MULTIPLIERLESS REALIZATION OF RECURSIVE DIGITAL FILTERS USING ALLPASS STRUCTURES

Tapio Saramäki and Mrinmoy Bhattacharya

Institute of Signal Processing
Tampere University of Technology
P. O. Box 553, Tampere, FIN 33101, Finland
E-mail: {ts, mrinmoy}@cs.tut.fi

ABSTRACT

Under certain conditions an odd-order low-pass or high-pass recursive digital filter can be decomposed into a sum of two all-pass filters with real coefficients. This decomposition has the attractive property that there exist for its implementation structures where both the number of delays and the number of multipliers are equal to the filter order, thereby making the overall implementation very efficient. This paper develops some all-pass filter structures that combine this advantage with those of some low-sensitivity substitution and transformation blocks for replacing unit delay elements. These combinations enable one to generate multiplierless implementations for odd-order recursive digital filters and even- order band-pass and band-stop filters. Utilizing these structures along with allowing some marginally insignificant deviations in the specifications such as in the pass-band and stop-band tolerances, the total number of nonzero bits for multiplier coefficients, i.e., those of shifts and adds and/or subtracts, becomes quite small, making this approach very attractive. Alternatively, the overall filter can be designed with marginally stricter tolerances than the desired specifications in such a manner that it meets the criteria after quantizing the filter coefficients.

1. INTRODUCTION

In multiplierless implementations of digital filters, the minimum number of signed powers of two (MNSPT) or canonic signed digits (CSD) representations of binary digits are extensively used for representing the multiplier coefficient values. An MNSPT representation of a coefficient value is given by $\sum_i a_i 2^{-t_i}$, where each a_i is either 1 or –1 and t_i is a positive or negative integer and the multiplication can be performed with the aid of bit shifts and adds and/or subtracts.

In the case of IIR filters, the structures such as a sum of two all-pass filters, including attractive lattice wave digital (LWD) filters, coupled with optimization methods have shown to yield good results for multiplierless implementations. These sums of all-pass filters are characterized by the attractive property that there exist structures with the number of required multipliers being equal to the filter order, thereby decreasing the number of multipliers compared with conventional realization forms. (We mention that due to paucity of space we are unable to list all the relevant and related references. However, they can be traced from the listed references).

Another interesting approach is the one that stems from designing an odd-order elliptic minimal Q-factor analog filter (EMQF) that has some special properties. When using the bilinear transformation, these filters can be implemented as a sum of two all-pass filters [5] along with an expanded design parameters space such as the pass-band (stop-band) tolerances, the edges, and the filter order.

In [1–4], the feasibility of implementing multiplierless recursive digital filters based on low-sensitivity structures has been demonstrated. It has been observed to be beneficial to develop the all-pass filter structures combining the advantages of the minimum number of multipliers and delays and the low pass-band sensitivity of the all-pass structure along with the advantages of those described in [1–4]. This combination would result in a gross reduction in the total number of nonzero bits (effectively the number of shifts and adds and/or subtracts required). On one hand, the number of multipliers would be less than in the case of the sum of two all-pass filters and, on the other hand, the number of nonzero bits per multiplier would be less due to low values of the modified coefficients. Of course, the transfer function to be realized would have to be of odd order and should fulfill the necessary conditions so as to be decomposable into a sum of the two all-pass filters [7]. Furthermore, by applying suitable transformations, realizations of multiplierless band-pass and band-stop filters become feasible. The next section demonstrates that such an approach is realizable.

2. PROPOSED STRUCTURES FOR THE IMPLEMENTATION

If the conditions given in [7] are fulfilled, then an odd-order transfer function $H(z)$ can be decomposed as

$$H(z) = \frac{1}{2}\big(H_0(z) + H_1(z)\big) , \quad (1)$$

where $H_0(z)$ and $H_1(z)$ are all-pass transfer functions with their orders differing by one. This necessitates the requirement of implementing first-order and second-order all-pass filter structures that could be cascaded as required for implementing the two channels. Also, in the cases of band-pass filters (BPFs) and band-stop filters (BSFs), there would be a need of using second- and fourth-order all-pass filter sections when transforming an odd-order low-pass filter (LPF) to an even-order

BPFs or BSFs to gain the advantage of reducing the number of multipliers.

