A Consideration on the Blind Estimation of Single-Input Double-Output System Using Orthogonal Direct Sum Decomposition of Received Signal Space

Nari TANABE†, Toshihiro FURUKAWA††, Shigeo TSUJII†††

†Graduate School of Information and System Engineering, Chuo University
1-13-27, Kasuga, Bunkyo-ku, Tokyo, 112-8551, Japan

††Department of Management Science, Faculty of Engineering, Science University of Tokyo
1-3, Kagurazaka, Shinjuku-ku, Tokyo, 162-0825, Japan

†††Department of Information and System Engineering, Faculty of Science and Engineering, Chuo University
1-13-27, Kasuga, Bunkyo-ku, Tokyo,112-8551, Japan

ABSTRACT

The subspace methods with second-order statistics based on PCA basically need to calculate the eigen-values and the eigen-vectors of the autocorrelation matrix of the received signal. However, the calculation of the eigen-values and the eigen-vectors of the matrix requires much computational complexity. In this paper, we propose a new algorithm based on PCA without solving the eigen-values and the eigen-vectors of the matrix. Moreover, we perform the proposed method under the condition that noise-variance is known, but we confirmed that the proposed method is effective to a certain degree when noise-variance is unknown. We show the effectiveness of the proposed method by numerical examples.

Key Words

PCA, eigen-value and eigen-vector, computational complexity

1. INTRODUCTION

On the blind identification based on second statistics, much method has been proposed in recent year[1][2]. In this paper, we propose a new algorithm based on Principal Component Analysis (PCA).

The method using PCA can get the impulse response vector with the fact that the signal-subspace is orthogonal to the noise subspace [2].

However, the method basically needs to calculate the eigen-values and the eigen-vectors of the autocorrelation matrix of the received signal. The calculation of the eigen-values and the eigen-vectors causes the increase of computational complexity, both the degradation of the accuracy and the convergence speed.

In this paper, we discuss a new algorithm to get the impulse response vector without solving the eigen-values and the eigen-vectors of the matrix with the estimation of the autocorrelation matrix of the received signal vector. This paper discusses the Signal-Input Double-Output (SIDO) system in which the sampling rate of the output signal is as twice as that of input one.

The proposed method is based on both the structural relationship between a matrix and a vector, and the fact that the subspace spanned by the column vectors of the impulse response matrix (the impulse response subspace) is equivalent to the column-space of the matrix, which is obtained by subtracting the autocorrelation matrix of the received signal vector from the diagonal matrix with its non-zero elements being the noise-variance.

This proposed method consists of the following: 1) With the orthogonality between the impulse response subspace and the noise-subspace, we will get the noise vector, which can be obtained by subtracting the vector projected onto the impulse response subspace of the received signal vector from that. 2) This noise vector has a relationship with the noise matrix. Hence, the noise matrix is composed of this noise vector. 3) Due to the orthogonality between the column-space of the noise matrix and the impulse response vector, we will get the desired impulse response vector, which can be obtained by subtracting the vector projected onto the column-space of the noise matrix of the received signal vector from that.

We perform the proposed method under the condition that noise-variance is known. However, we confirm that the proposed method is effective to a certain degree when noise-variance is unknown.

2. PROBLEM FORMULATION

2.1. Preparation

Consider a blind channel identification/estimation of a discrete-time signal-input multiple-output (SIMO) system. The ith component of output at time n is given by

$$x_n^{(i)} = \sum_{m=0}^{M} h_m^{(i)} d_{n-m} + b_n^{(i)}, \qquad i = 0, \cdots, L-1 \quad (1)$$

where the $h_m^{(i)}$ are the finite impulse responses (FIR) of subchannels to be estimated using observations $x_n^{(i)}$. We assume that the transmitted signal d_n is independent of the observed noise $b_n^{(i)}$.A model in (1) can be formulated by temporally oversampling the channel outputs at L time the baud rate.

