widetilde{\boldsymbol{P}}(j)E\{\boldsymbol{e}_r(j)\}\Bigg) + E\{\boldsymbol{e}_s\}. \quad (13)$$

Proof:
The total round-off error is the linear combination of round-off error and scaling error. Thus, this theorem is derived immediately. ■

Finally, from eqs. (12) and (13), the MSE of the total round-off error is represented as

$$E\left|\boldsymbol{e}_{or}\right|^2 = \frac{2}{K^2}\Bigg(w_v(N-1) + \sum_{j=1}^{N-1}\prod_{i=j}^{N-1} w_v(j-1)k(i)^2\Bigg) + \frac{1}{K^2}\Bigg|E\{\boldsymbol{e}_r(N)\} + \sum_{j=1}^{N-1}\widetilde{\boldsymbol{P}}(j)E\{\boldsymbol{e}_r(j)\}\Bigg|^2 + E\left|\boldsymbol{e}_s\right|^2. \quad (14)$$

3.3. MSE of Total Output Error

Theorem 2.6:
The MSE of the total CORDIC output $E\left|\boldsymbol{e}_o\right|^2$ can be expressed as

$$E\left|\boldsymbol{e}_o\right|^2 = E\left|\boldsymbol{e}_a(N)\right|^2 + E\left|\boldsymbol{e}_{or}\right|^2. \quad (15)$$

Proof:
The direction sequence $\sigma(i)$'s in the infinite precision system are different from those in the actual CORDIC system. The difference of $\sigma(i)$'s results in additional angle approximation error [5]. Hence, the angle approximation error is dependent on the round-off error in general. However, in the applications when the input angle is given, residual angle δ is deterministic, and the angle approximation error depends only on the input energy $E\left|\boldsymbol{v}(0)\right|^2$. Hence, two error sources are independent. In this case, it can be assumed that the angle approximation error is an additional error introduced by a linear transform C^{-1} after the termination of CORDIC operation. If the input angle is not given, the angle approximation error variance with $\alpha(N-1)$ is replaced by its upper bound $\alpha(N-1) + \sum_{i=0}^{N-1}\left|\epsilon_{\alpha(i)}\right| + |\epsilon_\theta|$, where $\epsilon_{\alpha(i)}$ and ϵ_θ are round-off errors of $\alpha(i)$ and θ, respectively [5], and is added to the round-off error under the independence assumption. Finally, the MSE of the total CORDIC output is the sum of angle approximation error and the round-off error.

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4. SIMULATION RESULTS

In the simulation, binary input vectors $\boldsymbol{v}(0)$'s are represented by two's complement with b fractional bits. For the simulation of MSE, 1000 sets of $\boldsymbol{v}(0)$'s are generated, which are random numbers uniformly distributed over $[-1, 1]$. Also, 50 rotation angles with double precision at every $1/100\pi$ (rad) over $[0, \pi/2]$ are used. For each angle and $\boldsymbol{v}(0)$, the simulator performs a complete CORDIC rotation operation. The result, in two's complement binary format, is converted into a double precision real number to be compared with the "theorem" derived in the previous section. The magnitude of the difference between these two results is taken as the quantization error corresponding to the given N, b, θ, and $\boldsymbol{v}(0)$. Tab. 1 shows the total MSE which is the sum of angle approximation error and round-off error. From the table, it is verified that the MSE derived in this paper closely matches with the error of actual CORDIC system. The statistical error analysis of MSE in this paper can help the designer estimate the SNR of the system as SNR (dB) $= 10\log_{10} E|\boldsymbol{v}_o(N)|^2/E|\boldsymbol{e}_o|^2$.

The proposed analysis can also be used for the analyses of modified CORDIC algorithms such as AR method, MVR-CORDIC, and EEAS scheme, which can improve the conventional one in terms of computational speed, accuracy, and complexity [3]. Their MSEs can also be evaluated using the similar method as the conventional CORDIC. The tightness of the analysis and simulation can be proved in the same manner as the conventional CORDIC, and the result is omitted here. Fig. 1 plots the predicted output errors versus 32 rotation angles for the case of $b = 16$. As shown in the figure, the proposed analysis can be used to compare the accuracy of modified CORDIC algorithms with that of the conventional CORDIC.