In the cases of LPFs it is obvious that an Nth-order elliptic filter would require N multipliers compared with approximately $(3N–1)/2$ multipliers required by conventional cascade-form structures as structures exist for implementing all-pass sections with minimum number of multipliers and delays. Similarly, if an Nth-order elliptic LPF with N odd is transformed into a $2N$th-order BPF or BSF implemented as a cascade of second-order sections, then one would require $3N$ multipliers. This figure would reduce to $(5N–1)/2$ multipliers in the case of substitution of unit delay by transformation block [4]. It should be mentioned at this stage that one would require only $2N$ multipliers (the minimum number of multipliers among the three cases) if one decomposes the prototype LPF into a sum of two all-pass filters first, followed by the substitution of unit delay by the transformation block.

The proposed first-order (second-order) and second-order (fourth-order) all-pass sections that combines the above-mentioned advantages are depicted in Figs. 3 and 4, respectively. We mention that the orders mentioned within parentheses are for transforming the prototype LPF to a BPF/BSF. The transformation blocks u^{-1} implemented as shown in Figs. 1 and 2 are given by

$$u^{-1} = z^{-1}/(1-kz^{-1}) \qquad (1)$$

and

$$u^{-1} = \frac{k_1(z^{-2} - \alpha z^{-1})}{1-\alpha(1-k_1k)z^{-1} - k_1kz^{-2}} \qquad (2)$$

for implementing low-pass (high-pass) filters and for transforming a LPF prototype to BPF or BSF, respectively. Here, $k_1 = -1$ for BPFs and $k_1 = 1$ for BSFs, and k is a number that can be represented by one bit or at the most two bits in the MNSPT form. Its absolute value is either equal to or less than unity. For example, k can be equal to ±1, ±0.5, etc. For most cases, $k = \pm1$ (or one bit) will suffice. Furthermore, α is given by

$$\alpha = \cos[(\omega_2+\omega_1/2)]/\cos[(\omega_2-\omega_1/2)] = \cos\omega_0, \qquad (3)$$

where ω_0, ω_1, ω_2, and ω_s are for BPFs and BSFs the center frequency, and the lower and upper passband edges, and the sampling frequency, respectively in the cases of BPF and BSF.

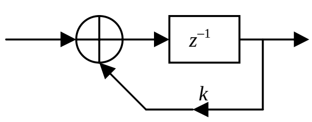

Fig. 1. Transformation block u^{-1} for LPFs.

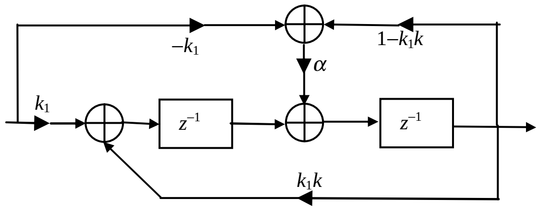

Fig. 2. Transformation block u^{-1} for BPFs/BSFs.

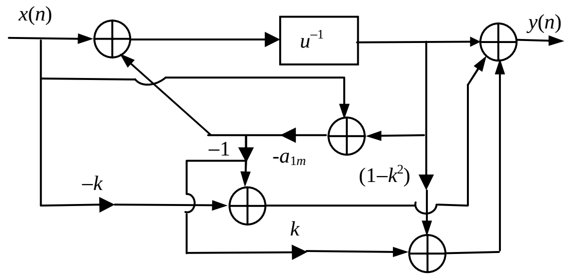

Fig. 3. The first-order (second-order) allpass structure.

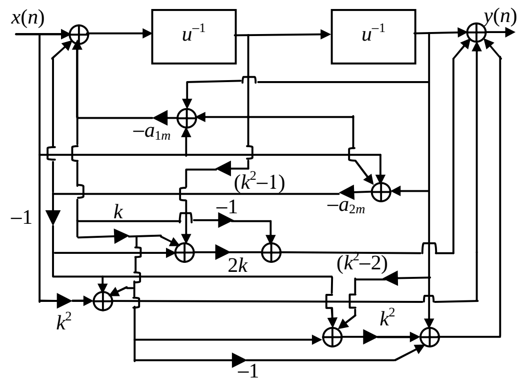

Fig. 4. The second-order (fourth-order) allpass structure.