Stacking N successive samples of the received signal sequence, i.e., $\mathbf{X}_n^{(i)} = [x_n^{(i)}, \cdots, x_{n-N+1}^{(i)}]^T$, we obtain

$$\mathbf{X}_n^{(i)} = \mathcal{H}_N^{(i)} \mathbf{D}_n + \mathbf{B}_n^{(i)} \quad (2)$$

where $\mathbf{B}_n^{(i)} \stackrel{\rm def}{=} [b_n^{(i)}, \cdots, b_{n-N+1}^{(i)}]^T (N\times 1)$, $\mathbf{D}_n = [d_n, \cdots, d_{n-N-M+1}]^T ((N+M)\times 1)$; matrix $\mathcal{H}_N^{(i)}$ is the $N\times(N+M)$ filtering matrix associated with the linear filter $\mathbf{H}^{(i)} \stackrel{\rm def}{=} [h_0^{(i)}, h_1^{(i)}, \cdots, h_M^{(i)}]^T$, defined as

$$\mathcal{H}_N^{(i)} = \begin{bmatrix} h_0^{(i)} & \cdots & h_M^{(i)} & 0 & \cdots & \cdots & 0 \\ 0 & h_0^{(i)} & \cdots & h_M^{(i)} & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & \cdots & 0 & h_0^{(i)} & \cdots & h_M^{(i)} \end{bmatrix}. \quad (3)$$

Hence, all received signal vector is expressed as

$$\begin{bmatrix} \mathbf{X}_n^{(0)} \\ \vdots \\ \mathbf{X}_n^{(L-1)} \end{bmatrix} = \begin{bmatrix} \mathcal{H}_N^{(0)} \\ \vdots \\ \mathcal{H}_N^{(L-1)} \end{bmatrix} \mathbf{D}_n + \begin{bmatrix} \mathbf{B}_n^{(0)} \\ \vdots \\ \mathbf{B}_n^{(L-1)} \end{bmatrix}. \quad (4)$$

Here, we express (4) as

$$\mathbf{X}_n = \mathcal{H}_N \mathbf{D}_n + \mathbf{B}_n. \quad (5)$$

A blind estimation procedure consist in estimating the $L(M+1)\times 1$ vector $\mathbf{H}$ of channel coefficients:

$$\mathbf{H} \stackrel{\rm def}{=} [\mathbf{H}^{(0)T}, \cdots, \mathbf{H}^{(L-1)T}]^T \quad (6)$$

from the sole observations of $\mathbf{X}_n$. In this paper, we will discuss the case with $L = 2$ in later section.

2.2. Principal Component Analysis

Since the additive measurement noise is assumed to be independent of the transmitted sequence, the autocorrelation matrix $\mathbf{R_X}$ $(LN\times LN)$ of the received signal vector $\mathbf{X}_n$ is given by

$$\mathbf{R_X} = \mathcal{H}_N \mathbf{R_d} \mathcal{H}_N^T + \sigma^2 \mathbf{I} \quad (7)$$

where $\mathbf{R_d}$ denotes the autocorrelation matrix of the transmitted signal vector $\mathbf{D}_n$, and σ^2 denotes the noise-variance. The source covariance matrix $\mathbf{R_d}$ has dimension $(M+N)\times(M+N)$, and is assumed to be full rank but otherwise unknown. The autocorrelation matrix $\mathbf{R_X}$ is the hermitian matrix. Therefore, this matrix is decomposed as following:

$$\mathbf{R_X} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T \quad (8)$$

where $\mathbf{\Lambda}$ is the diagonal eigen-values matrix, and $\mathbf{U}$ corresponds to the eigen-vectors matrix. Let $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{LN-1}$ denote the eigen-values of $\mathbf{R_X}$, and λ_i^s denote the eigen-values of $\mathcal{H}_N \mathbf{R_d} \mathcal{H}_N^T$. Since $\mathbf{R_d}$ is full rank, the signal part of the autocorrelation matrix $\mathbf{R_X}$, i.e., $\mathcal{H}_N \mathbf{R_d} \mathcal{H}_N^T$ has rank $M+N$, hence:

$$\begin{cases} \lambda_i = \lambda_i^s + \sigma^2 & (for\ \ i = 0, \cdots, M+N-1) \\ \lambda_i = \sigma^2 & (for\ \ i = M+N, \cdots, LN-1). \end{cases} \quad (9)$$

