INTERACTING MULTIPLE MODELS FOR SINGLE-USER CHANNEL ESTIMATION AND EQUALIZATION

M.H. Jaward and V. Kadirkamanathan

Department of Automatic Control and Systems Engineering
The University of Sheffield
Mappin Street, Sheffield S1 3JD, UK

ABSTRACT

In this paper, a blind sequence estimation algorithm based on interacting multiple model is introduced to estimate the channel and the transmitted sequence corrupted by ISI (intersymbol interference) and noise. The proposed algorithm avoids the exponential growth complexity caused by increasing channel memory length. The performance of the IMM (interacting multiple model) based equalizer is studied and compared with the well known algorithm like DDFSE (Delayed Decision-Feedback Sequence Estimation).

1. INTRODUCTION

The interest in this paper is the problem of detection of digital data in the presence of intersymbol interference (ISI) and additive noise. Throughout this work the assumption made is that after some processing (matched filtering, for instance), the continuous time received signals are sampled at the baud (symbol) rate. Thus results in a discrete time model of the channel. Our objective is to produce a reliable decision of the input sequence based on the received data in the absence of channel characteristics.

As discussed in [1], various approaches to data detection can be broadly divided into symbol by symbol and sequence estimation. The first class contains linear and decision-feedback detectors. These schemes have low complexity and undesirably high error rates. Another approach to data detection is given by maximum-likelihood sequence estimation (MLSE) [2]. The trellis-based Viterbi algorithm [3] solves the MLSE problem recursively when the memory of the channel is finite. The symbol error rate of the Viterbi algorithm is often much lower than error rates of the symbol by symbol detectors. However, the total storage (complexity) of the algorithm is proportional to the number of states of the trellis which grows exponentially with the channel memory length. When the channel memory becomes large, the algorithm becomes impractical. In this case reduced state algorithms like RSSE (Reduced-State Sequence Estimation) [4, 5] and DDFSE (Delayed Decision-Feedback Sequence Estimation) [6] are used. These algorithms assume some past decisions as correct while estimating several most recent symbols.

Another set of equalizers use a hidden Markov model (HMM) formulation for blind (or semi blind) equalization for input sequences governed by Markov chains. Either they use off-line [7] or on-line EM algorithm [8, 9] to maximize the Kullback-Leibler (KL) measure to calculate the HMM model. [10, 11] use a HMM estimator together with a sequence estimation for stochastic maximum likelihood (ML) equalization. While on-line methods overcome the memory and computational cost involved in the off-line EM algorithm based methods, they still need to use some kind of state reduction algorithm to reduce the state complexity of the state trellis [9]. Tugnait et al. [12] presents a comprehensive review of single-user channel estimation and equalization techniques. Here we propose an alternative approach which utilizes the interacting multiple model (IMM) algorithm.

The paper is organized as follows: In Section 2 we define our signal model and formulate the equalization problem as a state estimation under model uncertainty problem. In Section 3, we review the IMM algorithm. We derive our IMM based equalizer in Section 4. Simulation studies are presented in Section 5 and finally some conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

Let $d(k)$ denote the symbol emitted by the digital source at time kT, where T is the symbol duration. This discrete time signal is modulated, filtered, sent through the communication channel, filtered and demodulated. The resulting signal is continuous and is given by

$$y(t) = \sum_{n=0}^{N_h-1} h(t-nT).d(n) + v(t) \qquad (1)$$

where T is the symbol period, $v(t)$ is the additive white noise independent from the emitted symbols, $h(t)$ is the composite channel response encompassing the effects of the transmitting filter, reception filter, channel response and modulation/demodulation (which is assumed to be linear). The composite channel is assumed to be FIR, with a duration of approximately N_hT. In general, $d(t)$ can take on K possible values. For simplicity we use binary transmission ($K = 2$) and symbols transmitted are either -1 and 1. The extension to the general case is straightforward. Symbol rate sampling is used which results in an equivalent discrete-time representation,

$$y(k) = \sum_{n=0}^{N_h-1} h(k-n).d(n) + v(k) \qquad (2)$$

This can easily be extended to multi rate sampling. To overcome the phase ambiguity in the channel coefficients, differential decoding is used.

