GEARBOX FAULT DIAGNOSIS USING INDEPENDENT COMPONENT ANALYSIS IN THE FREQUENCY DOMAIN AND WAVELET FILTERING

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ABSTRACT: In this paper, we combine independent component analysis in the frequency domain (ICA-FD) and Morlet wavelet filtering for gearbox fault diagnosis. Collected vibration signals from a gearbox are separated into two components with ICA-FD. Morlet wavelet filtering is then applied to the separated components. The optimal shape parameter β of the basic Morlet wavelet is obtained by minimizing the wavelet entropy. Better diagnosis results are obtained with this combination than using wavelet filtering alone.

1. INTRODUCTION

Vibration signals from equipment form a time series. Various analysis techniques have been used on such a time series for fault detection of the equipment. Kurtosis may give a crude indication of defects. Fourier Transform has been used extensively for identification of different types of faults in equipment. When different faults give rise to harmonics and/or sidebands, cepstrum and envelope are more effective methods.

Independent Component Analysis (ICA) or Blind Source Separation (BSS) is a recently developed method of signal processing. It has been used for processing of mixtures of simultaneous speech signals that have been picked up by several microphones, brain waves recorded by multiple sensors, interfering radio signals arriving at a mobile phone, or parallel time series obtained from an industrial process [1]. For speech signals, Torkkola [2] applied the information theoretic approach proposed by Bell and Sejnowski [3] to convolutive mixtures. Lee *et al*. extended this approach [4, 5]. Smaragdis applied the information theoretic approach to audio signal separation in the frequency domain [6]. However, there exists an inherent permutation problem in applying ICA in the frequency domain (ICA-FD). The problem is that permutations of the separated sources all generate the same observed signals. Mitianoudis *et al.* used a nonstationary time varying scale parameter to solve this permutation problem [7]. For complex valued signals, Bingham and Hyvarinen proposed a fast fixed-point algorithm for ICA [8].

For application of ICA on rotating machine, Ypma *et al.* used a bilinear forms based convolutive mixture model for blind source separation of two coupling machines [9]. Gelle *et al*. applied BSS to acoustical and vibration analysis of rotating machines [10, 11]. Li *et al.* [12] studied noise signals from diesel engines using ICA. Lin and Zuo [13] applied ICA to fault diagnosis through feature separation for one dimensional time series.

2. INDEPENDENT COMPONENT ANALYSIS

Humans can focus their attention on a single source and discern other sound sources out of a mixture. This was termed the "cocktail party effect" by Cherry [14]. The ICA technique was developed to deal with the "cocktail party" problem. It is a statistical method for transforming the obtained multichannel signals into components that are statistically independent from each other as much as possible [15, 16].

2.1 ICA in the Time Domain [16]

Assume that we have collected two signals in the time domain, $x_1(t)$ and $x_2(t)$, from two separate sensors. Each of the two observed signals is a combination of two source signals, $s_1(t)$ and $s_2(t)$. Without considering the time delays for the source signals to reach the sensors, we have

$$\mathbf{x} = \mathbf{A}\mathbf{s}, \tag{1}$$

where $\mathbf{x} = (x_1, x_2)^T$, $\mathbf{s} = (s_1, s_2)^T$, $\mathbf{A}$ is the matrix representing the transform relationship between **s** and **x**. Hence, we can obtain:

$$\mathbf{s} = \mathbf{A}^{-1}\mathbf{x}. \tag{2}$$

Let $\mathbf{y} = \mathbf{W}\mathbf{x}$. If we can find **W** and make $\mathbf{W} = \mathbf{A}^{-1}$, then $\mathbf{y} = \mathbf{s}$. Now, the problem to be solved is how to find the matrix **W**. According to the fixed-point algorithm proposed by Hyvärinen [16], the following equations are used:

$$\mathbf{w}^{+} = \mathbf{w} - \lambda[E\{\mathbf{x}g(\mathbf{w}^T\mathbf{x})\}-\eta\mathbf{w}]/[E\{g'(\mathbf{w}^T\mathbf{x})\}-\eta], \tag{3}$$