Let us denote the unit-norm eigen-vectors associated with the eigen-values $\lambda_0, \cdots, \lambda_{M+N-1}$ by $\mathbf{S}_0, \cdots, \mathbf{S}_{M+N-1}$ and denote those corresponding to $\lambda_{M+N}, \cdots, \lambda_{LN-1}$ by $\mathbf{G}_0, \cdots, \mathbf{G}_{LN-M-N-1}$. Also define

$$\mathbf{S} = [\mathbf{S}_0, \cdots, \mathbf{S}_{M+N-1}] \qquad (LN\times(M+N)) \quad (10)$$

$$\mathbf{G} = [\mathbf{G}_0, \cdots, \mathbf{G}_{LN-M-N-1}] \quad (LN\times(LN-M-N)). \quad (11)$$

Using (9),(10) and (11), (8) is thus also rewritten by

$$\mathbf{R_X} = \mathbf{S}\mathrm{diag}(\lambda_0, \cdots, \lambda_{M+N-1})\mathbf{S}^T + \sigma^2 \mathbf{G}\mathbf{G}^T. \quad (12)$$

The columns of matrix $\mathbf{S}$ span the so-called signal-subspace, while the columns of $\mathbf{G}$ span its orthogonal complement, the noise-subspace. Therefore $\mathcal{R}(\mathbf{S}) = \mathcal{R}(\mathbf{G})^\perp$. Furthermore, considering relationship between (7) and (12), we have $\mathcal{R}(\mathbf{S}) = \mathcal{R}(\mathcal{H}_N)$. Hence, the following is given by

$$\mathcal{R}(\mathcal{H}_N) = \mathcal{R}(\mathbf{G})^\perp. \quad (13)$$

The method using PCA get the impulse response vector based on (13). But, it is necessary to calculate the eigen-values and the eigen-vectors once in it. The calculation of the eigen-values and the eigen-vectors causes the increase of computational complexity, furthermore it causes both the degradation of the accuracy and the convergence speed. So, it is desirable to get the impulse response vector without using the eigen-values and the eigen-vectors.

3. PROPOSED METHOD

In this section, we will present a new algorithm based on PCA without the usage of the eigen-values and the eigen-vectors. Firstly, we describe the condition of proposal method. Secondly, we get the impulse response subspace with estimation of the autocorrelation matrix of the received signal vector. Following that, we can obtain the impulse response vector.

3.1. Assumption in the Discussion

(i) Noise variance: Noise variance σ^2 is known.

(ii) Concerning the relationship with the order of each channel M, the number of samples N and the number of virtual channels L: We assume that $dim\mathcal{N}(\mathcal{H}_N^T) = dim\mathcal{N}(\mathcal{G}_i^T) = 1$. Hence, $N = M+1$, $L = 2$.

3.2. Relationship with Each Subspace

Since the noise-variance σ^2 is known, all the vectors $\mathbf{y}$ with which the noise-subspace $\mathcal{R}(\mathbf{G})$ is spanned are satisfied with following relation:

$$(\mathbf{R_X} - \sigma^2\mathbf{I})\mathbf{y} = \mathbf{0}. \quad (14)$$

We define this symmetric matrix as $(\mathbf{R_X} - \sigma^2\mathbf{I}) = \mathbf{A}$, so (14) is expressed using $\mathbf{A}$ as

$$\mathbf{A}\mathbf{y} = \mathbf{0}. \quad (15)$$

The noise-subspace $\mathcal{R}(\mathbf{G})$, which is spanned with all vector $\mathbf{y}$ in (15), is null space $\mathcal{N}(\mathbf{A})$, i.e., orthogonal complement of $\mathcal{R}(\mathbf{A}^T)$. In other word, $\mathcal{R}(\mathbf{A}^T) = \mathcal{R}(\mathbf{G})^\perp$, and $\mathcal{R}(\mathbf{G}) = \mathcal{N}(\mathbf{A})$. As matrix $\mathbf{A}$ is the

