SOME OBSERVATIONS LEADING TO MULTIPLIERLESS IMPLEMENTATION OF LINEAR PHASE FIR FILTERS

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ABSTRACT

This paper explores alternatives for implementing multiplierless implementation of linear-phase finite-impulse response (FIR) digital filters by converting coefficient values to minimum signed powers-of-two (MNSPT) or canonic signed digit (CSD) forms. Our observation is that if one is willing to accept some deviations in the given specifications, the required number of nonzero bits becomes quite low, making multiplierless implementation feasible. Alternatively, one may start with a filter that exceeds the given criteria, at the expense of a slightly increased filter order, and then quantize the coefficient values into the desired representation forms such that the given overall criteria are still met. In many cases, this results in an overall implementation where the total number of nonzero bits is significantly less than that obtained by using the initial design. A fairly exhaustive investigation suggests that less than three nonzero bits per multiplier are quite sufficient along with a reduction in number of arithmetic operations and an attendant increase in the rate of the data throughput.

1. INTRODUCTION

In a multiplierless implementation of a digital filter, generally minimum number of signed powers-of-two (MNSPT) or canonic signed digits (CSD) representations of binary digits are extensively used for representing the coefficient values. An MNSPT representation of a coefficient value is given by $\sum_i a_i 2^{-t_i}$, where each a_i is either 1 or –1 and t_i is a positive or negative integer. For instance, 1.93359375 can be realized as $2-2^{-4}-2^{-8}$. In this example case, the multiplication is achieved not by a nine-bit multiplier, but with aid of three bit shifts and two subtracts.

For finite-impulse response (FIR) digital filters, the major approach for multiplierless implementations comprises of optimizing [5–8, 11] the filter coefficient values such that the resulting filter meets the given criteria with its coefficients values being expressible in MNSPT or CSD forms. For filter design being basically a problem of approximation due to the tolerances in specifications, optimization methods are used to find the optimal transfer functions under the given constraints. While optimization methods are considered to be quite satisfactory, one may not assure or guarantee that the optimal solution will always be found under the given constraints. The solution can be unsatisfactory, for example, in terms of the filter order, the given word-length of the multipliers, or the specified number of shifts and adds (in the case of multiplierless implementation), or some combination of them. In such cases, some parameters or characteristics of the filter have to be relaxed to obtain an acceptable design.

Another approach is based on combining simple sub-filters [1, 2, 12] that can be implemented using only a few shifts and adds and/or subtracts. Although quite attractive, to make this approach as a viable one, a large database of such filters will have to be generated and some optimization method will have to be evolved in order to combine some of them to meet the desired specifications.

In [3, 4] the feasibility of implementing multiplierless recursive digital filters based on low-sensitivity structures has been demonstrated. Allowing a marginally insignificant deviation in the specifications, a gross reduction in the number of nonzero bits (effectively the number of shifts and adds and/or subtracts required) has been seen to be feasible without any increase in the filter order.

This paper investigates the feasibility of implementing multiplierless linear-phase direct-form FIR filters by allowing some deviations in the given specifications that would results in some increase in the filter length. We observe that a similar approach for multiplierless implementations is feasible where the increase in the filter length is offset by a gross reduction in the total number of nonzero bits and the number of arithmetic operations with an attendant increase in the data throughput.

At this stage we digress to point out that a filter being implemented is a sub-system of some system that the system designer wants to implement and the requirement is not exactly rigid but generally flexible within limits. The considerations are that of the performance and the cost (or the performance vis-a-vis the cost), and not necessarily the strict adherence to the initial specifications issued; most likely, a good amount of the design margin was included in the goal system. As such, the filter designer may find out all the options when consulting with the system designer.

2. IMPLEMENTATION

We consider the typical direct form structure [9, 10] for the linear-phase FIR filter system with transfer function given by

$$H(z) = \sum_{n=0}^{L-1} h(n) z^{-n} \tag{1}$$

where

$$h(n) = \pm h(L-1-n)\,. \qquad (2)$$

Both the symmetry (with the plus sign) and anti-symmetry (with the minus sign) conditions are incorporated in Eq. (2). The number of coefficients is $L/2$ or $(L+1)/2$ depending on the filter length is even or odd, respectively[1].

The steps leading to multiplierless implementation are carried out as follows:

(a) Initially, the filter is designed to meet the given specifications using the Remez multiple exchange algorithm described in [9, 10] and implemented in the Signal Processing Toolbox of Matlab.

