BILINEAR SIGNAL SYNTHESIS IN ARRAY PROCESSING

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ABSTRACT

Multiple source signals impinging on an antenna array can be separated by time-frequency synthesis techniques. Averaging of the time-frequency distributions of the data across the array permits the spatial signatures of sources to play a fundamental role in improving the synthesis performance. This improvement is achieved independent of the temporal characteristics of the source signals and without causing any smearing of the signal terms. Unlike the recently devised blind source separation methods using spatial time-frequency distributions, the proposed method does not require whitening or retrieval of the source directional matrix.

1. INTRODUCTION

Time-frequency distributions (TFDs) have been shown to be a powerful tool in nonstationary signal analysis and synthesis [1, 2, 3, 4]. The TFD in all its bilinear and higher order forms represents a powerful tool for high resolution angle-of-arrival (AOA) estimation and recovery of the source waveforms impinging on a multi-sensor receiver, specifically those of nonstationary temporal characteristics.

The existing array signal processing techniques for nonstationary source separation using bilinear transforms require the construction of spatial time-frequency distribution (STFD) matrices from the sensor data. It was shown in [5, 6] that the formula relating this matrix to that of the sources is identical to the relationship between the data spatial covariance matrix and the source correlation matrix. Therefore, blind source separation (BSS) can be performed using the source time-frequency (t-f) signatures, instead of their correlation functions.

In this paper, we introduce a new approach for t-f signal synthesis in multi-sensor receivers. This approach utilizes the sources' spatial structures to enhance their signatures in the t-f domain. This is achieved by averaging the time-frequency distributions of the data across the array. Bilinear signal synthesis methods can then be applied to the enhanced source t-f features to recover the signal waveform and its temporal characteristics. Unlike source separation techniques based on STFD, the proposed approach does not require whitening or retrieval of the source directional matrix, thereby, simplifies the signal recovery process. This is achieved independent of the temporal characteristics of the source signals and without causing any smearing of the signal auto-terms.

The paper is organized as follows. The signal model is presented in Section 2. The proposed array averaging technique for time-frequency signal synthesis is formulated in Section 3. Numerical simulations illustrating the performance of the proposed method are given in Section 4.

2. SIGNAL MODEL

Assume L source signals incident on an M-sensor array. The propagation delay between antenna elements is assumed to be small relative to the inverse of the transmission bandwidth, so that the received signals are identical to within a complex constant. The data received across the array is given by the narrowband model

$$\mathbf{x}(t) = \mathbf{y}(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad t = 1, \cdots, N, \quad (1)$$

where $\mathbf{x}(t) = [\mathrm{x}_1(t), \cdots, \mathrm{x}_M(t)]^T$ is the $M \times 1$ data snapshot vector and $\mathbf{s}(t) = [\mathrm{s}_1(t), \cdots, \mathrm{s}_L(t)]^T$ is the $L \times 1$ source signal vector at time instant t, respectively. The superscript ^{T} denotes the vector/matrix transpose. The $M \times 1$ vector $\mathbf{n}(\mathrm{t})$ is the stationary white noise vector. Moreover, $\mathbf{A}$ denotes the $M \times L$ mixing matrix,

$$\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_L]. \quad (2)$$

The columns of matrix $\mathbf{A}$ are the source spatial signatures (SSs), and are given by

$$\mathbf{a}_i = [\mathrm{a}_{i1}, \cdots, \mathrm{a}_{iM}]^T, \quad i = 1, \cdots, L, \quad (3)$$

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where a_{ij} is the jth component of the ith SS $\mathbf{a}_i$. To simplify the discussion, we exchange any possible scalar factor embedded in $\mathbf{a}_i$ to the source signal and assume that $||\mathbf{a}_i||_2 = M$. It is obvious that this exchange does not affect the data observed from the antenna array. For the purpose of subsequent derivation, we expand equation (1) using definitions (2) and (3) to obtain the received noise-free data vector. The data received at sensor k ($k = 1, 2, \cdots, M$) is given by

$$y_k(t) = \sum_{i=1}^{L} a_{ik} s_i(t). \tag{4}$$

3. PROBLEM FORMULATION

3.1. Array Averaged WVD

The discrete form of WVD of the signal $y(t)$ is given by [4]

