FAST KALMAN/LMS ALGORITHMS ON THE STRONG MULTIPATH CHANNEL

*Choi Jung Min, **Jung Su Kim, *Jae Hong Park, *Jong Wha Chong

*Hanyang University, Seoul, KOREA
Department of Electronics Engineering
CAD & Communication Circuit Lab.

ABSTRACT

We present a Decision Feedback Equalizer (DFE) based on the proposed hybrid structure, which consists of the fast Kalman and LMS algorithm, to guarantee the fast convergence speed, computational complexity and numerical stability on the strong multipath channels. The LMS algorithm recommended by ATSC for HDTV does not guarantee TOV(Threshold Of Visibility) on the strong multipath channels during the training period. Since the fast Kalman algorithm is not only numerically instable but it also has high computational complexity, we proposed a hybrid structure to solve both of the problems. The fast Kalman algorithm is applied during the training sequence for the fast convergence, and then LMS algorithm is used to reduce the computational complexity and avoid the instability problem for the payload data part. The simulation result demonstrates that our hybrid structure is numerically stable with the fast convergence speed and it overcomes the strong multipath channel.

1. INTRODUCTION

The basic purpose of equalization is to improve the performance of a high-speed communication system encountered with the channel Intersymbol Interference(ISI). This ISI is caused by the imperfect channel characteristics and seriously degrades the receiver performance. In order to improve the Signal-to-Noise-Ratio(SNR) and reduce the Symbol-Error-Rate(SER), some equalization techniques are applied to high-speed communication systems such as Cable Modem and HDTV system. Advanced-Television-Systems-Committee(ATSC), which provides the standard of the HDTV transmission and receiver system, recommends an adaptive decision feedback equalizer using Least Mean Square (LMS) algorithm [1]. In order to attain the reasonable error rate on the strong multipath channel, the number of equalizer taps is very large (over 200 taps) to be adequate. The drawback of the LMS algorithm is slow convergence speed[4][5][7]. Therefore the LMS algorithm with a short training period(820 field sync) is unlikely to guarantee the reasonable convergence characteristic on the severe multipath environment. Recently, Recursive Least Squares (RLS) algorithms are actively studied because of its fast convergence characteristics against the strong ghosts and fading channel such as a terrestrial and mobile environment [3][5]. The most widely used one of least-squares algorithm is the fast Kalman algorithm witch has a remarkably lower computational complexity($O(N)$) than that of conventional Kalman algorithm($O(N^2)$). In order to overcome the drawback of LMS algorithm, this paper focuses on the fast Kalman algorithm that is remarkably faster than the LMS convergence speed.

The Fast recursive least squares algorithms exploit the shifting property of most sequential estimation problems. In equalization, this property expresses the fact that the number of new samples entering and old samples leaving the equalizer is no N , but much smaller integer p each iteration. Although the fast Kalman algorithm is a nice solution to reduce the computational complexity burden of the conventional Kalman algorithm, these algorithms are unstable when implemented with finite precision arithmetic. Increasing the word-length does not solve the instability problem. The only effect of employing a longer word-length will take longer to diverge[10]. Therefore we use hybrid structure that consists of fast Kalman and LMS algorithm, which has stable characteristic, to guarantee fast convergence speed for training sequence part and be numerically stable for payload part.

In section 2, we discuss the data frame structure for HDTV. Section 3 describes the least square algorithm and data recombination for explaining the shift property of the fast Kalman algorithm, and section 4 describes the fast Kalman algorithm with DFE structure. Section 5 proposes the hybrid structure. In section 6, we prove our algorithm as showing the simulation result.

2. DATA FRAME FOR HDTV

Figure 1 shows how HDTV normal data are organized for transmission[1]. Each data frame consists of two data fields and each one contains 313 data segments. The first data segments of each data field are a unique synchronizing signal(data field sync) and include a training sequence used for the equalization in the receiver. A field sync segment which consisted of PN511, PN63, PN63, PN63, VSB mode signals and reserved signals(104) are transmitted in a binary form and provided as a training sequence[1].

As described above only 820 symbols except precode(12 symbol) are available for a training signal to detect the remaining 312 payload data segments. In this structure, training sequence is fixed to be sent every other 312 segments. The main issue for VSB data frame is to guarantee a reasonable error rate within a training period to protect the payload data from the error propagation.