(b) Performances, in terms of the degradations of the peak-to-peak passband ripple, expressed as A_p decibels, and the minimum stopband attenuation A_s in decibels, with a gradual reduction (up to a certain level) in the number of bits after quantization of the coefficient values is noted. Also, the total number of nonzero bits after quantizing coefficient values to MNSPT or CSD forms is noted.

(c) As mentioned earlier, there are parameters such as the filter length L, the passband ripple A_p, the stopband attenuation A_s, the passband edge ω_p , and the stopband edge ω_s, for which certain deviations may be allowed. Noting the pattern of degradations as in step (b) above one may allow certain deviations in the specifications and design a fresh filter with the same algorithm. For example, our observation shows that the rate of the degradation of stopband attenuation in the direct form realization is much more than the rate of that in the passband ripple. Hence, the specifications for the fresh filter the stopband attenuation should be much higher than that for the initial specifications.

(d) As in step (b), the number of nonzero bits to meet the initial specification is noted.

We note that that the total number of nonzero bits is considerably lower than that for the initial design and the results are better, as will be seen in the following section.

3. RESULTS AND DISCUSSIONS

Table 1 illustrates the results of the implementations for many representatives of linear-phase FIR filter designs. In this table, N_m, N_b, and N_{bm} indicate the number of coefficients, the total number of nonzero bits for the multipliers, and the average number of nonzero bits per multiplier coefficient, respectively. Figures 1, 2, 3, and 4 show the amplitude characteristics of some of the filters of Table 1.

For the purpose of elaborating the contents of Table 1, we consider in more details, the case of Filter 1 with the following three stop-band attenuation levels: 40 dB, 39 dB, and 33 dB. First, the filter designed with the revised specifications is compared with that meeting the initial criteria in the 40-dB attenuation case (the first row for Filter 1 performance results). In this case, it is seen that the first one reduces N_b from 47 to 29 compared with the second one at the expense of increasing the filter length by three. (32 compared with 29). In contrast in the 39-dB and 37-dB cases, the opposite is true, that is, the requirements for N_b are much less in for degraded performances of the initial designs than for those of the revised specifications that are associated with marginal decreases in the filter lengths. The choices are obvious (the values of L, N_m, and N_b are shown in bold and italicized letters for the chosen ones) and confirm our earlier statements. Similar observations can be made for all the filter examples in Table 1.

Further observation is made in respect of a substantial reduction in arithmetic operations like number of bit shifts and number of additions. The reduction can be computed as ($L_1 - L_2 + N_{b1} - N_{b2} - N_{m1} + N_{m2}$) additions and ($N_{b1} - N_{b2}$) bit shifts, where we have considered a case similar to the case of 40 dB stopband attenuation of Filter 1 (with subscript "1" refers to the initial design case and subscript "2" refers to the revised specification design case; we also mention that N nonzero bits imply (N–1) additions). This reduction in arithmetic operations is quite substantial in most cases and would provide an increase in throughput rate of data. We also observe that with increasing filter length, the reduction in arithmetic operations (for example, see the cases pertaining to Filter 4) increases.

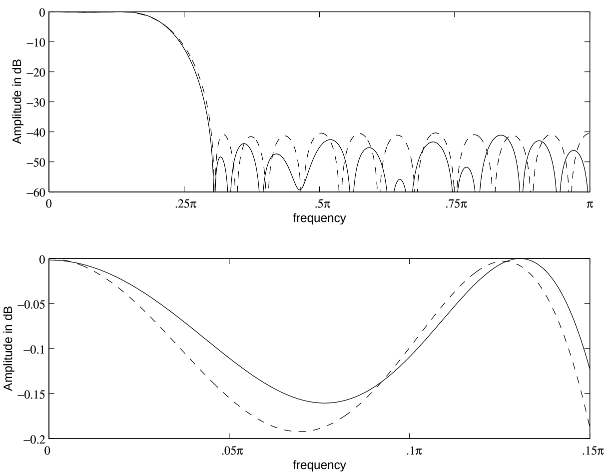

Fig. 1. Amplitude responses of Filter 1 for the initial design (dashed line) and the alternative feasible design (solid line) that requires twenty-nine nonzero bits.

