A PIECEWISE PARAMETRIC METHOD BASED ON POLYNOMIAL PHASE MODEL TO COMPENSATE IONOSPHERIC PHASE CONTAMINATION

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ABSTRACT

This paper addresses a parametric method based on High-Order Ambiguity Function (HAF) to solve the problem of phase contamination of HF skywave radar signals corrupted by the ionosphere. When signal-to-noise ratio and data sequence available satisfy the predefined conditions, the ionospheric phase contamination may be modeled by the polynomial phase signal. As a new parametric tool for analyzing polynomial phase signal, HAF is applied to estimate polynomial phase model parameters and reconstruct the disturbance signal. Using the estimated reconstructed signal, compensation can be performed before coherent integration and the original radar return spectrum can be restored. A piecewise scheme is proposed to track rapid variation of the phase contamination in HAF method, and it can remove the Doppler spread effect caused by the ionosphere nonstationarity. Simulation is used to demonstrate the efficiency of the proposed method.

1. INTRODUCTION

High-frequency skywave over-the-horizon radar can provide a range-coverage of up to 4,000km by means of the refraction within the ionosphere. But the signal contamination suffered in double ionospheric transit. Especially, Doppler spread mechanism renders the sea clutter spectrum distorted and resolution of coherent integration technique that is widely applied in the radar systems is degraded extremely [1].

The echo signal reflected by the sea surface has a pair of peaks in the Doppler domain, which is often known as the Bragg lines. In some applications of HF skywave radar such as remote sensing and surface surveillance, temporal nonlinear phase path variation often produces significant spread of the ionoshpheric propagated signals so that the Bragg lines and target echoes smear cross the Doppler frequency domain. Since the energy of the Bragg lines is so strong that slight spread of the first-order sea clutter spectrum can obscure the neighboring echo scattered by a slow moving surface vessels. The phase contamination is attributed to complex geophysical mechanisms [2,3].

To limit the Doppler spread effect and allow extended coherent integration time for good frequency resolution, it is necessary to estimate and compensate the raw radar signal by signal processing techniques before coherent integration. By virtue of the nonstationarity of the phase perturbation, instantaneous frequency estimation methods are introduced to solve this problem. Bourdillon and Parent *etc. al* have suggested some methods such as MESA (Maximum Entropy Spectrum Analysis) and energy weighted average phase differential estimator [4, 5, 6].

In this paper a piecewise parametric phase estimator based on high-order ambiguity function is proposed. HAF can identify the degree of the phase polynomial and estimate its coefficients. HAF-based estimator has some attractive properties, such as computationally efficiency, robustness for deviations from the signal model to some extent, and estimation accuracy that is very close to the CRB [7]. Due to these attributes we can realize an instantaneous phase estimation algorithm, which is used to compensate the phase of the modulation.

2. HIGH-ORDER AMBIGUITY FUNCTION

In this paper, we consider the following model for a complex sampled signal $s(n)$

$$s(n) = b_0 \exp\left\{ j \sum_{m=0}^{M} a_m (n\Delta)^m \right\} \quad (1)$$

where $0 \le n \le N-1$, N is sample number, Δ is the sample interval and coefficients a_m are real. That is a model with constant amplitude and polynomial phase of order M.

The M-th order high-order ambiguity function HAF_M is defined as the discrete Fourier transform of high-order instantaneous moment HIM_M. Let $s(n)$ be the form of (1). The operators $\mathrm{HIM}_1[s(n),\tau]$ and $\mathrm{HIM}_2[s(n),\tau]$ are defined by [8]

$$\mathrm{HIM}_1[s(n),\tau] = s(n) \quad (2)$$

$$\mathrm{HIM}_2[s(n),\tau] = s(n)s^*(n-\tau) \quad (3)$$

where the delay parameter τ and the order of the operator M are positive integers. And higher-order operators are defined by

$$\mathrm{HIM}_M[s(n),\tau] = \mathrm{HIM}_2[\mathrm{HIM}_{M-1}[s(n),\tau],\tau]. \quad (4)$$

Here the asymmetric definition is used. The following two lemmas can be resulted from these definitions and are useful for the subsequent development.

