



# ANALYSIS OF DIFFERENT PREDISTORTION STRUCTURES AND EFFICIENT LEAST-SQUARE ADAPTIVE ALGORITHMS

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**Abstract:** Adaptive predistortion is one of the most promising techniques to overcome the nonlinear of High Power Amplifier (HPA). However, the adaptation of predistorter is not satisfied, for many efficient Least-Square (LS) adaptive algorithm cannot be used. In this paper, we analyze previously published predistortion structures and their difficulties in adapting with these efficient algorithms. The proposed structure constructs the desired output of the predistorter, so many efficient LS adaptive algorithms can be used directly, which implies that we can obtain faster convergence and lower complexity in adaptive process. Simulation results using Recursive Least Square (RLS) algorithm in adaptation demonstrate about 30dB spectrum spreading improvement.

## 1. INTRODUCTION

Spectrally efficient modulation schemes are widely used in current radio communication services in order to increase system capacity. Linear modulation schemes, such as  $p/4$ -DQPSK and M-QAM with an appropriate pulse shaping, are spectrally efficient, but they present variations in amplitude and phase. This could cause amplitude and phase distortion after nonlinear power amplification, resulting intersymbol interference, adjacent channel interference, and so on. Predistortion techniques have been proposed as a potential solution to overcome the nonlinear distortion effects. Basically, these techniques aim to introduce “inverse” nonlinearities that can compensate the distortions generated by the nonlinear amplifier. The predistorter is required to be adaptive because of variations in power amplifier nonlinearity with time, temperature and different operating channels. However, the adaptation of this “inverse” model is not satisfied. Random search algorithm is employed, but it

suffers from the very slow convergence [1]. Another common method is to estimate the characteristics of HPA first, then find analytical solutions for the inverse function of the HPA [2-3]. This method is only efficient for low orders, as the order of nonlinearity grows higher, so does the computational complexity. LS adaptive algorithms offering “best” performance are expected in this field. However, these efficient algorithms cannot be used directly for the limitation of the predistortion structure. Although the modified Least Mean Square (LMS) algorithm is used in [4-5], it results the tedious derivative problem of the HPA output with respect to the input.

In this paper, we analyze previously published predistortion structures and their difficulties in using LS adaptive algorithms. A new architecture for HPA adaptive predistortion is presented that many efficient LS adaptive algorithms can be used directly. Theory analysis and simulation all demonstrate the validity of proposed scheme.

## 2. PREDISTORTION PROBLEM

The nonlinear characteristics of a power amplifier can be represented by their AM/AM and AM/PM effects. For an input baseband signal  $u_i(t)$

$$u_i(t) = r(t)e^{jq(t)} \quad (1)$$

the output of the power amplifier can be written as:

$$u_0(t) = G(r(t))e^{j(q(t)+f(r(t)))} \quad (2)$$

where,  $G(\cdot)$  and  $f(\cdot)$  denote the amplitude and phase transfer function of the power amplifier.

Assumed that  $F(\cdot)$  and  $y(\cdot)$  are used in order to compensate the amplitude and phase distortion introduced by the power amplifier respectively. The output of

the predistorter is

$$\mathbf{u}_d(t) = F(r(t))e^{j(q(t)+y(r(t)))} \quad (3)$$

It also is the baseband-equivalent input of the HPA, so we can obtain the output of HPA as

$$\mathbf{u}_0(t) = G(F(r(t)))e^{j(q(t)+y(F(r(t)))+f(r(t)))} \quad (4)$$

The goal of predistortion is to ensure the output of HPA is linear amplification of the input signal, i.e.,

$$\begin{cases} G(F(r(t))) = \mathbf{a} \cdot r(t) \\ \mathbf{y}(r(t)) + \mathbf{f}(F(r(t))) = 0 \end{cases} \quad (5)$$

where,  $\mathbf{a} \cdot r(t)$  is the desired linear response. The technique to derive  $F(\cdot)$  and  $\mathbf{y}(\cdot)$  is called as predistortion. The phase predistorter aims to identify the opposite of the phase shift  $\mathbf{f}(F(r(t)))$ , which is a simple identification problem, and will not be treated further here. Correcting for the AM/AM distortion-which is functional, not additive-is more difficult and is the focus of this paper.