Under the assumption that $h(k)$ is perfectly known, the optimum receiver is composed of a filter matched to the pulse $h(k)$ followed by a symbol rate sampler and a Viterbi decoder that searches for the path with minimum metric in the trellis diagram of a finite state machine of the equivalent channel.

Unfortunately, in many practical communication systems, we do not know the impulse response $h(k)$ or we need to use simplified state description of the channel in order to reduce excessive state complexity. The objective is to find the transmitted symbol sequence from the received sequence when the channel coefficients are not known. The suboptimal optimal approach to this makes use of data aided parameter estimators together with the sequence estimator.

If the transmitted symbols $d(k)$ were known a priori, then the estimation of channel coefficients, would be a conventional linear Gaussian problem, and could be solved using the Kalman filter. But in our case, with $d(k)$ unknown, the measurement model represented by (2) is dependent on a Markov switching process generated by the symbol sequence. This is the problem of state estimation with model uncertainty [13]. Hence we propose to solve the blind equalization problem using the interacting multiple models (IMM) [14, 15], one of several methods available for the Markov switching systems (Hybrid systems).

3. THE INTERACTING MULTIPLE MODEL ALGORITHM

Our blind sequence estimation can be formulated as a linear hybrid system and hence only the linear hybrid systems are considered here. The simplest linear hybrid system is described by a set linear models as

$$\theta(k) = F_j(k-1)\theta(k-1) + G_j(k-1)w_j(k-1) \quad (3)$$
$$y(k) = H_j(k)\theta(k) + v_j(k) \qquad \forall\, j \in S \quad (4)$$

where $S = \{1, \ldots, r\}$ is the set of possible modes. This model can handle changes in the system structure as well as in the noise statistics but in our case, as will be seen, changes occur only in the observation matrix, $H_j(k)$. The mode at time kT, $m(k)$ is assumed to be among the possible r modes. The mode transitions are governed by a first-order homogeneous Markov chain

$$P\{m_j(k+1)|m_i(k)\} = \pi_{ij} \qquad \forall\, j \in S \quad (5)$$

where π_{ij} is the transition probability from mode i to mode j and $m_i(k)$ symbolises that the mode at time kT is i. Let $m_i^s(k-1)$ denote the sth mode history up to and including time $(k-1)T$ with mode i at time $(k-1)T$. Then the mode probabilities are given by

$$\begin{aligned} P\{m_j(k)|Y^k\} &= \sum_s P\{m_j(k)|Y^k, m_i(k-1)\} \quad (6)\\ &\quad \times P\{m_i^s(k-1)|Y^k\} \\ &= \sum_s \pi_{ij} P\{m_i^s(k-1)|Y^k\} \end{aligned}$$

where both summations are over all possible mode histories and $P\{m_i^s(k-1)|Y^k\}$ can be calculated using Bayes formula. The number of mode histories through $(k-1)T$ is r^{k-1} since r number of modes are possible at each time. Thus the number of histories grows exponentially with time. To overcome this problem, fixed memory algorithms like GPB (Generalized Pseudo Bayes) [14] and IMM (Interacting Multiple Model) [14, 15] algorithms were introduced.

The IMM algorithm is a recursive algorithm consisting of four major steps: interaction (mixing), filtering, mode update and combination. In each cycle, the initial condition for the filter matched to a certain mode is obtained by interacting (mixing) the state estimates of all filters at the previous time under the assumption that this particular mode is in effect at the current time. This is followed by a prediction and update step, performed in parallel for each mode. The mode probability update is performed next for each modes. Then a combination of the updated state estimates for all filters yields the state estimate. The mode probability acts as the weighting in the interaction step and in the combination of states and covariances. Implementation details can be found in [14] and section (4).