$$\mathbf{w}_{new} = \mathbf{w}^{+}/\|\mathbf{w}^{+}\|, \tag{4}$$

$$\eta = E\{\mathbf{w}^T\mathbf{x}g(\mathbf{w}^T\mathbf{x})\}, \tag{5}$$

where **w** is the vector that makes up the row vector of matrix **W**; $\mathbf{w}^T$ is the transpose of **w**; $E\{.\}$ is the mathematical expectation; g(.) the derivative of $G(.)$; and λ is a step size parameter that is updated with each iteration. The initial value of the parameter λ is selected to be 1 first and decreased gradually (λ/10) until convergence is satisfactory. The function form G is selected to be $\log(0.1+|\mathbf{w}^T\mathbf{x}|^2)$ in this paper. Uniformly distributed random numbers are generated and used as the initial elements of **w** in this paper. When the largest element of $||\mathbf{w}^+|-|\mathbf{w}||$ is less than 0.0001, we stop the iteration and search for the next row vector **w** of matrix **W**. This is how the matrix **W** is obtained.

2.2 ICA in the Frequency Domain [8]

In the time domain, the ICA technique can solve the instantaneous mixing problem. However, this method does not work for the convolved mixing problem in the time domain, which is closer to real problem. According to the theory of Fourier transform, convolved mixing in the time domain corresponds to instantaneous mixing in the frequency domain. Hence, the ICA method has been used in the frequency domain. The difference is that the real valued signals in the time domain are changed to complex valued signals in the frequency domain. Accordingly, the algorithms can be expressed as:

$$\mathbf{w}^+ = E\{\mathbf{x}(\mathbf{w}^H\mathbf{x})^* g(|\mathbf{w}^H\mathbf{x}|^2)\} - E\{g(|\mathbf{w}^H\mathbf{x}|^2)+|\mathbf{w}^H\mathbf{x}|^2 g'(|\mathbf{w}^H\mathbf{x}|^2)\}\mathbf{w} \quad (6)$$

$$\mathbf{w}_{new}= \mathbf{w}^+/||\mathbf{w}^+||, \quad (7)$$

where $\mathbf{w}^H$ stands for the Hermitian of **w**, that is, **w** transposed and conjugated. The iterative procedure for refining the **w** vectors is the same as that in the time domain. The procedure for ICA-FD is as follows.

- Performing FFT of the inputs **x**(t) and obtaining **X**(f);
- Using learning rule (6) to find the unmixing matrix **W;**
- From **W**, we can obtain the estimated **S**(f);
- Performing Inverse Fast Fourier Transform (IFFT) of **S**(f) and obtaining the estimate of the original signals **s**(t);

3. WAVELET ANALYSIS

In contrast to the Fourier transform, the wavelet transform breaks the signal into its "wavelets", scaled and shifted versions of the mother wavelet. In comparison to the sine wave which is smooth and of infinite length, the wavelet is irregular in shape, which makes wavelets an ideal tool for analyzing signals of a non-stationary nature. Their local property lends them to analyzing signals with sharp changes. The translation and dilation operations applied to the mother wavelet are performed to calculate the wavelet coefficients, which represent the correlation between the wavelet and a localized section of the signal. The wavelet coefficients are calculated for each wavelet segment, giving a time-scale function relating the wavelets correlation to the signal.