In the 8-VSB HDTV system, a fast convergence is inevitably required because of the very short training sequence against a long multipath and strong ghosts fading channel. We describe the fast Kalman algorithm to solve the slow convergence speed of the LMS algorithm.

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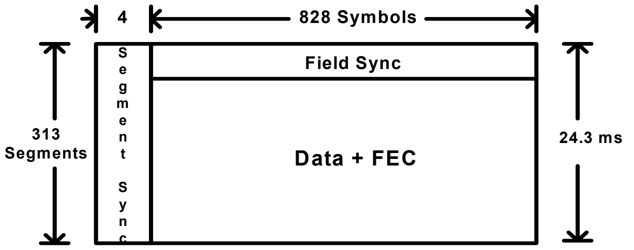

Figure1. VSB data frame

3. LEAST SQUARE ALGORITHM

The classical least squares problem is to find the adaptive filter coefficient at each time instant so as to minimize the accumulation of squared error between the filter output and the desired output up to that time instant. For example in an equalizer adjustment algorithm, suppose $d(1), d(2), \cdots, d(n)$ is a training sequence during the initial equalizer training period(field sync). Channel introduces intersymbol inter- ference and additive noise. We assume that the sequence $d(1), d(2), \cdots, d(n)$ transmitted over a channel and results in the received sequence are $y(1), y(2), \cdots, y(n)$. To correct the transmitted signal, the Decision Feedback Equalizer(DFE) applies the linear combination of received sequence $y(1), y(2), \cdots, y(n-N_1+1)$ and previously decoded values $d(n-1), d(n-2), \cdots, d(n-N_2)$. We define the following N-dimensional vector[11][12]:

$$X_N(n) = [y(n), y(n-1), \cdots, y(n-N_1+1) \quad d(n-1), d(n-2), \cdots, d(n-N_2)] \tag{1}$$

where $N = N_1 + N_2$.

We indicate the dimensionality of vectors and matrices by subscripts. The absence of subscript indicates scalar. $X_N(n)$ is an N-dimensional vector, $A_{NP}(n)$ is a matrix with N rows and P columns. Matrix transpose will be represented by a superscript T.

The least squares problem in DFE is to find the N-dimensional vector $C_N(n)$ of equalizer tap values, which minimizes the following sum of squared error:

$$\sum_{k=1}^{n} \lambda^{n-k} \cdot \left[e_N(k) \cdot e_N^*(k) \right]^2 \tag{2}$$

where the exponential forgetting factor λ is such that $0 \le \lambda \le 1$ and the a posteriori error residual, $e_N(n)$, is

$$e_N(n) = d(n) - C_N(n)^T \cdot X_N(n) \tag{3}$$

where $C_1(n), C_2(n), \cdots, C_N(n)$, are the tap gain of the forward filter, $C_{N_1+1}, \cdots, C_{N_1+N_2}$, are the tap gain of the feedback filter. Using the definition of the complex gradient operator[11], we can show that the optimum value of the vector $C_N(n)$ which minimizes the value of (2) is given by

$$C_N(n) = -R_{NN}^{-1} \cdot \gamma_{NN}(n) \tag{4}$$

where $R_{NN}(n)$ is a $N \times N$ correlation matrix given by

$$R_{NN}(n) = \sum_{k=0}^{n} \lambda^{n-k} \cdot X_n(k) \cdot X_n^H(k) + \delta I_{NN} \tag{5}$$

In practice, the parameter δ is fixed at a small positive constant to guarantee non-singularity of the matrix $R_{NN}(n)$. And the cross correlation vector $\gamma_{NN}(n)$ is given by

$$\gamma_{NN}(n) = \sum_{k=0}^{n} \lambda^{n-k} \left[d^*(k) \cdot X_N(k) \right] \tag{6}$$

This sequence $C_N(n)$ can be generated recursively as follows

$$C_N(n) = C_N(n-1) + k_N(n) \cdot e(n) \tag{7}$$

$$k_N(n) = R_{NN}(n)^{-1} \cdot X_N(n) \tag{8}$$

The inverse estimated covariance matrix $R_{NN}(n)^{-1}$ accelerates the equalizer's adaptation. The vector $k_N(n)$ can be generated by a recursive algorithm which yields the $N \times N$ matrix $R_{NN}(n)^{-1}$ without requiring explicit matrix inversion. The resulting algorithm is the special case of the conventional Kalman algorithm for equalizer adaptation. This algorithm's complexity arises from the $N \times N$ matrix used to compute $k_N(n)$. The fast Kalman algorithm to be presented here and is mathematically equivalent to it, but exploits the shifting property to compute the vectors $k_N(n)$ recursively, without needing to compute a $N \times N$ matrix.

