

# A MODIFIED OVERALL ON-LINE MODELING ALGORITHM FOR THE FEEDFORWARD MULTIPLE-POINT ANC SYSTEM

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## ABSTRACT

This paper proposes a modified overall on-line modeling algorithm for application in feedforward multiple-point active noise control (ANC) system, in which acoustic paths decoupling can be improved. First, the structure of the ANC system adopting the proposed method is discussed. Second, the update equations of the system modules employing the LMS algorithm are derived. Next, the off-line initialization procedures of the system modules together with the update equations are described. Computer simulations illustrate the performance of the feedforward multiple-point ANC system adopting the proposed method. Simulation results show that the system can achieve superb low-frequency noise reduction and the reconvergence of the system error is guaranteed even when changes are encountered into the primary and secondary paths.

## 1. INTRODUCTION

The fundamental concept of active noise control (ANC) is to reproduce an additional secondary source which is anti-phase to the original noise source [1]. Thus, it is critical to identify primary and secondary paths and equalize the secondary paths in order to produce an accurate secondary source. In general, equalization of secondary path is difficult to achieve since FIR filter with long tail is required in such a case. Worse still, the equalization is invalid if the secondary path is non-minimum-phase. According to the multiple-input/multiple-output inverse theorem (MINT), the exact equalization of secondary paths by FIR filters can be accomplished by using extra secondary paths produced by the activation of more loudspeakers [2]. Furthermore, the corresponding secondary path equalizers are of acceptable length.

Another concern of ANC is the time-varying property of all the acoustic paths in the system. Hence, on-line identification of primary and secondary paths and equalization of the secondary paths are desirable. The primary path identification and secondary path equalization can be achieved by the LMS and FXLMS algorithms respectively. However, the secondary path identification is not straightforward. There are mainly two approaches for the on-line secondary path identification, the additive random noise technique and the overall on-line modeling algorithm [3]. Although the performances of these two approaches are similar, the additive random noise technique increases the system error level, whereas exact path identification is not guaranteed by the overall on-line modeling algorithm.

In order to improve the on-line secondary path identification, a modified overall on-line modeling algorithm is developed in this paper. The proposed method introduces the Independent Equalization Process (IEP), in which on-line secondary path identification is accomplished independent of the system error, it implies that the convergence of each module is less correlated with each other.

Consequently, the acoustic paths decoupling can be improved in the system.

## 2. FEEDFORWARD ANC SYSTEM WITH MODIFIED OVERALL ON-LINE MODELING ALGORITHM

The  $M \times (N + 1) \times N$  feedforward multiple-point ANC system with the modified overall on-line modeling algorithm is shown in Fig. 1. Equivalently, there are  $M$  reference signals,  $(N + 1)$  loudspeakers and  $N$  error microphones. The primary paths and secondary paths are represented by  $P$  and  $S$  respectively, and the system modules  $\hat{P}$ ,  $\hat{S}$  and  $W$  denote respectively the primary path identifiers, secondary path identifiers and secondary path equalizers. Furthermore,  $P$  and  $S$  are assumed to be discrete-time multi-input-multi-output (MIMO) linear systems and  $\hat{P}$ ,  $\hat{S}$  and  $W$  are MIMO FIR filters.

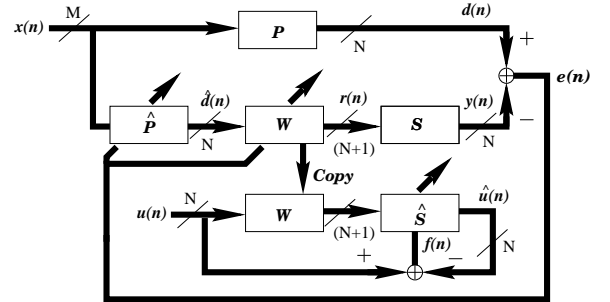


Fig. 1. The feedforward ANC system with the modified overall on-line modeling algorithm

In order to attenuate the noise level within the region monitored by the error microphones, the system modules should satisfy the following

$$\hat{P} \rightarrow P, \quad (1)$$

$$W \rightarrow S^{-1} \quad (2)$$

and

$$\hat{S} \rightarrow W^{-1} \rightarrow S \quad (3)$$

where  $\hat{P} \rightarrow P$  denotes the convergence of  $\hat{P}$  to  $P$  and  $S^{-1}$  denotes the inverse of  $S$ .