4. IMM ALGORITHM FOR ADAPTIVE EQUALIZATION

IMM algorithm can be used for sequence estimation by assuming the mode in effect is due to one of transmitted symbols. For binary transmission, the received signal comes from one of two symbols and hence the number of modes, r is equal to 2. We need r number of filters in parallel at a time as only one of r modes can be in effect. We define the state vector θ (coefficients of channel) as

$$\theta(k) = [\, h(0)\; h(1) \ldots h(N_h - 1)\,] \quad (7)$$

Let M be the set of symbols transmitted. We use a random walk model for the system model (3) and from (2), the observation matrix can be written as $H_j(k) = [d(k)\, d(k-1) \,..\, d(k-N_h+1)]$. Hence the hybrid system simplifies to,

$$\theta(k) = \theta(k-1) + w(k-1) \quad (8)$$
$$y(k) = H_j(k)\theta(k) + v(k) \qquad \forall\, j \in S \quad (9)$$

The algorithm can now be summarised as follows:

- Calculate the probability that the symbol corresponding to mode i was in effect at time $(k-1)T$ given that the mode j was in effect at kT conditioned on past received data, Y^{k-1}, for all $i, j = 1, \ldots, r$. Here symbols corresponding to i and j takes on values in M and the calculation is done for all $K = r$ values in M.

$$\begin{aligned} \mu_{i|j}(k-1|k-1) &= P(m_i(k-1)|m_j(k), Y^{k-1}) \\ &= \frac{1}{c} P(m_j(k)|m_i(k-1), Y^{k-1}) \\ &\quad .P(m_i(k-1)|Y^{k-1}) \\ &= \frac{1}{c} \pi_{ij}\mu_i(k-1) \quad i, j = 1, .., r \end{aligned}$$

  where c is the normalizing constant and $\mu_i(k-1)$ is the posterior symbol probability at time $(k-1)T$.

- Compute the mixed initial condition for the filter matched to each symbol at time kT using $\hat{\theta}^i(k-1|k-1)$, state estimate of the Kalman filter matched to mode i at time $(k-1)T$. The initial state estimate is given by

$$\begin{aligned} \hat{\theta}^{0j}(k-1|k-1) &= \sum_{i=1}^{r} \hat{\theta}^i(k-1|k-1) \quad (10)\\ &\quad .\mu_{i|j}(k-1|k-1) \;\; \forall\, j = 1, .., r \end{aligned}$$

and the corresponding covariance matrix is

$$
\begin{aligned}
&P^{0j}(k-1|k-1) \quad (11)\\
&= \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\{P^i(k-1|k-1)\\
&+(\hat{\theta}^i(k-1|k-1) - \hat{\theta}^{0j}(k-1|k-1))\\
&(\hat{\theta}^i(k-1|k-1) - \hat{\theta}^{0j}(k-1|k-1))'\}\\
&\forall\, j = 1,..,r
\end{aligned}
$$

where $\hat{\theta}^j(k|k)$ and $P^i(k|k)$ are the state estimate and covariance of the state estimate at time kT.

- The above two estimates, (10) and (11) are used as input to two Kalman filters matched to two symbols to obtain $\hat{\theta}^j(k|k)$ and $P^j(k|k)$. These are the outputs of the Kalman filter and the Kalman filter equations can be found in [16]. The likelihood corresponding to two filters,

$$
\begin{aligned}
\Lambda_j(k) \quad &= \quad p(y(k)|m_j(k), \quad (12)\\
&\hat{\theta}^{0j}(k-1|k-1), P^{0j}(k-1|k-1))
\end{aligned}
$$

are also computed. For Gaussian noise this reduces to

$$\Lambda_j(k) \quad \propto \quad \exp(-0.5\, e(k)R_e(k)^{-1}e(k)) \quad (13)$$

where $e(k)$ and $R_e(k)$ are respectively the innovation and innovation variance given by the Kalman filter.

- The probability that the mode j $(j = 1,..,r)$ is in effect is updated as follows

$$
\begin{aligned}
\mu_j(k) \quad &= \quad P(m_j(k)|Y^k) \quad (14)\\
&= \quad \frac{1}{\overline{c}}\Lambda_j(k)\sum_{i=1}^{r}\pi_{ij}\mu_i(k-1)\\
&\quad \forall\, j = 1,..,r
\end{aligned}
$$

where $\overline{c}$ is the normalization constant. The mode at time kT can now be estimated as

$$m(k) = \arg\max_j \Lambda_j(k) \quad (15)$$

Thus the symbol at kT is the symbol corresponding to mode $m(k)$.