$$W_{yy}(t,f) = \sum_{l=-\infty}^{\infty} y(t+l) y^*(t-l) e^{-j4\pi fl}, \tag{5}$$

where * denotes complex conjugation. Substituting (4) into (5), we can express the WVD of the signal at the kth sensor $y_k(t)$ as

$$W_{y_k y_k}(t,f) \;=\; \sum_{i=1}^{L}\sum_{j=1}^{L} a_{ik} a_{jk}^* W_{s_i s_j}(t,f), \tag{6}$$

where $W_{y_k y_k}(t,f)$ will herein be referred to as the auto-sensor WVD of $y_k(t)$. $W_{s_i s_j}(t,f)$ corresponds to the auto-source or cross-source WVD, depending on whether $i = j$, or $i \neq j$.

Averaging the auto-sensor WVDs over the array yields

$$\begin{aligned} \bar{W}(t,f) &= \frac{1}{M}\sum_{k=1}^{M} W_{y_k y_k}(t,f) \\ &= \sum_{i=1}^{L}\sum_{j=1}^{L} \left(\frac{1}{M}\mathbf{a}_j^H \mathbf{a}_i\right) W_{s_i s_j}(t,f). \end{aligned} \tag{7}$$

In equation (7), $\mathbf{a}_j^H \mathbf{a}_i$ is the inner product of the SSs $\mathbf{a}_i$ and $\mathbf{a}_j$. Define the spatial correlation coefficient

$$\beta_{ij} = \frac{1}{M}\mathbf{a}_j^H \mathbf{a}_i, \tag{8}$$

then equation (7) could be rewritten as

$$\bar{W}(t,f) = \sum_{i=1}^{L}\sum_{j=1}^{L} \beta_{ij} W_{s_i s_j}(t,f). \tag{9}$$

Equation (9) shows that $\bar{W}(t,f)$ is a linear combination of the auto-source and cross-source WVDs that typically appear in the WVD of a multicomponent signal. However, in (9), these autoterms and crossterms are weighted by constant values represented by the spatial correlation coefficients that have resulted from the inner product between the sources' SSs.

It is straightforward to show that for the ith and the jth sources,

$$|\beta_{ij}| \le 1, i \neq j \;\text{ and }\; \beta_{ij} = 1, i = j, \tag{10}$$

indicating that the multiplication constants in (9) associated with the auto-source WVDs are always greater than or at least equal to those for the cross-source WVDs. This property is the key offering of the array averaging process and is shown to improve the signal synthesis performance.

An interesting case is when all SSs are orthogonal, i.e., $\beta_{ij} = 0$ for any $i \neq j$. In this case,

$$\bar{W}(t,f) = \sum_{k=1}^{L} W_{s_k s_k}(t,f). \tag{11}$$

In (11), $\bar{W}(t,f)$ is solely the summation of the source signal autoterms. This underscores the fact that all source signal crossterms are entirely eliminated and only the autoterms are maintained, which is most desirable from the signal synthesis perspective. It is important to note that by the virtue of the inner product, the source directional information carried by its respective SS is lost in $\bar{W}(t,f)$.

It is easy to infer from above equations that the extent to which the crossterms are mitigated is TFD-blind, as it does not depend on any specific t-f kernel that might be used for averaging the data bilinear products in (9). Crossterm mitigation depends exclusively on the expression of β_{ij} which is determined by the channel and the source spatial structures. On the other hand, the integration of both types of spatial and t-f smoothing can, indeed, result in crossterm suppression that cannot be achieved by each type applied alone.

3.2. WVD-Based Synthesis

The WVD-based synthesis techniques could be found in [7, 8]. In this paper, we apply the method of *extended discrete-time Wigner distribution* (EDTWD), introduced in [8], to the output of array averaged WVD given in (9). The advantage of using the EDTWD lies in the fact that it does not require *a priori* knowledge of the source waveform, and thereby avoids the problem of matching the two "uncoupled" vectors (even-indexed and odd-indexed vectors).