Fig. 3 shows the realization the following first order allpass transfer function (or transformation to 2nd-order BPF/BSF):

$$H_{1st}(z) = \frac{a_1z+1}{z+a_1} \qquad (4)$$

and the relation of the modified coefficient is given by

$$a_{1m} = k + a_1. \qquad (5)$$

In Fig. 4 the realization of the following second-order allpass transfer function (or transformation to a 4th-order BPF/BSF section):

$$H_{2nd}(z) = \frac{a_2z^2 + a_1z + 1}{z^2 + a_1z + a_2} \qquad (6)$$

is shown and the modified coefficients are given by

$$a_{1m} = 2k + a_1 \text{ and } a_{2m} = k^2 + ka_1 + a_2. \qquad (7)$$

3. RESULTS AND DISCUSSIONS

Quite a few odd-order elliptic filters were realized as a sum of two all-pass filters with the aid of both the unmodified [6] and the modified structures as depicted in Figs. 3 and 4. Also, some even order BPFs and BSFs were realized by transforming odd order LPF prototypes to the desired transfer functions using the schemes of Figures 2, 3, and 4. Table 1 illustrates the implementation results for some of these filters. This table shows the pass-band and stop-band and tolerances and the required number of nonzero bits along with the type of structure used. Figure 5 shows the amplitude responses of Filter 2 of Table 1 in

Table 1. Requirement of nonzero bits for some filters realized as sum of allpass structures

Filter Characteristics	Details of performance achieved			
	Number of nonzero bits	Tolerances achieved		
		Pass-band	Stop-band	Comments
Filter 1: Low-pass; 5th-order:	(a) 26	0.6 dB	50.0 dB	Unmodified all-pass structures rae used; design is based on the initial specifications of the filter.
Pass-band edge = 0.1π	(b) 21	0.603 dB	49.4 dB	….."…..
Stop-band edge = 0.15π	(c) 20	0.605 dB	47.8 dB	….."…..
Pass-band ripple = 0.6 dB	(d) 18	0.631 dB	49.8 dB	….."…..
Stop-band attenuation = 50 dB	(e) 16	0.750 dB	48.6 dB	….."…..
Number of multipliers = 5	(f) 13	0.52 dB	51.26 dB	Modified all-pass structures are used; design is based on the revised specifications with pass-band ripple = 0.5 dB and stop-band attenuation = 51 dB.
Filter 2: Low-pass; 5th-order:	(a) 36	0.05 dB	50.0 dB	Unmodified all-pass structures are used; design is based on the initial specifications of the filter.
Pass-band edge = 0.025π	(b) 33	0.051 dB	49.99 dB	….."…..
Stop-band edge = 0.05π	(c) 30	0.054 dB	49.96 dB	….."…..
Pass-band ripple = 0.05 dB	(d) 27	0.055 dB	49.63 dB	….."…..
Stop-band attenuation = 50 dB	(e) 23	0.068 dB	43.7 dB	….."…..
Number of multipliers = 5	(f) 14	0.042 dB	51.8 dB	Modified all-pass structures are used; design is based on the revised specification with pass-band ripple = 0.03 dB and stop-band attenuation = 51 dB.
Filter 3: Band-pass: 10th-order	(a) 66	0.6 dB	50.0 dB	Unmodified all-pass structures are used; design is based on initial specifications of the filter.
Pass-band edges = 0.1π, 0.2π	(b) 56	0.6 dB	49.99 dB	….."…..
Stop-band edges = 0.08π, 0.24π	(c) 46	0.603 dB	49.65 dB	….."…..
Pass-band ripple = 0.6 dB	(d) 40	0.606 dB	47.8 dB	….."…..
Stop-band attenuation = 50 dB	(e) 38	0.631 dB	47.75 dB	….."…..
Number of multipliers = 10	(f) 37	0.522 dB	51.55 dB	Modified all-pass structures are used; design is based on the revised specification with pass-band ripple = 0.5 dB and stop-band attenuation = 51 dB.
Filter 4: Band-stop; 10th-order	(a) 77	0.6 dB	50.0 dB	Unmodified all-pass structures are used; design is based on initial specifications of the filter.
Pass-band edges = 0.1π, 0.25π	(b) 62	0.6 dB	49.99 dB	….."…..
Stop-band edges = 0.12π, 0.21π	(c) 45	0.6 dB	49.96 dB	….."…..
Pass-band ripple = 0.6 dB	(d) 38	0.602 dB	49.62 dB	….."…..
Stop-band attenuation = 50 dB	(e) 35	0.62 dB	49.2 dB	….."…..
Number of multipliers = 10	(f) 33	0.5 dB	50.8 dB	Modified all-pass structures are used; design is based on the revised specification with pass-band ripple = 0.5 dB and stop-band attenuation = 51 dB.