Let  $S_{k,l}(z)$  be the frequency response of the secondary path from the  $k$ th input of  $S$  to the  $l$ th output of  $S$ . Hence, the frequency response matrix of  $S$  is denoted as

$$\begin{aligned} S(z) &= \begin{bmatrix} S_{1,1}(z) & \dots & S_{N+1,1}(z) \\ \vdots & \ddots & \vdots \\ S_{1,N}(z) & \dots & S_{N+1,N}(z) \end{bmatrix} \\ &= \begin{bmatrix} S_1(z) & S_2(z) & \dots & S_{N+1}(z) \end{bmatrix}. \end{aligned} \quad (4)$$

It can be shown that the unique solutions for  $\mathbf{W}$  exist when  $[\mathbf{S}_1(z) \mathbf{S}_2(z) \dots \mathbf{S}_N(z)]$  and  $\mathbf{S}_{N+1}(z)$  are left coprime [4]. Let  $L_s$  be the length of the secondary path, then it can be shown that the length of the secondary path equalizer is given by

$$L_w = [N \cdot (L_s - 1) - 1]. \quad (5)$$

IEP consists of the internal generated random noise vector  $\mathbf{u}(n)$ , the copy of  $\mathbf{W}$  and  $\hat{\mathbf{S}}$ . The objective of IEP is to equalize the copy of  $\mathbf{W}$  by  $\hat{\mathbf{S}}$ . Here,  $\hat{\mathbf{S}}$  is driven by the output of the copy of  $\mathbf{W}$  activated by  $\mathbf{u}(n)$ . The difference between  $\mathbf{u}(n)$  and the output of  $\hat{\mathbf{S}}$  is adopted to monitor IEP. Equivalently, IEP is independent of the system error. Once  $\mathbf{W}$  has completed the identification of the inverse of  $\mathbf{S}$ ,  $\hat{\mathbf{S}}$  can converge to  $\mathbf{S}$  with the copy of  $\mathbf{W}$ . It follows that the convergence of  $\hat{\mathbf{P}}$  to  $\mathbf{P}$  is straightforward.

Even  $\mathbf{S}$  is varied in our system, as long as the phase difference between the frequency responses of  $\hat{\mathbf{S}}$  and  $\mathbf{S}$  is less than  $90^\circ$ ,  $\mathbf{W}$  can still converge to the new inverse of  $\mathbf{S}$  [5]. Thus, the convergence of  $\hat{\mathbf{S}}$  to  $\mathbf{S}$  is guaranteed. Furthermore, the off-line initialization of the system modules is implemented in the system in order to provide a more stable system performance and faster convergence of the system error.

### 3. UPDATE EQUATION OF EACH MODULE

Since all the acoustic paths may change in real-time application, the update equations of all the modules are derived in order to track the environment accordingly. Without loss of generality, assume the lengths of the primary path identifier, secondary path identifier and equalizer equal  $L_p$ ,  $L_s$  and  $L_w$  respectively, where  $L_p$  is the length of the primary path.

Define the  $i$ th reference noise vector at time  $n$  as

$$\mathbf{x}_i(n, L_p) = [x_i(n) \ x_i(n-1) \ \dots \ x_i(n-L_p+1)]^T, \quad (6)$$

the  $j$ th internal generated random noise vector at time  $n$  as

$$\mathbf{u}_j(n, L_w) = [u_j(n) \ u_j(n-1) \ \dots \ u_j(n-L_w+1)]^T, \quad (7)$$

the primary path identifier vector from the  $i$ th input of  $\hat{\mathbf{P}}$  to the  $j$ th output of  $\hat{\mathbf{P}}$  at time  $n$  as

$$\hat{\mathbf{p}}_{i,j}(n) = [\hat{p}_{i,j,0}(n) \ \hat{p}_{i,j,1}(n) \ \dots \ \hat{p}_{i,j,L_p-1}(n)]^T, \quad (8)$$

the secondary path equalizer vector from the  $j$ th input of  $\mathbf{W}$  to the  $k$ th output of  $\mathbf{W}$  at time  $n$  as

$$\mathbf{w}_{j,k}(n) = [w_{j,k,0}(n) \ w_{j,k,1}(n) \ \dots \ w_{j,k,L_w-1}(n)]^T, \quad (9)$$

the secondary path identifier vector from the  $k$ th input of  $\hat{\mathbf{S}}$  to the  $l$ th output of  $\hat{\mathbf{S}}$  at time  $n$  as

