SUBSPACE DETECTORS FOR STOCHASTIC PROCESS SHIFT KEYING

Arnt-Børre Salberg and Alfred Hanssen

University of Tromsø, Physics Department,
Electrical Engineering Group, N-9037 Tromsø, Norway
E-mail:{arnt,alfred}@phys.uit.no

ABSTRACT

We present a new digital modulation technique that introduces covertness in digital communications in a simple fashion. The basic principle is to transmit realizations of a stochastic process in such a manner that the transmitted waveform appears noiselike. The transmitted waveform is expressed in a subspace formalism, allowing for an elegant geometrical interpretation of the waveform, and a simple and accurate subspace detector for the receiver. The effect of inter-symbol-interference (ISI) is also studied, and a simple zero-forcing subspace detector is suggested. The technique is demonstrated by numerical simulations, and it shows that our simple subspace detectors yield high-quality and reliable receivers.

1. INTRODUCTION

An obvious way of introducing covertness in digital communications, is to ensure that the transmitted waveform appears like wideband noise. Spread-spectrum techniques e.g. [1], apply a known quasi-stochastic spreading sequence to increase the bandwidth of the transmitted signal, and thus obtain some degree of privacy. Limitations are imposed by the need to be strictly synchronous with the transmitter.

Salberg and Hanssen in [2], [3] proposed the following low-probability-of-intercept method for encoding digital information. Transmit a realization of a stochastic process $X_0(t)$, $0 \le t < T_s$ to represent bit zero, and a realization of another stochastic process $X_1(t)$, $0 \le t < T_s$ to represent bit one. Here T_s is the symbol duration. Thus, rather than altering aspects of a deterministic signal, realizations of two different stochastic processes are transmitted. This has the effect that two subsequent equal source bits have different transmitted waveforms. In addition, two different source bits have similar waveforms, due to the fact that they are close in a statistical sense. The transmitted waveform representing a bit string will thus appear noiselike, and it contains no repetitions or periodicities. Moreover, the waveform contains no discontinuities, so the pulse length is also hidden. Since the transmitted baseband waveform is noiselike, a transmission would not attract the attention of unfriendly receivers. It is obvious that this signaling method adds an extra (physical) layer of security in digital communication, thus reducing the risk of eavesdropping.

2. NOTATION

Using the same notation as in [4], we let the vectors $\boldsymbol{\gamma}, \boldsymbol{\phi} \in \mathbb{L}^2$ represent the continuous-time, finite-energy signals $\gamma(t)$ and $\phi(t)$, respectively. The usual inner product of these vectors is defined as

$$\langle \boldsymbol{\gamma}, \boldsymbol{\phi} \rangle = \int_{-\infty}^{\infty} \gamma(t)\phi(t)\,dt. \quad (1)$$

Given an indexed set of signals $\mathcal{F} = \{\boldsymbol{\phi}_k\}_{k=1}^{K} \subset \mathbb{L}^2$, $\boldsymbol{\phi}_k \in \mathbb{L}^2$, let $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_K]^T \in \{\mathbb{L}^2\}^K$. We find the multi-linear inner product taking $\{\mathbb{L}^2\}^K \times \mathbb{L}^2$ into $\mathbb{R}^K$ to be

$$\{\langle \boldsymbol{\gamma}, \boldsymbol{\Phi} \rangle\}_i = \langle \boldsymbol{\gamma}, \boldsymbol{\phi}_i \rangle = \int_{-\infty}^{\infty} \gamma(t)\phi_i(t)\,dt, \quad (2)$$

where $\{\cdot\}_i$ denotes the ith element. For future reference, define $\mathcal{S}^\tau$ to be a delay operator in $\mathbb{L}^2$, i.e. $\mathcal{S}^\tau \boldsymbol{\gamma} = \gamma(t-\tau)$.

3. STOCHASTIC PROCESS SHIFT KEYING

The transmitted waveform for an infinite duration *Stochastic Process Shift Keying* (SPSK) signal suggested in [2],[3] can be written as

$$X(t) = \sum_{n=-\infty}^{\infty} \sum_{k=1}^{K} x_n(k)\phi_k(t - nT_s), \quad (3)$$

where $\phi_k(t)$ is the kth basis function, and $x_n(k)$ is the kth element of the random vector $\mathbf{x}_n = [x_n(1), \ldots, x_n(K)]^T$. The distribution of $\mathbf{x}_n$ is determined by the nth symbol, and $\mathbf{x}_n$ is thus a symbol vector. The received continuous time pulse $\mathbf{r} \in \mathbb{L}^2$ representing the nth symbol is [4]

$$\mathbf{r} = \sum_{k=1}^{K} x_n(k)\boldsymbol{\phi}_k + \mathbf{n}, \quad (4)$$

where $\boldsymbol{\phi}_k \in \mathbb{L}^2$ corresponds to $\phi_k(t)$, and $\mathbf{n}$ is an additive white Gaussian noise process. For the detection problem we form a vector $\mathbf{y}_n = G(\mathbf{r})$, $G : \mathbb{L}^2 \to \mathbb{R}^K$, by correlating $\mathbf{r}$ against the elements of a basis $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_K]^T$, $\mathbf{b}_i \in \mathbb{L}^2$,

$$\mathbf{y}_n = G(\mathbf{r}) = \langle \mathbf{r}, \mathbf{B} \rangle. \quad (5)$$