Lemma 1:

$$\mathrm{HIM}_M[s(n),\tau] = \prod_{q=0}^{M-1}\left[s^{\$_q}(n-q\tau)\right]^{\binom{M-1}{q}} \quad (5)$$

where

$$s^{\$_q}(n) = \begin{cases} s(n), & \text{q is even,} \\ s^*(n), & \text{q is odd,} \end{cases} \quad (6)$$

Lemma 2:

$$\mathrm{HIM}_M[s(n),\tau] = \mathrm{HIM}_2[\mathrm{HIM}_2[\cdots \mathrm{HIM}_2[s(n),\tau],\cdots,\tau],\tau] \quad (7)$$

where the operator HIM_2 is applied M-1 times.

The following theorem shows that applying the operator of order M to a constant-amplitude polynomial-phase signal of the same order transforms the broadband signal into a single tone with a frequency related to the *m*-th order coefficient a_m.

Theorem 1: Let s(n) be a polynomial-phase signal of order M given in (1). Then, for all positive integers τ

$$\mathrm{HIM}_M[s(n),\tau] = \exp\{j(\omega_0 n\Delta + \varphi_0)\} \quad (8)$$

where $(M-1)\tau \le n \le N-1$ and

$$\omega_0 = M!(\tau\Delta)^{M-1}a_M \quad (9)$$

$$\varphi_0 = (M-1)!(\tau\Delta)^{M-1}a_{M-1} - 0.5(M-1)M!(\tau\Delta)^M a_M \quad (10)$$

The operator HIM_M has the following properties:

Property 1:

$$\mathrm{HIM}_M[s_1(n)s_2(n),\tau] = \mathrm{HIM}_M[s_1(n),\tau]\mathrm{HIM}_M[s_2(n),\tau]. \quad (11)$$

Property 2: For every real b_0

$$\mathrm{HIM}_M[b_0,\tau] = b_0^{2^{M-1}}. \quad (12)$$

Property 3: For $K = 0,1,\ldots,M-2$

$$\mathrm{HIM}_M\left[\exp\left\{j\sum_{m=0}^{K}a_m(n\Delta)^m\right\},\tau\right] = 1. \quad (13)$$

Property 4:

$$\mathrm{HIM}_M\left[\exp\left\{j\sum_{m=0}^{M-1}a_m(n\Delta)^m\right\},\tau\right] = \exp\{j\varphi_0\} \quad (14)$$

where $\varphi_0 = (M-1)!(\tau\Delta)^{M-1}a_{M-1}$.

The proofs of these properties are given in literature [8]. From property 3 and 4 it can be concluded that applying the operator of order M to a polynomial-phase signal of order lower than M, will produce a constant.

Then the high-order ambiguity function of order M can be described as follow:

$$\mathrm{HAF}_M[s(n),\omega,\tau] = \sum_{n=(M-1)r}^{N-1}\mathrm{HIM}_M[s(n),\tau]\exp\{-j\omega n\Delta\} \quad (15)$$

So we can conclude that $\mathrm{HAF_M}$ of polynomial-phase signal of the same order will produce a single spectral line at non-zero frequency. By locating this line and estimating its frequency, we can obtain an estimate of the highest order polynomial coefficient. And applying $\mathrm{HAF_M}$ to a polynomial-phase signal with order lower than M will result a spectral line at zero frequency (DC).

3. IONOSPHERIC PHASE DECONTAMINATION ALGORITHM

In this section, an ionospheric phase decontamination algorithm is addressed. Unlike those methods based on instantaneous frequency estimation techniques, the proposed algorithm is phase estimation method. It is not necessary to estimate the instantaneous frequency of the nonstationary phase distortion and then integrate the phase of the correction signal to compensate. The phase of correction signal can be estimated in a straightforward way.

In practice, the first step is always to determine whether the observed signal is phase-contaminated or not. If the ionosphere is stationary, it is obvious that the Bragg lines locate at the fixed frequencies in the Doppler domain as described by Barrick [9]. Therefore, if the Bragg lines are not at the expected frequencies, we can conclude by and large that the phase modulation exists.