### 3. PREDISTORTION STRUCTURE AND ADAPTIVE ALGORITHM

The performance of adaptive predistortion technique is mainly determined by adaptive algorithm, whereas the performance of adaptive algorithm is measured by the accuracy of the obtained solution, convergence speed, computational complexity, and so on. Efficient LS adaptive algorithms, the “best” tradeoff among these factors, have been designed [6]. The LMS and RLS algorithms are the most celebrated examples, so the following analysis is based on these two algorithms, but the conclusions are also applied to other LS algorithms.

A number of adaptive predistortion structures have been proposed. These structures can be classified into two main categories: direct and indirect.

#### 3.1 Direct Structure

A direct structure for an adaptive predistorter in a digital baseband-equivalent transmission system is shown in Fig.1 [1].

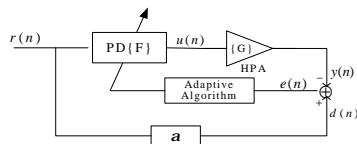


Fig.1. Direct structure for adaptive predistortion

Consider a memoryless HPA that the nonlinear characteristics can be approximated by a M-order polynomial

$$\hat{y}(n) = G(u(n)) = \sum_{i=1}^M g_i (u(n))^i = \mathbf{g}_M^T \mathbf{U}_M \quad (6)$$

where,  $\mathbf{g}_M = [g_1, g_2, \dots, g_M]^T$  is the coefficients vector of HPA,  $\mathbf{U}_M = [u(n), u(n)^2, \dots, u(n)^M]$  is the input extension vector. The corresponding PD model can be expressed as

$$u(n) = F(r(n)) = \sum_{i=1}^M f_i (r(n))^i = \mathbf{f}_M^T \mathbf{R}_M \quad (7)$$

where,  $\mathbf{f}_M = [f_1, f_2, \dots, f_M]^T$ ,  $\mathbf{R}_M = [r(n), r(n)^2, \dots, r(n)^M]$ .

We can determine the optimum coefficients  $\mathbf{f}_M$  of the PD by minimizing the mean square error. For LMS algorithm, the cost function is defined by

$$V(\mathbf{f}_M) = e^2(n) \quad (8)$$

So the optimum coefficients related to this criterion can be derived as follows:

$$\begin{aligned} \mathbf{f}_M(n) &= \mathbf{f}_M(n-1) + \mathbf{m} \nabla V(\mathbf{f}_M) \\ &= \mathbf{f}_M(n-1) + \mathbf{m} e(n) \frac{\partial \hat{y}(n)}{\partial \mathbf{f}_M} \\ &= \mathbf{f}_M(n-1) + \mathbf{m} e(n) \frac{\partial \hat{y}(n)}{\partial u(n)} \mathbf{R}_M(n) \end{aligned} \quad (9)$$

where,  $\mathbf{m}$  is a positive step size.  $\partial \hat{y}(n) / \partial u(n)$  is the derivative of the HPA output with respect to input. The difference between (9) and traditional LMS algorithm is the factor  $\partial \hat{y}(n) / \partial u(n)$ , which is not known with this structure. Hence LMS algorithm cannot be used, so does RLS algorithm.

#### 3.2 Indirect Structure

Another commonly used adaptive predistortion structure is indirect one, as in Fig.2.

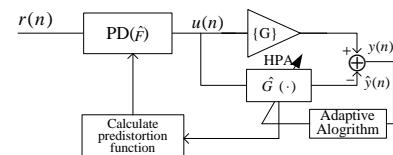


Fig.2. Indirect structure for adaptive predistortion

It presents a two-step method. In the first step, the input and output of the HPA are available so that we can use LMS or RLS to identify the distortion model. In the second step, the parameters of predistorter model can be computed directly [2-3]. Substituting (7) into (6), we obtain

$$\hat{y}(n) = \sum_{i=1}^M g_i (u(n))^i = \sum_{i=1}^M g_i (\sum_{j=1}^M f_i (r(n))^j)^i \quad (10)$$

On the other hand, the desired response is

$$\hat{y}(n) = \mathbf{a} \cdot r(n) \quad (11)$$

Matching the corresponding coefficients of the right-hand side of (10) and (11) and neglecting the higher order terms, we can get the relation of the coefficients of PD with HPA, so the coefficients of the predistorter can be obtained. This method is only efficient for low orders, as the order of nonlinearity grows higher, so does the computational complexity. Furthermore, if the “truncation” effect of the distortion model were severe, the solution bias would arise.