As an example of the application of the proposed error analysis, the output MSE of an $2K/4K/8K$-point FFT processor based on the CORDIC processor is analyzed. Detailed architecture and practical implementation of $2K/4K/8K$-point FFT processor can be found in [8]. The small DFT modules use the conventional complex multiplier based on Booth algorithm. However, in the twiddle factor multiplications for larger transforms such as $2K$, $4K$, and $8K$ point DFT, CORDIC processor is more efficient since it does not require large ROM for storing many twiddle factors. Fig. 2 shows close match between the analysis and simulation in terms

Table 1. Comparisons between simulation and analysis with respect to the fractional bits b and iterations N.

b	N	Simulation	Analysis	Difference %
16	13	$7.29159\text{e}{-09}$	$7.28858\text{e}{-09}$	0.0412603%
	14	$2.02484\text{e}{-09}$	$2.02522\text{e}{-09}$	-0.0184523%
	15	$7.96293\text{e}{-10}$	$7.97837\text{e}{-10}$	-0.193942%
	16	$5.60222\text{e}{-10}$	$5.59224\text{e}{-10}$	0.178053%
32	29	$1.60902\text{e}{-18}$	$1.61080\text{e}{-18}$	-0.110723%
	30	$5.40555\text{e}{-19}$	$5.40945\text{e}{-19}$	-0.0721537%
	31	$2.69712\text{e}{-19}$	$2.69116\text{e}{-19}$	0.220931%
	32	$2.17780\text{e}{-19}$	$2.17662\text{e}{-19}$	0.0541165%

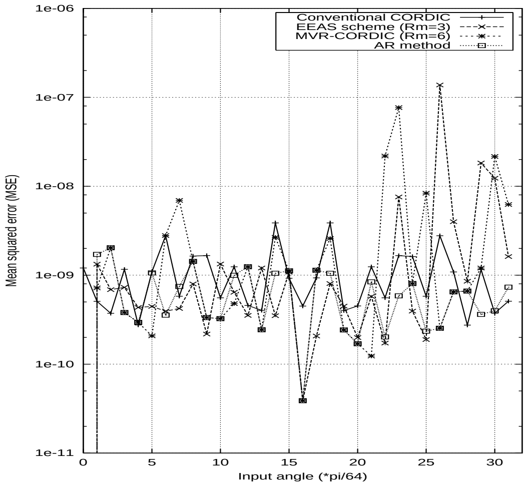

Fig. 1. Comparisons between the conventional CORDIC algorithm ($N = 16$) and the modified CORDIC algorithms. (Rm: predefined iteration number of modified schemes).

of MSE at one sample of the output. Also, a simple design example of the FFT processor is shown in Tab. 2. As shown in the table, in order to have the same accuracy as the case that all complex multipliers employ the Booth algorithm, the iteration number larger than 18 is needed. From the analysis, the number of bits needed for certain MSE or peak SNR can be obtained.

5. CONCLUSIONS

In this paper, fixed-point error analysis of the CORDIC processor has been presented using error and variance propagation formula. Using the model, total quantization error of the CORDIC algorithm has been derived in terms of MSE. Simulation shows the tightness of the derived MSE with the simulation results. As an example of the application of proposed error model, the output MSEs of the modified CORDIC algorithm and an FFT processor employing the CORDIC processor have been analyzed. The result shows close match between the analysis and simulation.

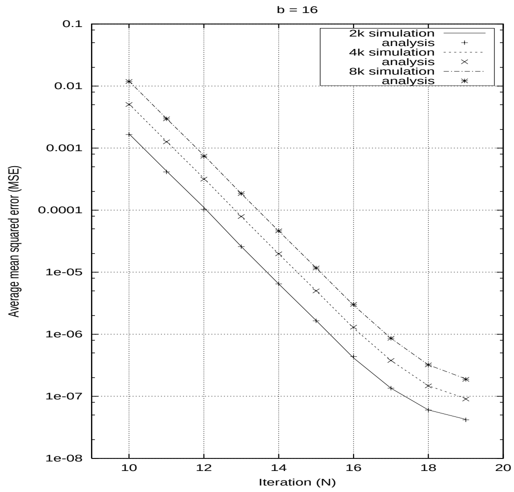

Fig. 2. Average MSE at one sample in the $2K/4K/8K$-point FFT processor. ($b = 16$).

Table 2. Design example of the CORDIC processor in $2K$-point FFT processor. (Booth algorithm: $2.89\text{e}{-08}$ ($b = 16$)).

N vs b	16	17	18	19
16	$4.37\text{e}{-07}$	$4.12\text{e}{-07}$	$4.05\text{e}{-07}$	$4.04\text{e}{-07}$
17	$1.35\text{e}{-07}$	$1.09\text{e}{-07}$	$1.03\text{e}{-07}$	$1.01\text{e}{-07}$
18	$6.02\text{e}{-08}$	$3.40\text{e}{-08}$	$2.74\text{e}{-08}$	$2.58\text{e}{-08}$
19	$4.17\text{e}{-08}$	$1.52\text{e}{-08}$	$8.52\text{e}{-09}$	$6.85\text{e}{-09}$

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