$$\hat{\mathbf{s}}_{k,l}(n) = [\hat{s}_{k,l,0}(n) \ \hat{s}_{k,l,1}(n) \ \dots \ \hat{s}_{k,l,L_s-1}(n)]^T, \quad (10)$$

the  $j$ th output vector of  $\hat{\mathbf{P}}$  at time  $n$  as

$$\hat{\mathbf{d}}_j(n, L_w) = [\hat{d}_j(n) \ \hat{d}_j(n-1) \ \dots \ \hat{d}_j(n-L_w+1)]^T, \quad (11)$$

the  $l$ th output of  $\mathbf{P}$  at time  $n$  as  $d_l(n)$ , the impulse response of secondary path from the  $k$ th input of  $\mathbf{S}$  to the  $l$ th output of  $\mathbf{S}$  at time  $n$  as  $s_{k,l}(n)$ , the  $k$ th output of  $\mathbf{W}$  at time  $n$  as  $r_k(n)$ , the  $l$ th output of  $\mathbf{S}$  at time  $n$  as  $y_l(n)$ , the  $l$ th output of  $\hat{\mathbf{S}}$  at time  $n$  as  $\hat{u}_l(n)$ , the  $l$ th system error at time  $n$  as  $e_l(n)$  and the  $l$ th error of the IEP at time  $n$  as  $f_l(n)$  where  $i = 1, \dots, M$ ,  $j = 1, \dots, N$ ,  $k = 1, \dots, (N+1)$  and  $l = 1, \dots, N$ .

Equivalently, we have

$$e_l(n) = d_l(n) - y_l(n), \quad (12)$$

$$f_l(n) = u_l(n) - \hat{u}_l(n), \quad (13)$$

$$\hat{d}_j(n) = \sum_{i=1}^M [\hat{\mathbf{p}}_{i,j}^T(n) \cdot \mathbf{x}_i(n, L_p)], \quad (14)$$

$$r_k(n) = \sum_{j=1}^N [\mathbf{w}_{j,k}^T(n) \cdot \hat{\mathbf{d}}_j(n, L_w)], \quad (15)$$

$$y_l(n) = \sum_{k=1}^{N+1} [s_{k,l}(n) * r_k(n)], \quad (16)$$

$$\hat{u}_l(n) = \sum_{j=1}^N \sum_{k=1}^{N+1} [\mathbf{u}_{j,w_{j,k}}^T(n) \cdot \hat{\mathbf{s}}_{k,l}(n)] \quad (17)$$

where

$$\mathbf{u}_{j,w_{j,k}}(n) = [u_{j,w_{j,k}}(n) \ \dots \ u_{j,w_{j,k}}(n-L_s+1)]^T \quad (18)$$

and

$$u_{j,w_{j,k}}(n) = \mathbf{u}_j^T(n, L_w) \cdot \mathbf{w}_{j,k}(n). \quad (19)$$

In order to track the primary paths by  $\hat{\mathbf{p}}_{i,j}(n)$ 's, the leaky LMS algorithm is applied to minimize the following error

$$\zeta_{\hat{\mathbf{p}}}(n) = \sum_{l=1}^N [e_l(n)]^2 + k_{\hat{\mathbf{p}}}(n) \sum_{i=1}^M \sum_{j=1}^N \hat{\mathbf{p}}_{i,j}^T(n) \cdot \hat{\mathbf{p}}_{i,j}(n) \quad (20)$$

where  $k_{\hat{\mathbf{p}}}(n)$  is the weighting factor defined as

$$k_{\hat{\mathbf{p}}}(n) = \alpha \cdot \sum_{l=1}^N \hat{E} [e_l^2(n)], \quad (21)$$

$\hat{E} [e_l^2(n)]$  is an estimate of  $E [e_l^2(n)]$  and  $\alpha$  is a positive constant. Here,  $\hat{E} [e_l^2(n)]$  can be estimated by

$$\hat{E} [e_l^2(n)] = (1 - \gamma) \hat{E} [e_l^2(n-1)] + \gamma e_l^2(n) \quad (22)$$

where  $\gamma$  is the smoothing factor.