For the second step, another method for computing the parameters of predistorter is adaptively updated. The advantage of this structure over that of the direct structure is that the gradient of the HPA distortion – required in (9) – is available now, since the transfer function of HPA has been estimated. It implies that LMS algorithm can be used, but the computation of  $\partial \hat{y}(n) / \partial u(n)$  is complicated, particularly when we consider the memory effect of the HPA. Furthermore, RLS algorithm still cannot be used in this structure. According to the development of RLS algorithm, the cost function  $V(\mathbf{f}_M)$  is defined by

$$V(\mathbf{f}_M) = \sum_{k=0}^M \mathbf{I}^{n-k} [e^2(n)] \quad (12)$$

where,  $\mathbf{I}$  is the forgetting factor. The optimum parameters of predistorter model are obtained by setting the gradient of  $V(\mathbf{f}_M)$  equal to 0, i.e.,

$$\begin{aligned} \nabla V(\mathbf{f}_M) &= -2 \sum_{k=0}^M \mathbf{I}^{M-k} e(n) \frac{\partial \hat{y}(n)}{\partial \mathbf{f}_M} \\ &= -2 \sum_{k=0}^M \mathbf{I}^{M-k} (d(n) - G(\mathbf{R}_M^T(n) \mathbf{f}_M(n-1))) \frac{\partial \hat{y}(n)}{\partial u(n)} \mathbf{R}_M(n) \\ &= 0 \end{aligned} \quad (13)$$

It is obvious that the coefficients  $\mathbf{f}_M$  cannot be solved from (13), so the RLS algorithm cannot be used.

#### 4. PROPOSED STRUCTURE

From above analysis, we found that the efficient LS algorithm can not be used in previous structures just due to the fact that the expired output of predictor is unknown, so does the gradient information. So, we construct a new predistorter structure shown in Fig.3.

The proposed scheme is a two-step process, too. First, the difference between  $u(n)$  and  $x(n)$  is used to adapt the parameters of  $\hat{P}$ , the model of another “inverse” of HPA distortions. Compared to the predistorter,  $\hat{P}$  is called as “postdistorter”. Here, we must emphasize that

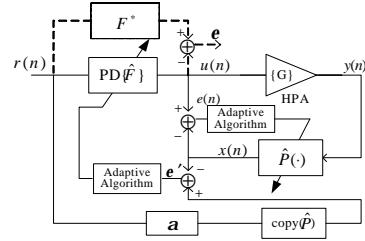


Fig.3. Proposed structure for adaptive predistortion

$\hat{P}$  is different from the predistorter model  $\hat{F}$  for nonlinear filters cannot be permute, i.e.,  $\hat{P}(G(\cdot)) \neq G(\hat{P}(\cdot))$ . It is obvious that the input observation and desired output responses of  $\hat{P}$  are available so that LMS or RLS algorithm can be used directly. Second, the desired system responses  $\mathbf{a} \cdot r(n)$  are filtered by the copy of  $\hat{P}$ , and the results act as the desired output of  $x(n)$ . The output error  $\mathbf{e}'$ , supposed as the difference of the output of ideal predistorter and that of the estimated model, are used to identify the parameters of predistorter model  $\hat{F}$ .