- Finally calculate the channel coefficient estimates using the following mixture equations

$$\hat{\theta}(k) = \sum_{j=1}^{r}\hat{\theta}^j(k|k)\mu_j(k) \quad (16)$$

5. SIMULATION RESULTS

The performance of the proposed algorithm was examined using simulations and compared to the well known reduced state sequence algorithms like RSSE which for binary transmission is equivalent to the DDFSE. The transmitted sequence was an independent and identically distributed (*iid*) binary sequence. The transition probability matrix, π_{ij} is given by

$$\pi_{ij} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \quad (17)$$

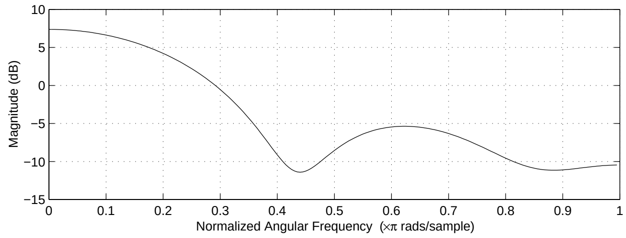

Fig. 1. Frequency response of equivalent channel

A two-ray channel is selected for simulation with channel coefficients,

$$h \;=\; [.620\ .560\ .480\ .460\ .220] \quad (18)$$

The resulting frequency response is plotted in figure 1. In DDFSE, channel estimation was done on a per-survivor basis as outlined in [17, 18] and the resulting algorithm is denoted by PSP DDFSE. Here the effective channel memory is truncated to 2. For DDFSE, the decoding delay was large enough to avoid any performance degradation due to unmerged survivors. For comparison these two algorithms were simulated assuming the correct transmitted sequence is available at the receivers. These idealized cases serve as the lower bounds for the two cases. The simulated bit error performance is plotted in figure 2. Twenty simulation runs of length 20000 symbols were used to obtain an average BER. This thus reflects the effects of estimator convergence.

As seen in figure 2, performance of the proposed method is generally better to that of DDFSE, with similar performance at lower SNRs. But the DDFSE incurs a significantly larger computational cost. For our example, it needs four filters but the proposed algorithm requires only two filters, independent of the channel memory length. In general, the number of filters required in the proposed method is K, whereas the DDFSE requires a K^V (where V is the truncated channel length) number of filters. The convergence of channel coefficients is shown in figure 3 where the mean square estimation (MSE) error at each SNRs are plotted. The MSE error was obtained by taking the mean square error for each of the 20 simulations and then averaging it out for each SNRs. As seen in the figure, the IMM based algorithm performs better than the PSP DDFSE.

6. CONCLUSIONS

In this paper, an IMM based channel estimator and equalizer has been derived. The algorithm avoids the exponential growth of the state complexity with respect to the channel memory. It also utilizes fewer number of filters than the DDFSE scheme and hence is computationally more efficient.

The results on the simulations also demonstrate the superior BER performance and channel estimation accuracy of the IMM based scheme over the DDFSE. The proposed algorithm is an attractive proposition in terms of both computation and performance for channels with long channel response and/or for systems which uses bandwidth efficient coding where a large input alphabet is used.