In the field of rotating machine fault diagnosis, researchers use wavelet transform to analyze faults in gearbox and rolling element bearings [17,18,19]. In wavelet family, Morlet wavelet is often used in the gearbox diagnosis because it is comparatively alike to the impulse symptoms of gearbox faults. A Morlet wavelet is defined as

$$\psi(t)=\exp(-\beta^2 t^2/2+j\pi t) . \quad (8)$$

Considering the real part, we can use the following basic wavelet

$$\psi(t)=\exp(-\beta^2 t^2/2)\cos(\pi t). \quad (9)$$

A daughter wavelet can be obtained by using the scale parameter a and shift parameter b.

$$\psi_{a,b}(t)=\exp[-\beta^2(t-b)^2/(2a^2)]\cos[\pi(t-b)/a] \quad (10)$$

Lin *et al* [18] used *wavelet entropy* to search for the optimal shape parameter β of the basic Morlet wavelet. The wavelet entropy can be calculated by

$$En = -\sum d_i \log d_i , \quad (11)$$

where $d_i=c_i/\sum c_j$, $\{c_i\}_{i=1\sim M}$ is the class of the wavelet coefficients, $c_i=\psi(t)*x(t)$, i.e. the convolution between signal x(t) and the wavelet function $\psi(t)$. The steps need to find the optimal β is as follows.

Step 1 Selecting the initial $\beta = 0.1$;
Step 2 Calculating the wavelet entropy En ;
Step 3 Let $\beta=\beta+\eta$, the step length $\eta = 0.1$;
Step 4 Repeating *step 2* and *step 3*, if $\beta = 20$, go to *step 5;*
Step 5 Finding the minimal En.

4. FAULT DIAGNOSIS WITH BOTH ICA-FD AND WAVELET FILTERING

4.1. Gearbox Fault Analysis with Wavelet Filtering only

A gearbox is a typical rotating machine. Gear tooth failure is the main type of fault in a gearbox. The signals from a gearbox are often complicated since there are many components in a gearbox. Some traditional methods for signal processing such as FFT have been used together with wavelet filtering.

In our experiment, we used two accelerometers to obtain signals from a gearbox dynamics simulator. The experimental setup is shown in Figure 1. Damage on one of the teeth of the gear on shaft 1 was introduced. The rotating frequency of shaft 1, f_{r1}, is 20Hz. The signals were sampled at 5120Hz. The rotating frequency of shaft 2 is 6.67Hz. The calculated gearmesh frequency, f_{m1}, of gears a' and b should be 320Hz. For a gear with a damaged tooth, the gearmesh frequency modulated by shaft rotating frequency should be observed [20]. Figure 2 shows the experimental results in the frequency domain.

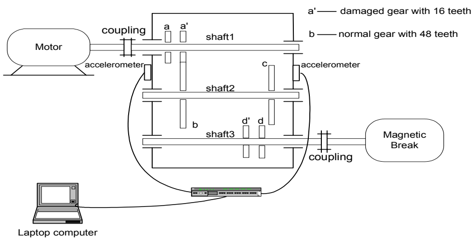

Figure 1 Gearbox Dynamics Simulator

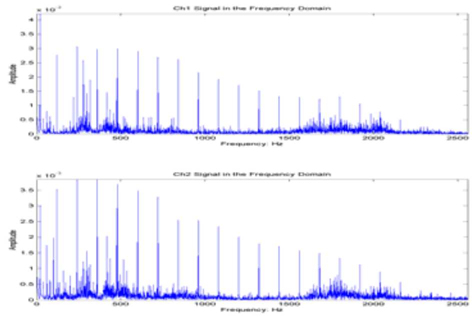

Figure 2 The spectrum plots in Channel 1 and Channel 2

From Figure 2, we can observe that multiples of 120Hz are distinctive. However, the source of this symptom is unknown to us. We are unable to find any indication of the introduced tooth damage on the gear on shaft 1. The spectrum plots after Morlet wavelet filtering are shown in Figure 3. We got the optimal shape parameter $\beta = 0.3$ by minimizing wavelet entropy. We selected the scale parameter $a = 2^n$ with n = 0,1,2,3,4,5.