The proposed signal synthesis technique for multi-antenna receivers is fundamentally different from other techniques that integrate array signal processing with the bilinear distributions, e.g., the spatial time-frequency distributions. The proposed technique does not require whitening or retrieval of the source directional matrix, thereby, simplifies the signal recovery process. Further, averaging TFDs across the array produces a t-f weighting function applied in an equivalent single sensor WVD problem. This function decreases the noise levels, reduces the interactions of the source signals, and mitigates the cross-terms. It does so independent of the temporal characteristics of the source signals and without causing any smearing of the signal auto-terms.

4. SIMULATIONS

In this section, we provide computer simulations to demonstrate the improvement gained by the proposed technique in the reduction or elimination of crossterms. Signals are incident on an eight-sensor ULA ($M = 8$) with inter-element spacing of half-wavelength. The additive noise is zero mean, Gaussian distributed, spatially and temporally white process. The length of the signal sequence is set to $N = 128$.

In the first example, three chirp signals, $\mathrm{s}_1(t)$, $\mathrm{s}_2(t)$ and $\mathrm{s}_3(t)$, arrive at the array with AOAs of $-20°$, $0°$ and $20°$, with the respective start and end frequencies given by $(0.9\pi, 0.5\pi)$, $(0.66\pi, 0.26\pi)$, and $(0.5\pi, 0.1\pi)$. In the t-f plane, the source signals have parallel signatures. The crossterm of $\mathrm{s}_1(t)$ and $\mathrm{s}_3(t)$ also forms a chirp-like crossterm structure whose frequency starts from 0.7π and ends with 0.3π, and therefore lies closely to the t-f signature of $\mathrm{s}_2(t)$. Figure 1 depicts the WVD of the signals at the reference sensor (sensor #1) for the case of noise-free environment. It is clear that the t-f signature of all signal autoterms and crossterms are parallel in the t-f domain. The crossterms produced from the three source signals are even more dominant than the source autoterms. In the single sensor receiver, it becomes difficult to distinguish the source autoterms from the crossterms without any *a priori* knowledge of the sources. Using the above values of AOAs, we obtain $\left|\frac{1}{8}\beta_{ij}\right| < -13\text{dB}$, $i \neq j$, indicating that the sources spatial signatures are weakly correlated, and the array averaging process could result in a substantial reduction in the crossterms. Figure 2 shows the corresponding array-averaged WVD. Due to the reduction in the cross-terms by more than 13dB, the t-f signatures of the sources are distinctively exhibited in the new plots. In particular, the crossterm from $\mathrm{s}_1(t)$ and $\mathrm{s}_3(t)$ ceased to become an interfering effect in identifying the adjacent signal source $\mathrm{s}_2(t)$. In effect, averaging the WVDs across the array significantly reduces the crossterms, whereas the three signals' autoterms remain intact.

Next, we add 5dB noise to the data at each array sensor so that the input SNR is -5dB. Figures 3 and 4 depict both the reference-sensor WVD and the array-averaged WVDs. It is evident that the noise obscures both the signal autoterms and crossterms of the WVD at a single sensor. It is difficult, therefore, to retrieve the desired signal if we only synthesize from a single sensor. Upon averaging, both noise and crossterms are sufficiently reduced to clearly manifest the individual source t-f signature. Subsequently, the signals could be recovered if we place the appropriate masks in the t-f region and then perform least square synthesis. Figure 5 shows the WVD of the synthesized signal $\mathrm{s}_2(t)$ after array averaging. Figure 6 displays the real part of the original signal $\mathrm{s}_2(t)$ and its synthesized versions, $\hat{\mathrm{s}}_2(t)$, using the STFD and the proposed method. It is clear that the result from the array averaging technique is closer to the original signal than the STFD-synthesized signal.

5. CONCLUSION

A synthesis technique using array-averaged quadratic distributions was proposed for multi-sensor receivers. It was shown that this synthesis approach is fundamentally different form the one recently devised using spatial time-frequency distributions. In the latter, the source spatial signatures need to be first estimated before the sources could be separated. The key attraction of the proposed approach is that it naturally extends bilinear signal synthesis to array processing. In doing so, it capitalizes on the spatial dimension to reduce the cross-terms without smearing the auto-terms, which could not be done using the t-f smoothing operation via reduced interference distributions.