Applying the leaky LMS algorithm can ensure the convergence of the system error while the divergence of the output of  $\hat{\mathbf{p}}_{i,j}(n)$  is avoided. This is very critical in our application before the convergence of all the modules. Since the excess system error power is proportional to the weighting factor [6],  $k_{\hat{\mathbf{p}}}(n)$  is chosen to be proportional to  $\sum_{l=1}^N \hat{E} [e_l^2(n)]$  in our application. Thus,  $k_{\hat{\mathbf{p}}}(n)$  can contribute less to the excess system error power after the convergence of the system error. In addition,  $\alpha$  should be kept small for an acceptable system performance.

It can be shown that the update equation is given by

$$\begin{aligned} \hat{\mathbf{p}}_{i,j}(n+1) &= [1 - \mu_{\hat{\mathbf{p}}}(n) k_{\hat{\mathbf{p}}}(n)] \hat{\mathbf{p}}_{i,j}(n) + \mu_{\hat{\mathbf{p}}}(n) \cdot \\ &\quad \sum_{l=1}^N \sum_{k=1}^{N+1} e_j(n) [\mathbf{x}_{i,w_{j,k}}(n) * s_{k,l}(n)] \end{aligned} \quad (23)$$

where  $\mu_{\hat{\mathbf{p}}}(n)$  is the step size,

$$\mathbf{x}_{i,w_{j,k}}(n) = [x_{i,w_{j,k}}(n) \ \dots \ x_{i,w_{j,k}}(n-L_p+1)]^T \quad (24)$$

and

$$x_{i,w_{j,k}}(n) = \mathbf{x}_i^T(n, L_w) \cdot \mathbf{w}_{j,k}(n). \quad (25)$$

Since one of our objective is to remove the secondary path effect, the exact equalization of the secondary paths is assumed for updating  $\hat{\mathbf{p}}_{i,j}(n)$ . Hence, (23) can be simplified as

$$\begin{aligned} \hat{\mathbf{p}}_{i,j}(n+1) &= [1 - \mu_{\hat{\mathbf{p}}}(n) k_{\hat{\mathbf{p}}}(n)] \hat{\mathbf{p}}_{i,j}(n) + \\ &\quad \mu_{\hat{\mathbf{p}}}(n) e_j(n) \mathbf{x}_i(n, L_p) \end{aligned} \quad (26)$$

where  $k_{\hat{p}}(n)$  is modified as

$$k_{\hat{p}_j}(n) = \alpha \cdot \hat{E} [e_j^2(n)] \quad (27)$$

since the error signal involved in (26) depends on  $e_j(n)$  only.

By the similar argument,  $w_{j,k}(n)$ 's are also updated by the leaky LMS algorithm which aims at minimizing

$$\zeta_w(n) = \sum_{l=1}^N [e_l(n)]^2 + k_w(n) \sum_{j=1}^N \sum_{k=1}^{N+1} w_{j,k}^T(n) \cdot w_{j,k}(n) \quad (28)$$

where

$$k_w(n) = \beta \cdot \sum_{l=1}^N \hat{E} [e_l^2(n)] \quad (29)$$

and  $\beta$  is a positive constant which should be kept small for lower system error level. Thus, it can be easily shown that

$$w_{j,k}(n+1) = [1 - k_w(n)\mu_w(n)] w_{j,k}(n) + \mu_w(n) \cdot \sum_{l=1}^N e_l(n) \hat{d}_{j,\hat{s}_{k,l}}(n) \quad (30)$$

where  $\mu_w(n)$  is the step size of update equation,

$$\hat{d}_{j,\hat{s}_{k,l}}(n) = [\hat{d}_{j,\hat{s}_{k,l}}(n) \quad \dots \quad \hat{d}_{j,\hat{s}_{k,l}}(n - L_w + 1)]^T \quad (31)$$

and

$$\hat{d}_{j,\hat{s}_{k,l}}(n) = \hat{d}_j^T(n, L_s) \cdot \hat{s}_{k,l}(n). \quad (32)$$

$\hat{s}_{k,l}(n)$  is identified through IEP by minimizing the error

$$\zeta_s(n) = \sum_{l=1}^N [f_l(n)]^2. \quad (33)$$

Based on the LMS algorithm, the copies of  $w_{j,k}(n)$ 's from (30) and take  $\mu_s(n)$  as the step size, we have

$$\hat{s}_{k,l}(n+1) = \hat{s}_{k,l}(n) + \mu_s(n) f_l(n) \sum_{j=1}^N u_{j,w_{j,k}}(n). \quad (34)$$

#### 4. OFF-LINE INITIALIZATION OF SYSTEM MODULES

It is often possible to conduct the off-line initialization of the system modules before the start of real-time operation. Hence, faster convergence of the system error and more stable system performance can be accomplished. Furthermore, re-initialization is also necessary during the real-time processing once the modules cannot track the abrupt changes in acoustic paths. Thus, off-line initialization is implemented in our system as shown in Fig. 2. Similar method can be found in [3].