In many other estimation and prediction applications, the input vectors $X_N(n)$ are such that $X_N(n+1)$ is obtained from $X_N(n)$ by shifting its components, introducing p new components and deleting the p oldest components. For example the $p = 2$ new elements $y(n)$, $d(n)$ enter the decision feedback equalizer at time $n+1$, while the elements $y(n-N_1)$ and $d(n-N_2)$ leave.

We defined a p-dimensional vector $\xi_p(n)$ which is the p new elements and a vector $\rho_p(n)$ which is the p deleted old elements at time $n+1$. For the linear equalizer, $\xi_1(n) = y(n)$ and $\rho_1(n) = y(n-N)$. For the decision feedback equalizer

$$\xi_2(n) = \begin{bmatrix} y(n+1) \\ d(n) \end{bmatrix}, \rho_2(n) = \begin{bmatrix} y(n-N_1+1) \\ d(n-N_1) \end{bmatrix}. \tag{9}$$

We also define an extended vector $\bar{X}_M(n)$, with $M = N + p$ dimension, which contains the elements of $\xi_p(n)$ appended in proper order to the elements of $X_N(n)$. For example, in the case of the decision feedback equalizer, with $p = 2$

$$\bar{X}_M(n) = [y(n+1), y(n), \cdots, y(n-N_1+1) \quad d(n), d(n-1), \cdots, d(n-N_2)] \tag{10}$$

In general, there are obvious permutation matrices S_{MM} and Q_{MM} which rearrange the elements of the extended vector $\bar{X}_M(n)$ to display $\xi_p(n), X_N(n), \rho_p(n)$ and $X_N(n+1)$ in simple partitioned form[13]:

$$S_{MM} \cdot \bar{X}_M(n) = \begin{vmatrix} \xi_p(n) \\ \cdots\cdots \\ X_N \end{vmatrix}, Q_{MM} \cdot \bar{X}_M(n) = \begin{vmatrix} X_N(n+1) \\ \cdots\cdots\cdots\cdots \\ \rho_p(n) \end{vmatrix} \tag{11}$$

Each row and column of S_{MM} and Q_{MM} has a single 1 and

$$S_{MM}^{-1} = S_{MM}^{T} \text{ and } Q_{MM}^{-1} = Q_{MM}^{T} \tag{12}$$

S_{MM} and Q_{MM} are identity matrices in the special case of the linear equalizer[12].

4. FAST KALMAN ALGORITHM

The fast algorithm for computing the vector sequence $\{k_N(n)\}$, which is used to update the equalizer's coefficient $\{C_N(n)\}$, is specified below. Before we apply the fast Kalman equations, we should initialize and define some quantities [11].

- $N \times p$ matrices $A_{Np}(n) = 0_{Np}$ and $B_{Np}(n) = 0_{Np}$
- $p \times p$ matrix $E_{pp}(n)$ with initial value $E_{pp}(0) = \delta I_{pp}$
- M dimensional vector $\overline{k}_M(n)$ where $M = N + p$
- p dimensional vector $\alpha_p^b(n), \alpha_p^a(n) \beta_p(n)$ and $\mu_p(n)$
- N dimensional vector $m_N(n)$, $k_N(1) = 0_N$

And all $x(n) = 0$ for $n \le 0$

$$\alpha_p^a(n) = \xi_p(n) + A_{Np}(n-1)^T \cdot X_N(n) \tag{13}$$

$$A_{Np}(n) = A_{Np}(n-1) - k_N(n) \cdot \alpha_p^a(n) \tag{14}$$

$$\alpha_p^b(n) = \xi_p(n) + A_{Np}(n)^T \cdot X_N(n) \tag{15}$$

$$E_{pp}(n) = E_{pp}(n-1) + \alpha_p^b(n) \alpha_p^a(n)^T \tag{16}$$

$$\overline{k}_M(n) = S_{MM}{}^T \begin{bmatrix} E_{pp}(n)^{-1} \cdot \alpha_p^b(n) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ k_N(n) + A_{Np}(n) \cdot E_{pp}(n)^{-1} \cdot \alpha_p^b(n) \end{bmatrix} \tag{17}$$