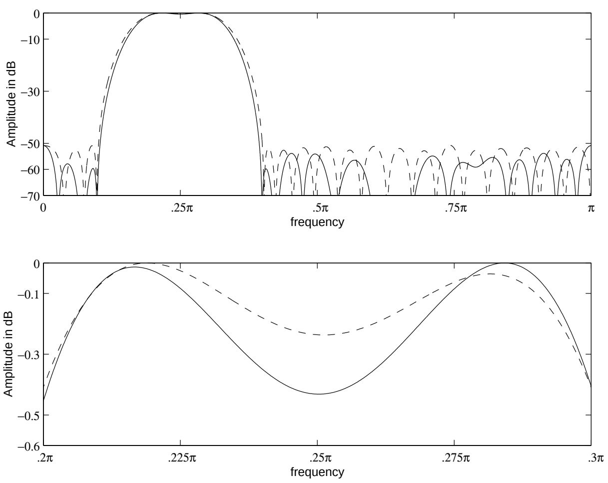

Fig. 2. Amplitude responses of Filter 5 (bandpass) for the initial design (dashed line) and the alternative feasible design (solid line) that requires fifty-three nonzero bits.

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[1] Strictly speaking, for a linear-phase FIR filter of odd length L and possessing an anti-symmetric impulse response, the central impulse response value is zero, thereby reducing the number of multipliers by one.

Table 1. Results of some representative FIR filters indicating the options.

Filter specifications	Feasibility from initial design						Alternatives feasible from revised design						Revised design values of	
	L	N_m	N_b	N_{bm}	A_p dB	A_s dB	L	N_m	N_b	N_{bm}	A_p dB	A_s dB	A_p dB	A_s dB
Filter 1: Low-pass: $\omega_p = 0.15\pi$, $\omega_s = 0.3\pi$, and $A_p = 0.2$ dB as well as $A_s = 40$ dB, 39 dB, and 37 dB	29 " "	15 " "	47 ***37*** ***31***	3.13 2.46 2.07	0.19 0.205 0.195	40.35 39.9 37.0	***32*** 28 27	***16*** 14 14	***29*** 49 51	1.81 3.5 3.64	0.16 0.2 0.192	41.2 39.32 37.3	0.15 0.21 0.2	45.0 39.0 37.0
Filter 2: Low-pass: $\omega_p = 0.2\pi$, $\omega_s = 0.3\pi$, and $A_p = 0.5$ dB as well as $A_s = 50$ dB, 39 dB, and 35 dB	42 " "	21 " "	78 ***41*** ***34***	3.71 1.95 1.62	0.42 0.485 0.48	51.0 39.5 35.82	***44*** 36 31	***22*** 18 16	***59*** 65 41	2.68 3.61 2.56	0.49 0.456 0.49	52.1 39.4 35.5	0.5 0.5 0.5	55.0 39.0 35.0
Filter 3: Low-pass: $\omega_p = 0.1\pi$, $\omega_s = 0.2\pi$, and $A_p = 0.3$ dB as well as $A_s = 60$ dB, 50 dB, and 49 dB	54 " "	27 " "	107 ***72*** ***63***	3.96 2.67 2.33	0.275 0.275 0.288	60.3 53.4 49.88	***62*** 47 "	***31*** 24 "	***83*** 103 "	2.68 4.29 "	0.272 0.297 "	62.25 50.2 "	0.3 0.3 "	75.0 50.0 "
Filter 4: Low-pass: $\omega_p = 0.1\pi$, $\omega_s = 0.15\pi$, and $A_p = 0.1$ dB as well as $A_s = 50$ dB and 45 dB	104 "	52 "	177 ***108***	3.40 2.07	0.093 0.098	50.5 45.2	***113*** 92	***57*** 46	***135*** 176	2.36 3.82	0.095 0.099	52.01 45.1	0.09 0.1	60.0 50.0
Filter 5: Band-pass: $\omega_{p_1} = 0.2\pi$, $\omega_{p_2} = 0.3\pi$, $\omega_{s_1} = 0.1\pi$, $\omega_{s_2} = 0.4\pi$, and A_p = 0.5 dB, as well as A_s = 50 dB, 48 dB and 45 dB	42 " "	21 " "	71 ***55*** ***44***	3.38 2.62 2.09	0.496 0.498 0.499	50.7 48.78 45.03	***49*** 41 40	***25*** 22 20	***53*** 70 72	2.12 3.18 3.6	0.45 0.5 0.5	50.9 48.1 45.2	0.5 0.5 0.5	60.0 48.0 45.0
					Absolute error less than						Absolute error less than		Absolute error (Design value)	
Filter 6: Hilbert transformer: $\omega_{p_1} = 0.1\pi$, $\omega_{p_2} = 0.9\pi$, $\omega_{s_1} = 0$, $\omega_{s_2} = \pi$.	31 "	8 "	36 ***21***	4.5 2.62	0.00278 0.006		***39*** 29	***10*** 7	***25*** 27	2.5 3.86	0.00265 0.0055		0.000683 0.00545	
Filter 7: Full-band differentiator:	32	16	66	4.12	0.00604		***36***	***18***	***38***	2.11	0.00601		0.005065	
Filter 8: Partial-band differentiator: magnitude is 0.4π at $\omega_p = 0.4\pi$, and $\omega_s = 0.45\pi$.	51 "	26 "	116 ***53***	4.46 2.04	0.0234 0.024		***57*** …	***29*** …	***77*** …	2.66 …	0.019 ….		0.0173 ……	