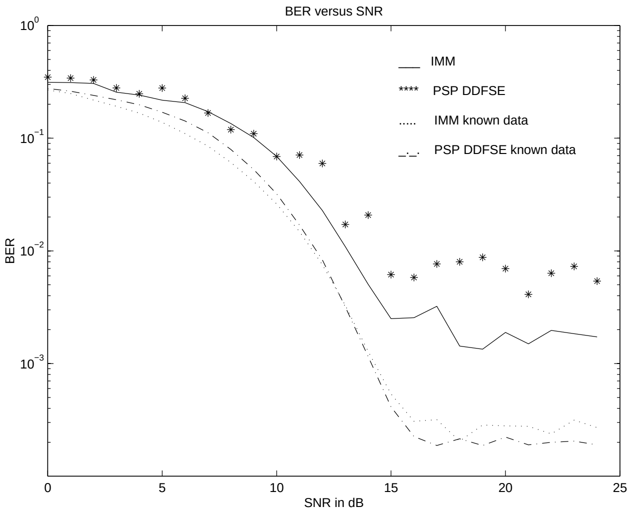

Fig. 2. Probability of symbol error (BER) with varying signal to noise ratio

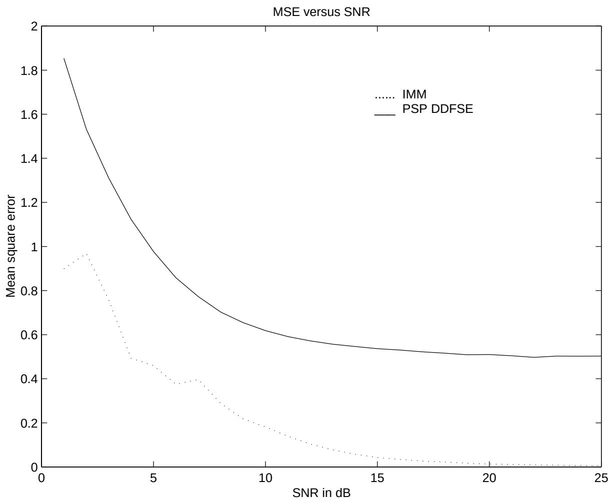

Fig. 3. Estimation error for channel coefficients

7. REFERENCES

[1] J.G. Proakis, *Digital communications*, McGraw-Hill, Inc, 3rd edition, 1995.

[2] G.D. Forney, "Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. on Information Theory*, vol. 18, no. 3, pp. 363–378, May 1972.

[3] G.D. Forney, "The Viterbi algorithm," *Proceedings of the IEEE*, vol. 61, no. 3, pp. 268–278, March 1973.

[4] M.V. Eyuboglu and S.U.H. Qureshi, "Reduced- state sequence estimation with set partitioning and decision feedback," *IEEE Trans. Commun.*, vol. 36, no. 1, pp. 13–20, Jan. 1988.

[5] M.V. Eyuboglu and S.U.H. Qureshi, "Reduced- state sequence estimation for coded modulation on intersymbol interference channels," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 989–995, Aug. 1989.

[6] A. Duel-Hallen and C. Heegard, "Delayed decision-feedback sequence estimation," *IEEE Trans. Commun.*, vol. 37, no. 5, pp. 428–436, May. 1989.

[7] G.K. Kaleh and R. Vallet, "Joint parameter estimation and symbol detection for linear or nonlinear unknown channels," *IEEE Trans. Communciations*, vol. 42, no. 7, pp. 2406–2413, July 1994.

[8] V. Krishnamurthy and L.B. White, "Blind equalization of FIR channels with Markov inputs," in *Proc. IFAC ACASP'92*, Grenoble, France, July 1992, pp. 633–638.

[9] V. Krishnamurthy and J.B. Moore, "On-line estimation of hidden markov model parameters based on the kullback-leibler information measure," *IEEE Trans. Signal Processing*, vol. 41, no. 8, pp. 2557–2575, August 1993.

[10] H.A. Cirpan and M.K. Tsatsanis, "Stochastic maximum likelihood methods for semi-blind channel estimation," *IEEE Signal Processing Letters*, vol. 5, no. 1, pp. 21–24, January 1998.

[11] H.A. Cirpan and M.K. Tsatsanis, "Blind receivers for nonlinearly modulated signals in multipath," *IEEE Trans. Signal Processing*, vol. 47, no. 2, pp. 583–586, February 1999.

[12] J.K. Tugnait, L. Tang, and Z. Ding, "Single-user channel estimation and equalization," *IEEE Signal Processing Magazine*, pp. 16–28, May 2000.

[13] J.K. Tugnait, "Detection and Estimation for Abruptly Changing Systems," *Automatica*, vol. 18, no. 5, pp. 607–615, 1982.

[14] Y. Bar-Shalom and X.R. Li, *Estimation and Tracking: Principles, Techniques and Software*, Artech House, Boston, MA, 1993.

[15] Y. Bar-Shalom, K.C. Chang, and H.A.P. Blom, "Tracking a maneuvreing target using input estimation versus the interacting multiple model algorithm," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 25, no. 2, pp. 296–300, April 1989.

[16] J.V. Candy, *Signal Processing: The model-based approach*, McGraw-Hill Book Company, 1987.

[17] R. Raheli, A. Polydoros, and C. Tzou, "Per-survivor processing: A general approach to MLSE in uncertain environements," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 354–364, Feb./March/April 1995.

[18] G.L. Stuber, *Principles of Mobile Communication*, Kluwer Academic Publishers, 1996.