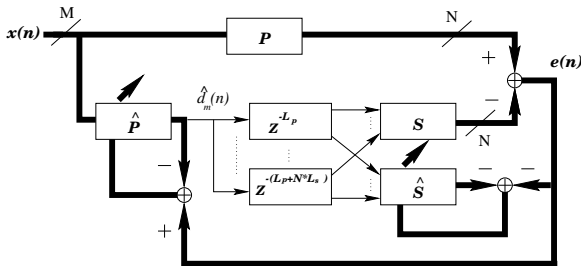


Fig. 2. The off-line initialization of the system modules

During the initialization process, only one of the outputs of  $\hat{P}$ ,  $\hat{d}_m(n)$ , is adopted for training  $\hat{S}$ , hence the complexity for updating  $\hat{S}$  can be reduced. In addition, all  $w_{j,k}(n)$ 's are initialized as zero except for  $w_{m,k}(n)$ 's, which are initialized as the pure delay block

$$w_{m,k}(n) = z^{-[L_p+(k-1)L_s]}. \quad (35)$$

The purpose of this assignment is to uncorrelate the input signals to  $S$ . Furthermore, the common delay element  $z^{-L_p}$  in all the pure delay blocks can ensure the uncorrelation within the outputs of  $P$  and  $S$ . As a result,  $e(n)$  mainly consists of the mixture of the uncorrelated outputs of  $P$  and  $S$ , it implies that the application of the LMS algorithm for tracking the paths is valid. After  $\hat{s}_{k,l}(n)$ 's have completed the path identification,  $w_{j,k}(n)$ 's are initialized by IEP with the copies of  $\hat{s}_{k,l}(n)$ 's.

In order to achieve the off-line initialization, the primary path identifiers, secondary path identifiers and equalizers aim at minimizing respectively the errors

$$\zeta_{e_{off,p}}(n) = \sum_{j=1}^N [e_j(n) - \hat{d}_j(n)]^2, \quad (36)$$

$$\zeta_{e_{off,s}}(n) = \sum_{l=1}^N [-e_l(n) - \hat{y}_l(n)]^2 \quad (37)$$

and

$$\zeta_{e_{off,w}}(n) = \sum_{l=1}^N [u_l(n) - \hat{u}_l(n)]^2 \quad (38)$$

where

$$\hat{y}_l(n) = \sum_{k=1}^{N+1} \hat{d}_{m,\hat{s}_{k,l}}(n - [L_p + (k-1)L_s]). \quad (39)$$

According to the LMS algorithm, the update equations of the system modules are derived as

$$\hat{p}_{i,j}(n+1) = \hat{p}_{i,j}(n) + \mu_p(n) [e_j(n) - \hat{d}_j(n)] x_i(n, L_p), \quad (40)$$

$$\begin{aligned} \hat{s}_{k,l}(n+1) &= \hat{s}_{k,l}(n) - \mu_s(n) [e_l(n) + \hat{y}_l(n)] \cdot \\ &\quad \hat{d}_m(n - [L_p + (k-1)L_s], L_s) \end{aligned} \quad (41)$$

and

$$\begin{aligned} w_{j,k}(n+1) &= w_{j,k}(n) + \mu_w(n) \sum_{l=1}^N [u_l(n) - \hat{u}_l(n)] \cdot \\ &\quad u_{j,\hat{s}_{k,l}}(n) \end{aligned} \quad (42)$$

where

$$u_{j,\hat{s}_{k,l}}(n) = [u_{j,\hat{s}_{k,l}}(n) \quad \dots \quad u_{j,\hat{s}_{k,l}}(n - L_w + 1)]^T \quad (43)$$

and

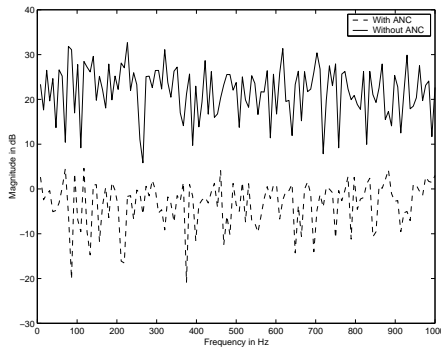
$$u_{j,\hat{s}_{k,l}}(n) = u_j^T(n, L_s) \cdot \hat{s}_{k,l}(n). \quad (44)$$