Data filtering before estimation is an indispensable stage in most existing algorithms to remove the unexpected component. The filter bandwidth f_B must be chosen carefully and it must be narrow enough to reject unwanted components but large enough to contain the component of interest. Generally the frequencies of the Bragg lines are used as the center frequency of filtering because they are much stronger than ordinary target echoes. But under some special conditions, other prominent echoes such as the backscatter of island and big surface vehicle can also be applied [2].

Now, filtered data can be applied in estimation of the phase contamination function. Using HAF method, the highest order used must be determined firstly. The operating range of the HAF algorithm is defined by [8]

$$MK(M,SNR) \le \frac{N}{25} \quad (16)$$

$$K(M,SNR) = \prod_{q=0}^{M-1}\left[\sum_{i=0}^{\binom{M-1}{q}}\left(\binom{\binom{M-1}{q}}{i}\right)^2 i!\left(\frac{1}{SNR}\right)^i\right] - 1 \quad (17)$$

From (16) and (17), the estimation of higher order parameter has higher signal-to-noise ratio requirement.

For the radar return from a certain range bin, there is not enough signal-to-noise ratio to perform the estimation of higher polynomial phase parameters. Through computation from expression above and a number of experiments, a model whose order is no more than five has been used.

The delay parameter τ in (3) is an important factor affecting the error accuracy. Care must be taken in selecting the parameter because it is limited by

$$\tau^{M-1} \le \frac{\pi}{M!\,|a_M|\Delta^M} \tag{18}$$

and

$$\tau \le \frac{N}{M-1} \tag{19}$$

Formula (18) can be seen as the constraint of Nyquist criterion from (9) while formula (19) can be derived from Lemma 1 where $n - q\tau$ cannot exceed N.

The HAF-based phase decontamination algorithm can be described as:

1) Initializing the observed signal by letting $m = M$ and the m-th iteration sequence $z^m(n) = y(n)$ where y(n) is the data sequence received from the sensor array;
2) Selecting a proper delay parameter τ_M and computing the M-th order polynomial parameter by $\hat{a}_m = \arg\max_{\omega}\left\{DPT_m\left[z^m(n), \omega, \tau_m\right]\right\} \Big/ \left(m!\,(\tau_m\Delta)^{m-1}\right)$;
3) Getting the *(m-1)*-th iteration sequence by $z^{m-1}(n) = z^m(n)\exp\left\{-j\hat{a}_m (n\Delta)^m\right\}$;
4) Letting $m = m - 1$ and turning step 2) to execute until $m < 1$;
5) Estimating the parameter a_0 and b_0 by ML method, $\hat{a}_0 = \arg\left\{\sum_{n=0}^{N-1} z^0(n)\right\}$ and $\hat{b}_0 = \left|\sum_{n=0}^{N-1} z^0(n)\right| \Big/ N$.

4. EXPERIMENTAL RESULTS

To illustrate the algorithms clearly, in the experiments we use a sinusoid phase model as the contamination function, which is defined as

$$G(n) = b_0 \exp\{-j\beta \sin(2\pi\gamma n)\} \quad 0 \le n \le N-1 \tag{20}$$

where b_0 is an amplitude constant, β is the modulation index, and γ is the sinusoid modulation frequency. The Doppler spread due to the phase model can be classified to two divisions: one is primary caused by the modulation index β and the other by γ. The first kind of spread mechanism is easier estimated on account of the slowly fluctuating phase. The Doppler spread due to the change of sinusoid modulation frequency can be eliminated by segmenting the data sequence to a few short sub-sequences and then estimating them respectively [10].

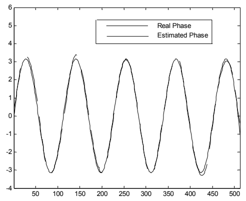
Fig.1 illuminates the resulting spectrum from the corruption model of (20) with $b_0 = 1$, $\beta = \pi$, and $\gamma = 0.02$. In this test, the filtering bandwidth f_B is 0.18Hz, the delay parameter τ is N/M for M=2,3 and N/(M+2) for $M \ge 4$, the highest order M is 3, the segment number is 16, the FFT length in HAF algorithm is 1024 (by zero-padding), and the land clutter at DC is applied as the filter central frequency criterion. The forward and backward sliding window average method is also used in data sequence segmenting and the data overlapped rate is 50%. The estimation of the phase contamination function is given in Fig.1 (a). It should be noted that at the cross point of two adjacent segments the accuracy is worse than other region. The error at the terminals of the data sequence depends on the estimation accuracy attribute of the polynomial phase model, and the detail has been described in [11].