6. REFERENCES

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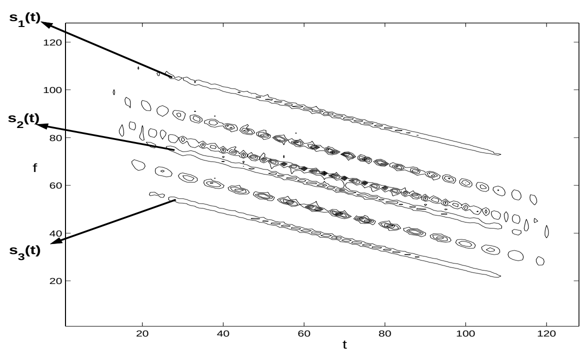

Fig. 1 WVD in noise-free environment at the reference sensor.

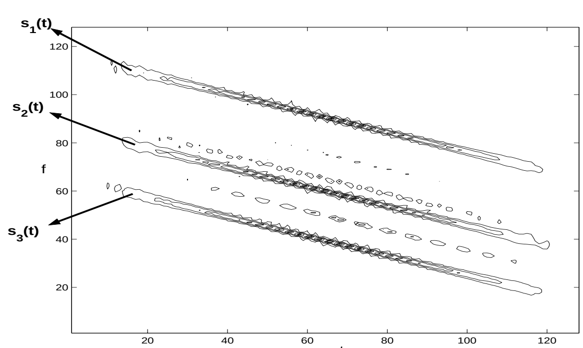

Fig. 2 Array-averaged WVD in noise-free environment.

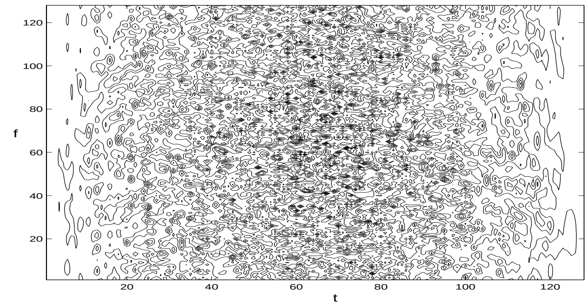

Fig. 3 WVD of the corrupted signals at the reference sensor.

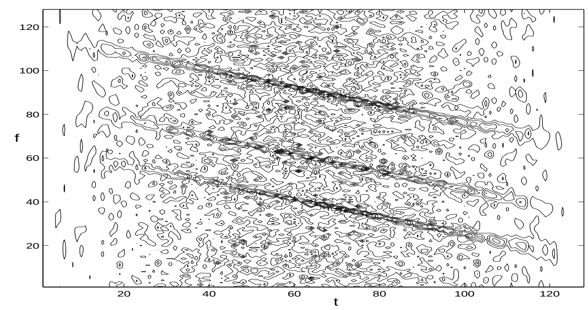

Fig. 4 Array-averaged WVD of the corrupted signals.

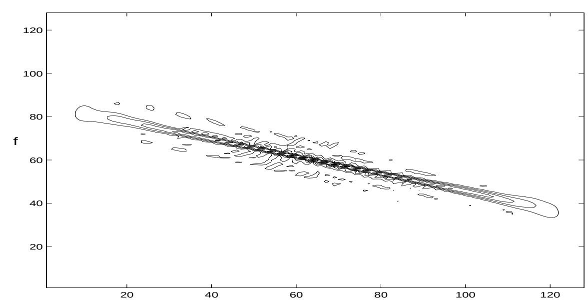

Fig. 5 WVD of synthesized $\hat{s}_2(t)$.

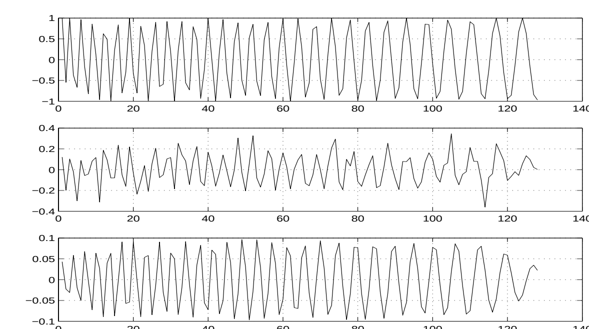

Fig. 6 (top) Real part of original $s_2(t)$; (middle) Real part from the STFD-recovered $\hat{s}_2(t)$; (bottom) Real part from the array averaged $\hat{s}_2(t)$.