$$Q_{MM} \cdot \overline{k}_M(n) = \begin{bmatrix} m_N(n) \\ \cdots\cdots \\ \mu_p(n) \end{bmatrix} \tag{18}$$

$$\beta_p(n) = \rho_p(n) + D_{Np}(n-1)^T \cdot X_N(n+1) \tag{19}$$

$$B_{Np}(n) = \frac{B_{Np}(n-1) - m_N(n) \cdot \beta_p(n)^T}{I_{pp} - \mu_p(n) \cdot \beta_p(n)} \tag{20}$$

$$k_N(n+1) = m_N(n) - B_{Np}(n) \cdot \beta_p(n) \tag{21}$$

$$e(n) = d(n) - C_N(n)^T \cdot X_N(n+1) \tag{22}$$

$$C_N(n+1) = C_N(n) + k_N(n+1) \cdot e(n) \tag{23}$$

5. NEW HYBRID FAST KALMAN/LMS STRUCTURE

The decision feedback equalizer has a better SER performance than a linear equalizer on the multipath fading channel. Since the linear equalizer does not work well on the channels with spectral nulls which cause noise enhancement, the DFE which can compensate dispersive channels without as much as noise enhancement, is widely used. In Grand Alliance standards, a DFE based on the LMS algorithm is recommended for equalizing terrestrial HDTV channels. The advantage of the LMS algorithm is with low computational complexity and numerical stability. In spite of these merits, it has slow convergence characteristics, which does not guarantee TOV during the period of 820 training symbols on the strong multipath fading channel. Therefore we need the fast convergent algorithm such as the fast Kalman algorithm ($O(N)$) which has a greatly lower computational complexity than that of the conventional Kalman algorithm ($O(N^2)$).

Unfortunately, the fast Kalman algorithm has two major drawbacks that are high computational complexity compared with the LMS algorithm and numerically instable problem when implemented with finite precision arithmetic. In order to solve the both of the problem, we proposed the hybrid fast Kalman/LMS structure. In the Fig.2, we apply the fast Kalman algorithm for fast convergence during the period of 820 training symbols, and then we change the fast Kalman algorithm to the LMS algorithm for the Forward Error Correction(FEC) and payload data part to avoid both problems of the high computational complexity and instability. The following table1 shows the computational complexity and numerical stability of the LMS, conventional Kalman and fast Kalman algorithm.

Table 1. Complexity and Stability for a linear channel

Algorithm	Multiplication of Linear	Multiplication of DFE	Stability
Fast Kalman	$7N+3$	$14N+12$	X
Conventional Kalman	$3N^2+3N$	$3N^2+3N$	X
LMS	$2N+1$	$2N+1$	O

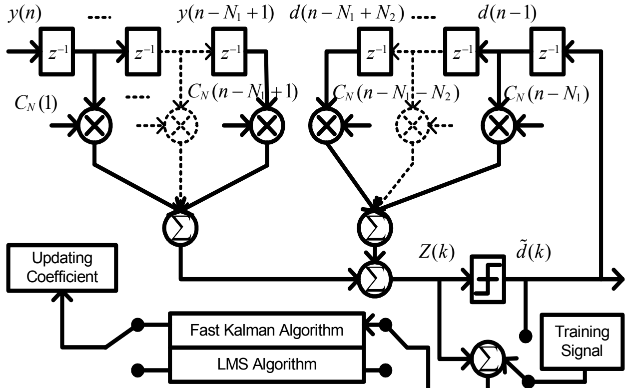

Figure 2. Proposed Hybrid fast Kalman/LMS Structure

6. SIMULATION RESULT

In order to evaluate the equalization performance of our hybrid structure, we examine it on the basis of Grand Alliance 8VSB terrestrial transmission system by computer simulation. On the simulation, we consider the adaptive equalization of two Brazil Lab test channels in the point of stability and convergence speed. The environment of Brazil C(Fig.3) channel is pedestrian channel at 3 miles/h and Brazil D(Fig.3) channel is urban channel at 50 miles/h for simulation. Fig.4,5 are simulated with the same fixed point system(Number system : Two's complement representation, word-length : 38, fractional-part : 24 for error part which is most significant register to control the system) on the each described Brazil C and D channel. Fig.4 a-1,b-1 are describe the MSE(Mean

Square Error) and Fig.4 a-2,b-2 show constellation of equalizer output. Fig.4 b-1,2 and Fig5 b-1,2 prove that our structure performs very well and guarantees the numerical stability and low computational load compared with the Fig.4 a-1,2 and Fig.5 b-1,2 which are simulated using the only fast Kalman algorithm during the training and payload data part. Fig.6 describes the SER curve of proposed algorithm. The simulation result of the LMS algorithm is not presented because it does not guarantee the TOV(14.9dB) on the strong multi path channels.