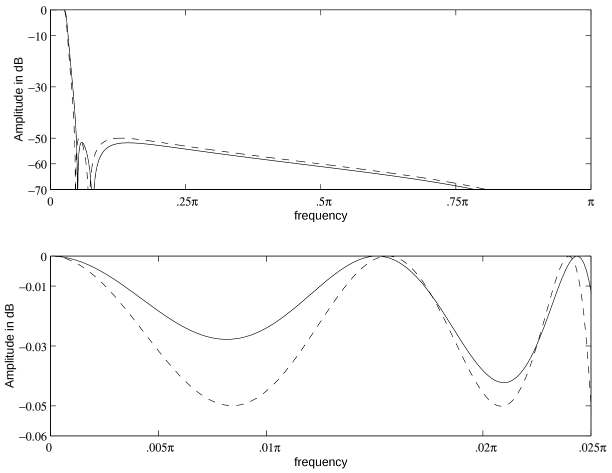

Fig. 5. Amplitude responses for Filter 2 in Table 1 in two cases. The dashed and solid lines show the initial design realized with the unmodified all-pass structure and the realization with revised specifications and modified all-pass structures requiring fourteen nonzero bits.

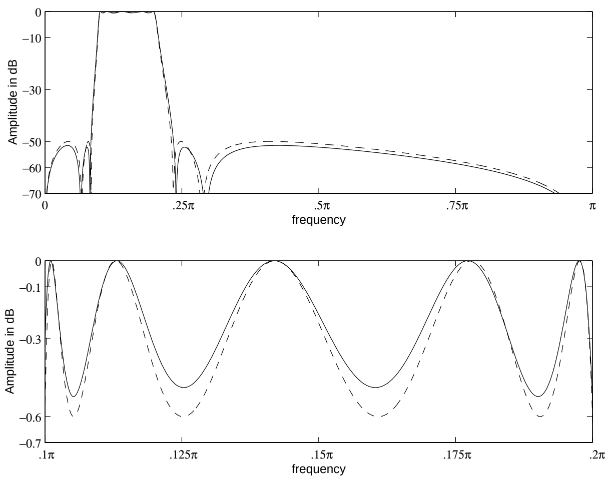

Fig. 6. Amplitude responses for the BPF's in Table 1 in two cases. The dashed and solid lines represent the initial design realized with the unmodified allpass structure and the realization with the revised specification and modified allpass structures requiring thirty-seven nonzero bits, respectively.

two cases. The dashed and solid line show the responses for the initial design realized with unmodified all-pass structure and the realization with revised specifications and using modified all-pass structures requiring fourteen nonzero bits only for five multipliers, respectively. Figure 6 depicts the results of implementation of Filter 3 in Table 1.

According to the results of Table 1, the low-sensitivity property of the sum of the proposed all-pass filter structures is well evidenced. It is seen from the slow degradation of pass-band and stop-band tolerances with the reduction of number of nonzero bits (that reflects the quantization levels).

It was observed that in the cases of BPFs or BSFs, by allowing marginal deviations in the band-edges (by reducing the number of nonzero bits for α−multipliers only), an additional reduction in the number of nonzero bits can be achieved. For example, in the case of the BPF, 27 nonzero bits (a reduction of two bit for each of the five α−multipliers) leads to the filter with the pass-band edges being located at 0.1177π and at 0.2181π.

4. CONCLUSIONS

This paper has demonstrated how a concept developed earlier by the authors (using low-sensitivity structures for multiplierless implementations of recursive filters) for utilization in multiplierless implementation of odd-order recursive digital filter can be used in developing low-sensitivity all-pass filter structures. The resulting structures have been used for implementing odd-order low-pass filters as a parallel connection of two all-pass filters. In addition, by employing the appropriate transformations to the odd-order LPF prototype attractive multiplierless realization of BPF/BSF are shown to be feasible. Utilizing the resulting overall structures, in addition to a reduction in the total number of multiplier coefficients, a gross reduction in total number of nonzero bits is achievable, making this approach very attractive. Several examples have indicated that utilizing the approach outlined, multiplierless realizations can be achieved by using around three and half nonzero bits per multiplier on the average, without any increase in the filter order. Future work is devoted to applying optimization techniques to further reducing the number of nonzero bits.

5. ACKNOWLEDGEMENT

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6. REFERENCES

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