To verify the correctness of this assumption, we consider that an ideal predistorter  $F^*$  exists, as dashed line depicted in Fig.3, and then the output error  $\mathbf{e}$  can be used to identify the model  $\hat{F}$  easily.  $\mathbf{e}$  can be expressed by:

$$\mathbf{e} = F^*(r(n)) - \hat{F}(r(n)) \quad (14)$$

The ideal predistorter  $F^*$  is defined by:

$$G(F^*(r(n))) = \mathbf{a} \cdot r(n) \quad (15)$$

From fig.3, we have

$$\mathbf{e}' = \hat{P}(\mathbf{a} \cdot r(n)) - \hat{P}(G(\hat{F}(r(n)))) \quad (16)$$

Substituting (15) into (16), we get:

$$\mathbf{e}' = \hat{P}(G(F^*(r(n)))) - \hat{P}(G(\hat{F}(r(n)))) \quad (17)$$

From the fist step of our proposed scheme, we can approximate  $x(n)$  as

$$x(n) = \hat{P}(G(u(n))) = u(n) \quad (18)$$

Thus, we may express the equation (17) as follows:

$$\mathbf{e}' = F^*(r(n)) - \hat{F}(r(n)) = \mathbf{e}$$

Therefore, It is reasonable that  $\mathbf{e}'$  are considered as  $\mathbf{e}$  to estimate the parameters of predistorter model, and many efficient LS adaptive algorithms can be used directly.

#### 5. SIMULATION RESULTS

In this section, a computer simulation is used to dem-

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onstrate the validity of the proposed predistortion structure for compensating of the nonlinear HPA. The commonly used saleh model is used for HPA [7]. The AM/AM distortion introduced by the amplifier is depicted with dotted line in Fig.4.

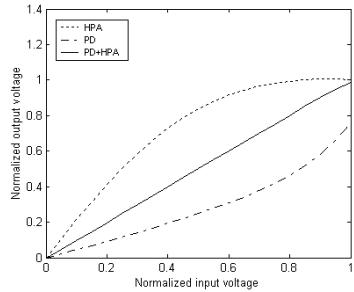


Fig.4 AM/AM characteristics of amplifier, predistorter and predistorter-amplifier

Proposed structure is applied to 16-QAM input signals with a maximum envelope value of 1, which is equal to the maximum saturated output voltage. The shaping filter is a raised-cosine filter with roll-off factor equal to 0.5. The order of postdistorter and predistorter all are set to seven with only odd terms. The desired linear model  $\mathbf{a}$  is chosen to 1.

Considering that the eigenvalues spreads are very large for nonlinear filter, RLS algorithm is employed in our simulation. In fig.4, the dash-dot line represents the amplitude response of the resulting predistorter, and the solid line represents the composite response of the PD+HPA. In fig.5, we plot the spectra of the transmitted signal in the cases with and without predistorter and the ideal case.

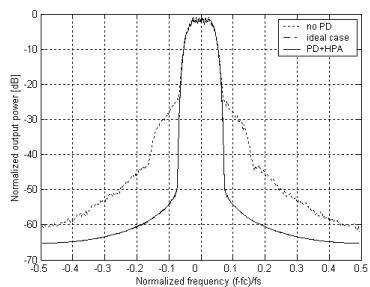


Fig.5 Power spectrum of filtered 16-QAM data signal with and without predistortion (fc: carrier frequency; fs: sample frequency)

The figure shows that the transmitted spectrum with linearisation follows same to that in the ideal linear case and spectrum spreading was reduced about 30dB compared to that without predistortion.

## 6. CONCLUSION

Previously mentioned predistortion structures are not suitable for efficient LS adaptive algorithms. This is because that the desired output of the predistorter is unknown, so does the error's gradient information in. To apply the efficient LS algorithms, we present a two-step predistortion structure where the postdistorter is estimated first, and then the predistorter is identified from the output error, which is the difference of the output of the postdistorter and that of desired system responses filtered by the copy of the postdistorter's model. For these twice-adaptive processes, the efficient adaptive algorithms, such as LMS and RLS, can be used directly. The feasibility of the proposed scheme is approved by algebra analysis. A computer simulation based on the memoryless polynomial model and RLS adaptive algorithm is also demonstrated the validity of the scheme.

Furthermore, it is important to note that although the discussion of this paper is based on the polynomial model of HPA and LMS, RLS algorithm, the conclusions are also suited for other nonlinear models and some other efficient LS adaptive algorithms.

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