Table 1: The computational complexity

Algorithm	Computational Complexity
Proposed Method	$4(LN)^3$
Moulines' Method Using Jacobi	$24(LN)^3 \sim 40(LN)^3$
Moulines' Method Using QR	$10(LN)^3 \sim 18(LN)^3$

symmetry matrix, the following is given by $\mathcal{R}(\mathbf{A}) = \mathcal{R}(\mathbf{G})^{\perp}$. Those relations yields the following:

$$\mathcal{R}(\mathbf{A}^T) = \mathcal{R}(\mathbf{A}) = \mathcal{R}(\mathcal{H}_N) = \mathcal{R}(\mathbf{S}) = \mathcal{R}(\mathbf{G})^{\perp}. \quad (16)$$

Well, the following structural relation

$$\mathbf{G}_i^T \mathcal{H}_N = \mathbf{H}^T \mathcal{G}_i = \mathbf{0} \quad (17)$$

is known[2], where the noise vector $\mathbf{G}_i(LN \times 1)$ is to the noise matrix $\mathcal{G}_i(L(M+1) \times (M+N))$ what the impulse response vector $\mathbf{H}$ is to the impulse response matrix $\mathcal{H}_N(See.(3)-(6))$.

In the following discussion, we realize blind estimation based on (16) and (17). Here, we represent $\mathbf{A}_{(n)}$ as

$$\mathbf{A}_{(n)} = \mathbf{R}_{x(n)} - \sigma^2 \mathbf{I} \quad (18)$$

where $\mathbf{R}_{x(n)}$ is the autocorrelation matrix of the received signal vector at time n. Therefore $\mathbf{R}_{x(n)}$ is expressed as

$$\mathbf{R}_{x(n)} = \frac{n-1}{n}\mathbf{R}_{x(n-1)} + \frac{1}{n}\mathbf{X}_{(n)}\mathbf{X}_{(n)}^T. \quad (19)$$

3.3. Blind Estimation

The proposed method can get the impulse response vector based on (16) and (17).

First, we will get the noise vector $\mathbf{G}_{i(n)}$, which can be obtained by projecting the received signal vector $\mathbf{X}_{(n)}$ at time n onto orthogonal complement of column-space of matrix $\mathbf{A}_{(n)}$. In general, the orthogonal projection matrix $\mathbf{P}_{(n)}$ onto column-space of matrix $\mathbf{A}_{(n)}$ is defined as

$$\mathbf{P}_{(n)} = \mathbf{A}_{(n)}\mathbf{A}_{(n)}^{+} \quad (20)$$

where $\mathbf{A}_{(n)}^{+}$ is referred to as the pseudo-inverse of $\mathbf{A}_{(n)}$. Using **QR** factorization based on the Gram-Schmidt orthogonalization, $\mathbf{A}_{(n)}$ is expressed as

$$\mathbf{A}_{(n)} = \mathbf{Q}_{(n)}\mathbf{R}_{(n)}. \quad (21)$$

Here, considering the condition of proposal method, we find that matrix $\mathbf{Q}_{(n)}$ has a dependent column-vector. We can get rid of a dependent column-vector $\mathbf{q}_{i(n)}$, which is satisfied with the condition that column-vector $\mathbf{q}_{i(n)}$ of the matrix $\mathbf{Q}_{(n)}$ is min $||\mathbf{q}_{i(n)}||$, from matrix $\mathbf{Q}_{(n)}$ with the Gram-Schmidt orthogonalization. Hence, $\mathbf{P}_{(n)}$ may thus be given by

$$\mathbf{P}_{(n)} = \mathbf{Q}'_{(n)}\mathbf{Q}'^{T}_{(n)}. \quad (22)$$

Considering above this and (16), $\mathbf{G}_{i(n)}$ given by

$$\mathbf{G}_{i(n)} = (\mathbf{I} - \mathbf{Q}'_{(n)}\mathbf{Q}'^{T}_{(n)})\mathbf{X}_{(n)} \quad (23)$$

where the noise vector $\mathbf{G}_{i(n)}$ is orthogonal complement to the column-space of $\mathbf{A}_{(n)}$.