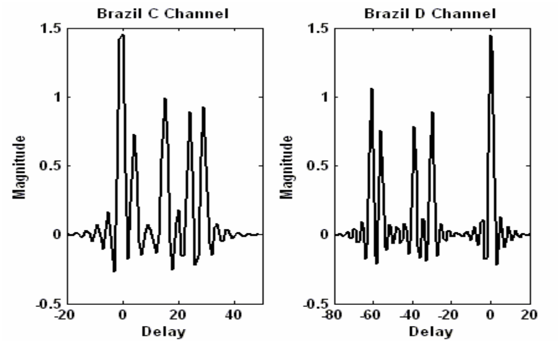

Figure 3. Channel Impulse Response of Brazil C,D

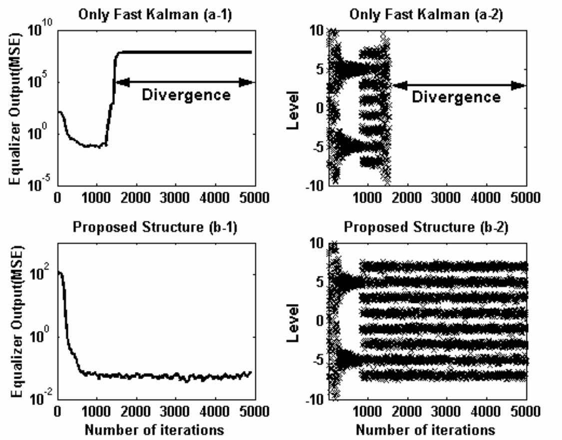

Figure 4. MSE and Constellation for the Brazil C Channel

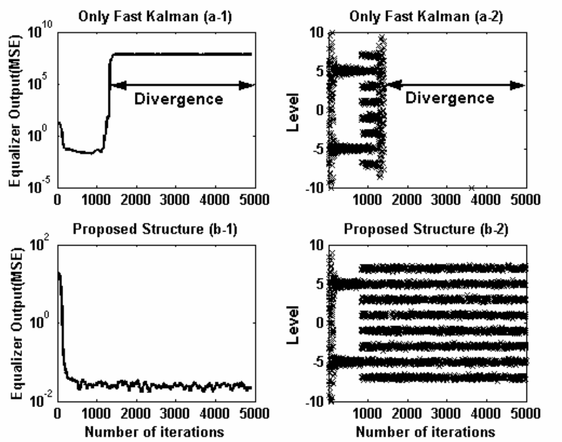

Figure 5. MSE and Constellation for the Brazil D Channel

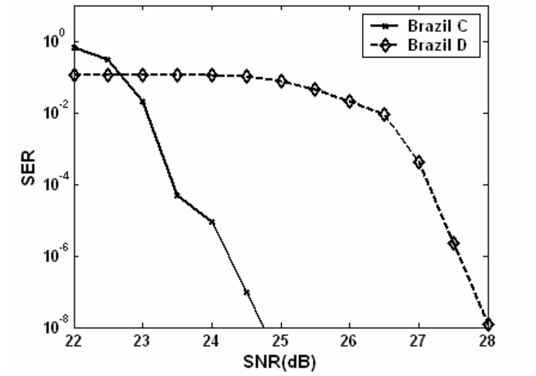

Figure 6. SER curve of Proposed Structure

6. CONCLUSION

In this paper, we proposed hybrid fast Kalman/LMS algorithm which is applicable to the DFE of the strong and long multipath channel. From the simulation result, we observe that our structure is numerically stable and less computational effort compared with using the only fast Kalman algorithm. And our structure also satisfies the convergence speed during the training sequence on the pedestrian(at 3miles/h) and urban(at 50 miles/h) channel for terrestrial and mobile communication. In the future, we will apply the blind equalizer to adjust time varying channel without training sequence for the payload data part.

7. REFERENCES

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