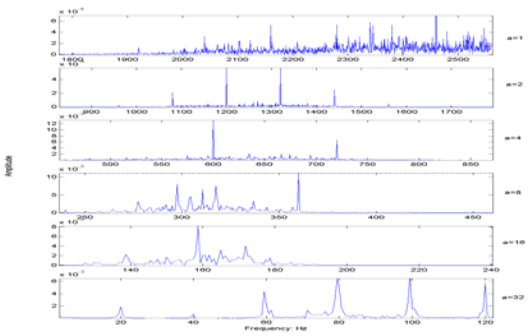

Figure 3 The spectrum plots after Morlet wavelet filtering

In the subplot a=32 of Figure 3, we can observe multiples of shaft 1 rotating frequency k*20Hz for k=1,2,3,4,5,6. In the subplot a=8, the peak spectral line is 360Hz. We expect to see some obvious indications of the gear with a damaged tooth, i.e., the gearmesh frequency with sidebands. However, they do not exist in Figure 3 either.

4.2. Gearbox Fault Analysis with both ICA-FD and Wavelet Filtering

Now, we consider performing ICA-FD first followed by wavelet filtering since the signals are complicated. We hope to be able to separate complicated signals from two channels into two relatively simple signals. Then we hope that one of them can provide us more distinctive information about the fault of gear tooth damage. Figure 4 shows the results after performing ICA in the frequency domain.

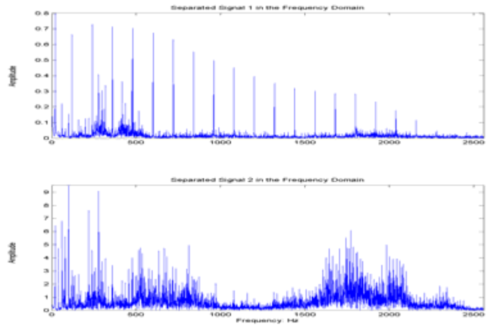

Figure 4 The spectrum plots after performing ICA-FD

The signals are separated into two components. The separated signal 1 contains mostly components of k*120Hz. We used Morlet wavelet filtering to each of these two separated signals. The result from signal 1 is not indicative of the introduced fault at all. The results from the separated signal 2 are shown in Figure 5.

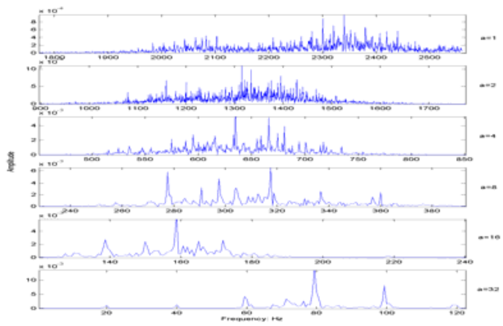

Figure 5 The spectrum plots after performing ICA-FD and Morlet wavelet filtering

In subplot a=8 of Figure 5, we can observe that the peak spectral line is 317.5Hz, which is very close to the calculated gearmesh frequency of 320 Hz. The other two distinctive spectral lines are 277.5Hz and 297.5Hz, which are $f_{m1} - 2f_{r1}$ and $f_{m1} - f_{r1}$, respectively, where f_{m1} is the gear mesh frequency of the gear on shaft 1 and f_{r1} is the rotating frequency of shaft 1. We can also observe two spectral lines at 337.5Hz and 357.5Hz on the right hand side of 317.5Hz. They are $f_{m1}+f_{r1}$ and $f_{m1}+2f_{r1}$, respectively. The symptom of a gear tooth fault, the gearmesh frequency modulated by the rotating frequency, is obviously shown in this subplot of a = 8. This shows that using both ICA-FD and wavelet filtering is more effective for identifying this fault than using wavelet filtering alone.

5. CONCLUSIONS

In this paper, the ICA in the frequency domain and Morlet wavelet filtering are used in combination to analyze the vibration signals from a gearbox simulator. A comparison was carried out between the results of Morlet wavelet filtering alone and with the combined method. The results indicate that the ICA technique in the frequency domain is promising for fault diagnosis of a rotating machine such as a gearbox together with other analysis techniques. However, it is still inconclusive and much further work is needed for the industrial application. For rotating machines, the signals from rotating components are often complicated. The combination applications of several signal-processing techniques can provide us more and distinctive fault information.

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