Next, we know a close relationship between the noise vector $\mathbf{G}_{i(n)}$ and the noise matrix $\mathcal{G}_{i(n)}$ about the element of the matrix $(See.(17))$. Hence the noise matrix $\mathcal{G}_{i(n)}$ is composed of the noise vector $\mathbf{G}_{i(n)}$.

Finally, with the orthogonality between the column-space of the noise matrix $\mathcal{G}_{i(n)}$ and the impulse vector $\mathbf{H}_{(n)}$, we will get the desired impulse response vector $\mathbf{H}_{(n)}$ with similar way to $(20)-(23)$. The comparison with the computational complexity and the procedures of the proposed Method are illustrated in Table 1 and Table 2, where matrix $\mathbf{Q}'_{(n)}$ and $\mathbf{Q}''_{(n)}$ are column full rank in Table 2.

Table 2: The proposed algorithm for computation

- Initial value: $n = 0$
  $R_{x(0)} = 0$
- Iteration: for $n = 1$ to $n = m$
  1) $R_{x(n)} = \frac{n-1}{n}R_{x(n-1)} + \frac{1}{n}X_{(n)}X_{(n)}^T$
  2) $A_{(n)} = R_{x(n)} - \sigma^2 I$
  3) Normalize each column-vectors of $A_{(n)}$
  4) QR decomposition of $A_{(n)}$
     $A_{(n)} = Q_{(n)}R_{(n)}$
  5) Define $Q'_{(n)}$ as the matrix in which the column-vector $||q_{i(n)}||$ with minimum norm are removed from matrix $Q_{(n)}$
  6) $G_{i(n)} = (I - Q'_{(n)}Q'^{T}_{(n)})X_{(n)}$
  7) Noise matrix $\mathcal{G}_{i(n)}$ is conposed of the noise vector $G_{i(n)}$
  8) QR decomposition of $\mathcal{G}_{i(n)}$
     $G_{i(n)} = Q''_{(n)}R''_{(n)}$
  9) $H_{(n)} = (I - Q''_{(n)}Q''^{T}_{(n)})X_{(n)}$

4. SIMULATION 1

4.1. Simulation Condition

- A performance measure:

$$\text{NEE} = 10 \log_{10} \frac{||\mathbf{H} - \widetilde{\mathbf{H}}||^2}{||\mathbf{H}||^2} \quad [\text{dB}].$$

- The signal to noise rate (SNR):

$$\text{SNR} = 10 \log_{10} \left[\sum_{i=0}^{L} \frac{E\{||\mathbf{H}^{(i)}\mathbf{d}^{(i)}(t)||^2\}}{E\{||b^{(i)}(t)||^2\}} \right]. \quad (24)$$

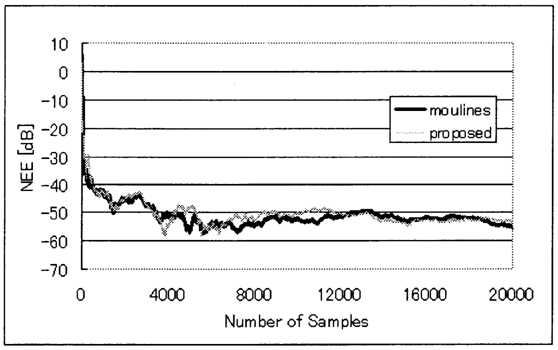

Figure 1: Comparison with the proposed and the traditional method

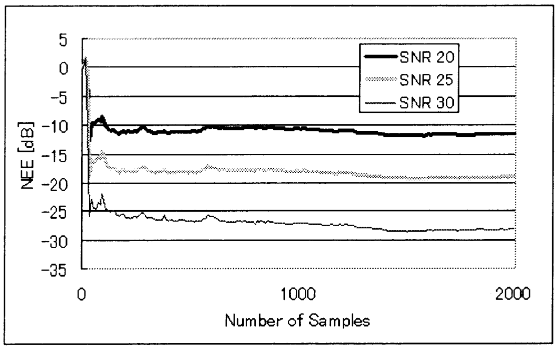

Figure 2: The proposed method of the noise-variance unknown

- The transmitted signal d_n is as follows:
  i) colored signal: d_n is generated by