#### 5. SIMULATION RESULTS

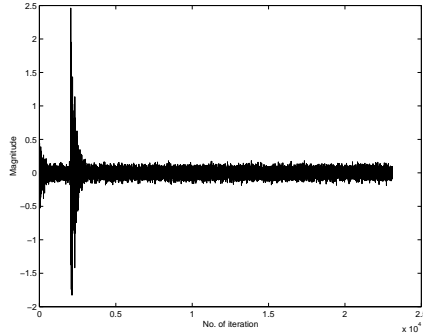
The objective of feedforward ANC system is to attenuate the low-frequency noise, thus sampling frequency of 4kHz is applied to prevent aliasing. In order to verify the applicability of the proposed method to ANC system, the performance of the  $2 \times 3 \times 2$  feedforward ANC system with this method is being evaluated by the computer simulations. The reference white noises and internal generated white noises with the magnitudes of 2 and 1 are chosen respectively. Some of the secondary paths are minimum-phase and

some are non-minimum-phase. The measurement noises with the magnitude of 0.1 are added to the error microphones. The order of the primary and secondary paths, primary and secondary path identifiers and secondary path equalizers are 16,16,16,16 and 32 respectively. Normalized step sizes are adopted in order to ensure the stability and convergence of all the update equations. In addition,  $\alpha$ ,  $\beta$  and  $\gamma$  are chosen for an acceptable system performance.

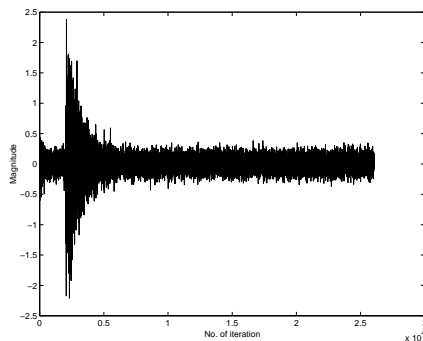
First, the off-line initialization process is carried out to obtain all the initial status of the system modules. Next, the simulation of the operation of our system is conducted with those initial values. Fig. 3 illustrates the simulation results of an error signal at one of the error microphones before and after the activation of our system. From the plot, it is observed that superb low-frequency noise attenuation can be achieved by applying our system.



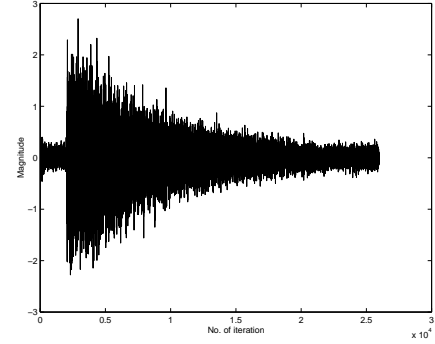
**Fig. 3.** Error signal at one of the error microphones before and after applying our system



**Fig. 4.** The system error reconverges after the changes in the primary paths



**Fig. 5.** The system error reconverges after the changes in the secondary paths



**Fig. 6.** The system error reconverges after the changes in the both paths

Second, reconvergence of the system error is evaluated by varying the primary and secondary paths. Fig. 4 shows the simulation result of our system error with changes in the primary paths. It can be observed that system error diverges initially when the changes are encountered into the paths. However, the system error can reconverge with a similar residual noise level within a short period of time. Next, the performance of our system is investigated by varying the secondary paths. From Fig. 5, similar phenomenon is observed except for the longer reconvergence time. It is predictable since the convergence of the secondary path identifiers and equalizers is necessary before the system error reconverges. Fig. 6 illustrates the simulation result of our system when both primary and secondary paths are changed. It can be observed that much longer time is required before the system error reconverges.

## 6. CONCLUSION

In this paper, a modified overall on-line modeling algorithm is proposed for improved acoustic paths decoupling. The structure of our system is designed such that less correlation exists within the convergence of the system modules. The update equations of the system modules are derived in order to minimize the system error. Next, the off-line initialization of all the modules are implemented for faster convergence of the system error. The corresponding update equations of the modules are also derived. Simulation results show that the feedforward multiple-point ANC system using the proposed method can remain a stable performance when abrupt changes are occurred in the acoustic paths.

## 7. REFERENCES

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