Note: For an odd length Hilbert transformer, every alternate multipliers are zero-valued.

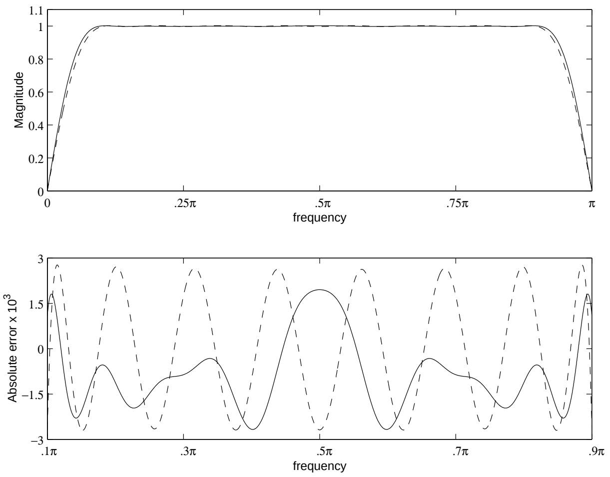

Fig. 3. Amplitude responses of the Filter 6 (Hilbert transformer) for the initial design (dashed line) and the alternative feasible design (solid line) that requires twenty-five nonzero bits.

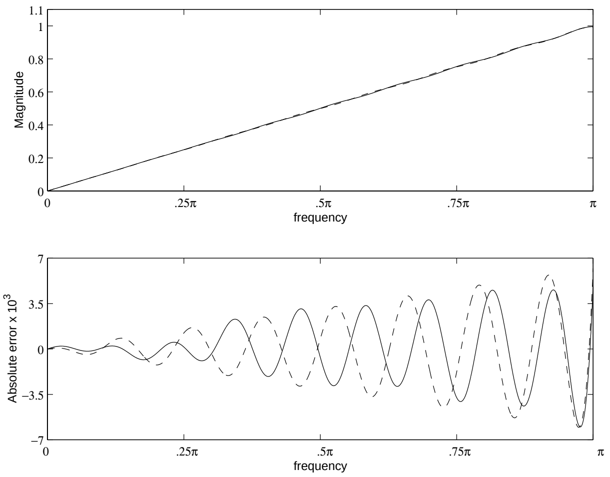

Fig. 4. Amplitude responses of the Filter 7 (full band differentiator) for the initial design (dashed line) and the alternative feasible design (solid line) that requires thirty-eight nonzero bits.

4. CONCLUSIONS

This paper has shown that the multiplierless implementation of FIR filters, using the approach outlined above and evidenced over a large spectrum of FIR filters, is a feasible and attractive proposition. One can either accept deviations in the passband and stopband tolerance specifications compared with the initial infinite-precision design or one can start with a design with stricter specifications followed by the coefficient values being quantized to a level such that the given overall criteria are met. In both cases, some increase in the filter length is involved that is offset by a gross reduction in the total number of nonzero bits. In addition, there is a substantial reduction in the total number of arithmetic operations like number of additions and bit shifts, leading to an increase in the data throughput. Our analysis indicates that utilizing this approach, multiplierless realizations with less than three nonzero bits per multiplier can be achieved at the expense of about ten percent increase in the filter length. Furthermore, it has been seen that there is some reduction in the leakage of the energy through the stopband. Future work is devoted to applying optimization techniques for further reducing the number of nonzero bits.

5. ACKNOWLEDGEMENT

This work was supported by the Academy of Finland, project No. 44876 (Finnish Centre of Excellence program (2000-2005)).

6. REFERENCES

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