$$d_n = \mathbf{C}(z)u_n, \quad \mathbf{C}(z) = [\mathbf{F}(z)]^7$$
$$\mathbf{F}(z) = 1 + 0.1z^{-1} - 0.3z^{-2} + 1.0z^{-3} + 0.4z^{-4} - 0.1z^{-5}$$

where u_n is white Gaussian signal (average $\overline{d}_n = 0$, variance$\sigma_d^2 = 1/12$).

White Gaussian noise is added to the output of channel; the output SNR is set to 20 [dB] using (24). According to section 3.1, the number of virtual channels is $L = 2$; the width of the temporal window is $N = 5$; the order of each channel is $M = 4$. The channel coefficients are listed below.

$$\mathbf{H} = [\mathbf{H}^{(0)T}, \mathbf{H}^{(1)T}]^T$$
where
$$\mathbf{H}^{(0)T} = [1.0,\ 3.0,\ 1.0\ ,\ 2.0,\ 1.5]$$
$$\mathbf{H}^{(1)T} = [\ 2.0,\ 1.0,\ 2.0,\ 3.0,\ 4.0].$$

With this above condition, we perform simulation of the proposed method under the condition that noise variance σ^2 is known.

4.2. Simulation Result 1

The simulation result is shown in Figure 1, where Moulines' line in Figure 1 is theoretical value. We can find the proposed method effective from Table 1 and Figure 1.

5. THE CASE OF UNKNOWN NOISE-VARIANCE

In order to confirm that the proposed method is effective to a certain degree in the case of unknown noise-variance, we perform the proposed method in the case unknown noise-variance σ^2 ($A_{(n)} = R_{x(n)}. See\ Table 2.\ 2)$)

5.1. Simulation Result 2

The result of the proposed method by simulation is shown in that Figure 2, the simulation condition in the case is equals to simulation of proposed method (See section 4.1, but noise-variance is unknown). Hence, we can confirm that the proposed method is effective to a certain degree from the viewpoint of parameter estimation when noise-variance is unknown. The detailed explanation about the proposed method is omitted due to the limitation of the pages.

6. CONCLUSION

In this paper, we proposed a new algorithm, which can get the impulse response vector without calculating the eigen-values and the eigen-vectors with estimation value of the autocorrelation matrix of the received signal vector.

Furthermore, we perform the proposed method under the condition that noise-variance is known, but we confirmed that the proposed method is effective to a certain degree when noise-variance is unknown.

We showed the proposed algorithm was useful by computer simulations. It is expected that the proposed algorithm decrease computational complexity without sacrificing the degradation of the accuracy and the convergence speed.

7. REFERENCES

[1] Lang Tong, Guanghan Xu, Thomas Kailath : "Blind Identification and Equalization Based on Second-Order Statistics : A Time Domain Approach," IEEE Transactions on Information Theory, Vol.40, No.2, pp.340-349, Mar. 1994.

[2] Eric Moulines, Pierre Duhamel: "Subspace Methods for the Blind Identification of Multichannel FIR Filters," IEEE Transacations on signal processing, vol.43 , No.2, pp516-525, Feb. 1995.

[3] Jacob H. Gunther, A. Lee Swindlehurst, Member : "On the Use of Kernel Structure for Blind Equalization," IEEE Transacations on signal processing, vol.48 , No.3, pp799-809, Mar. 2000.

[4] G. H. Golub, C. F. Van Loan, Matrix Computations, 3rd ed. Baltimore, MD: Johns Hopkins Univ. Press, 1996.

[5] Strang, Gilbert, Linear Algebra and Its Applications, 3rd ed. Hb College. Press, 1988.

[6] W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, Numerical Recipes in C, 2nd ed. Cambridge Univ. Press, 1993.