The significant Doppler spread is observed in Fig.1 (b), and the sea and land clutter dominated in energy can easily smear across the neighboring weak target. Under the condition with considerable spread, target detection is difficult to be accomplished. The corrected spectrum after estimation and compensation using HAF method is shown in Fig.1 (c). It can be observed that the spread spectrum peaks have been sharpened clearly and the target close to the positive first-order Bragg line can be visualized. Obviously the probability to detect the surface vehicle will increase.

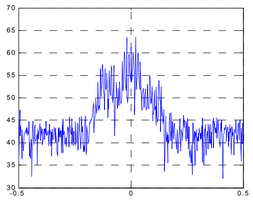
The error accuracy variation with the segment number is given in Fig.2 (a). When SNR is constant, the error gets to a minimum at about 16 with different highest orders. A consideration should be noted that it is not necessary to divide the observed sequence to several segments. Only when the Doppler spread due to the phase fast fluctuation dominates in the corrupted spectrum, segmenting becomes an effective scheme. Derived from (16), the validity of the HAF method is related to SNR, available sample number N and the highest order M of HAF. If the segment number becomes large and SNR and M are not changed, the data length must be short, and which would damage the estimation performance of the HAF algorithm. However, long data length cannot show the detail of the phase contamination, especially variation at short interval. There is a trade-off between tracking the non-stationarity of the ionosphere and achieving good estimation accuracy.

A model with higher order can approximate the real continuous phase more accurately, but it requires more SNR, or else the HAF-based method would fail. When the phase contamination function fluctuates slowly, a lower order model may be sufficient.

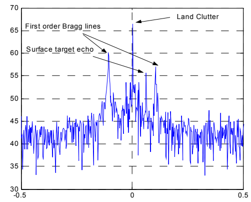
The error accuracy variations with the parameters shown in (20) are given in Fig.2 (b) and (c) respectively.



(a) estimated phase



(b) spectrum before correction



(c) spectrum after correction

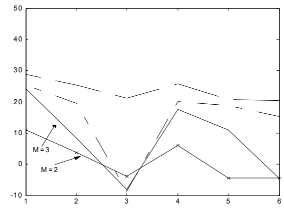
Fig.1 (a) estimated phase; (b) spectrum before correction; (c) spectrum after correction

From these figures, we can conclude that the HAF-based method is more efficient for β - type spread than for γ - type case. With the increasing of the parameter γ, the error sum increases also and when $\gamma > 0.03$, the error sum is too large to estimate the phase. And even the parameter β is over 2π and $\gamma = 0.01$ (where the spread is much wider than the spread with $\gamma = 0.03$ and $\beta = \pi$), the estimated result can still be accepted.

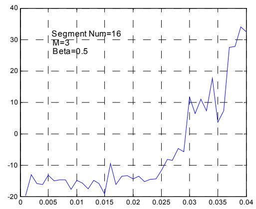
5. CONCLUSION

It is shown in the paper that the parametric method that modeling and estimating the ionospheric phase contamination by polynomial phase signal is valid. When the whole phase modeling cannot be realized, a segment scheme may be applied. A trade-off relation is discussed on the choice of parameters, and after choosing good estimation accuracy can be achieved. The spectrum quality after compensation is improved significantly.

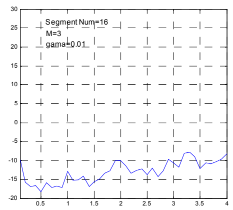
The variation of parameters of the phase contamination model is also discussed and corresponding analysis is given.



(a) Error sum Vs. segment number(log2(N)-1)



(b) Error sum Vs. γ



(c) Error sum Vs. β

Fig.2 (a) error accuracy with segment number (dashed line for M=5, dot-dashed line for M=4, solid line for M=3 and x-marked line for M=2); (b) error accuracy with parameter γ ($\beta = \pi$, M=3, and segment number is 16); (c) error accuracy with parameter β ($\gamma = 0.01$, M=3, and segment number is 16)

6. REFERENCES

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