Thus, $\mathbf{y}_n$ is a coordinate vector of $\mathbf{r}$ with respect to the basis $\mathbf{b}_1, \ldots, \mathbf{b}_K$, when the nth symbol is transmitted. The decoding can be made simple if $\mathbf{B}$ are chosen to be orthonormal to signaling waveforms $\boldsymbol{\Phi}$, i.e.

$$\langle \boldsymbol{\Phi}, \mathbf{B} \rangle = \mathbf{I}_K, \quad (6)$$

where $\mathbf{I}_K$ denotes the $K \times K$ identity matrix. In that case the received vector is

$$\mathbf{y}_n = \mathbf{x}_n + \mathbf{n}. \quad (7)$$

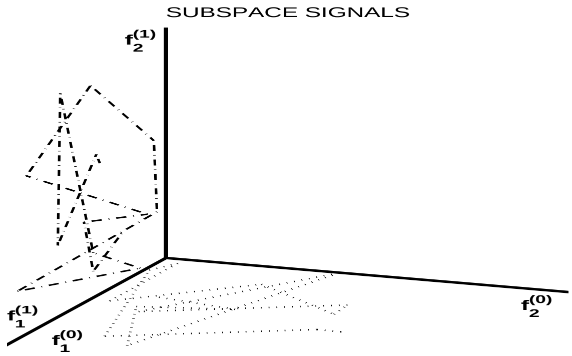

Fig. 1. Trajectories of the subspace signals $\mathbf{x}_0$ (dotted) and $\mathbf{x}_1$ (dash-dotted).

3.1. Parallel transmission scheme

In a parallel transmission scheme the K basis functions (signaling waveforms) $\phi_1(t), \ldots, \phi_K(t)$ are different (often orthonormal) waveforms, and are transmitted simultaneously in time over the channel. The wavelet-based orthonormal modulation code set used in [3] is thus a parallel transmission scheme.

3.2. Serial transmission scheme

In the case of serial transmission the K basis functions can be chosen as time delays of a common chip pulse $\phi_k(t) = \psi(t-(k-1)T_c)$, where the chip period $T_c = T_s/K$. The symbol vector $\mathbf{x}_n = [x(nK), x(nK+1), \ldots, x(nK+K-1)]^T$ is converted into a serial chip sequence $x(l)$ before transmission, and the transmitted SPSK signal can then be written as

$$X(t) = \sum_{l=-\infty}^{\infty} x(l)\psi(t-lT_c), \tag{8}$$

and the received time pulse $\mathbf{r} \in \mathbb{L}^2$ representing the nth symbol is

$$\mathbf{r} = \sum_{k=1}^{K} x(nK+k-1)\boldsymbol{\phi}_k + \mathbf{n}. \tag{9}$$

3.3. Redundancy matrices

We have chosen to define the symbol vector as [3]

$$\mathbf{x}_i = \mathbf{F}_i \mathbf{s}_i, \quad i = 0, 1, \tag{10}$$

where $\mathbf{s}_i$ is a $M \times 1$ random vector drawn from a multivariate probability density $p_{\boldsymbol{\theta}_i}(\mathbf{s})$, $\boldsymbol{\theta}_i$ is a relevant parameter vector, $\mathbf{F}_i$ is a $K \times M$ *redundancy matrix* which introduces redundancy in the symbol vector. The symbol vector $\mathbf{x}_n$ representing the nth symbol is known to lie in the M dimensional linear subspace $\langle \mathbf{F}_i \rangle$ spanned by the columns of $\mathbf{F}_i = [\mathbf{f}_1^{(i)}, \ldots, \mathbf{f}_M^{(i)}]$. This is illustrated in Fig. 1 where the dotted line is the trajectory of the subspace signal $\mathbf{x}_0$, and the dash-dotted line is the trajectory of the subspace signal $\mathbf{x}_1$, $M = 2$, and $K = 3$. From the figure we see the randomness of the signals $\mathbf{x}_0$ and $\mathbf{x}_1$, and that $\mathbf{x}_i$ is in the subspace spanned by the columns of $\mathbf{F}_i = [\mathbf{f}_1^{(i)}, \mathbf{f}_2^{(i)}]$.

3.4. Normalization

To ensure that a every symbol has equal energy, normalization of the noise vector $\mathbf{s}_n$ is necessary. The normalized noise vector is

$$\mathbf{s}_n^{norm} = \sqrt{E_b}\frac{\mathbf{s}_n}{||\mathbf{s}_n||}, \tag{11}$$

and the trajectories of the subspace signals will now be on a hyper sphere centered in origin, and with radius equal to $\sqrt{E_b}$.

4. CHANNEL EFFECTS

The SPSK signal $X(t)$ is transmitted over a channel having an impulse response $\mathbf{c} \in \mathbb{L}^2$. The received pulse corresponding with the nth transmitted pulse is now

$$\mathbf{r} = \sum_{k=1}^{K} x_n(k)\mathbf{u}_k + \mathbf{n}, \tag{12}$$

where $\mathbf{u}_k \in \mathbb{L}^2$ corresponds to the time signal $u_k(t) = c(t) * \phi_k(t)$, and $*$ denotes the convolution operator.

4.1. Parallel transmission

Assume that the overall channel $h(t)$ has a time support length equal to λT_s, $\lambda > 1$, and define $L = \lceil \lambda \rceil$, where $\lceil \cdot \rceil$ denotes the ceiling-integer. Furthermore, in the case of zero ISI, $\mathbf{x}_n$ can be recovered after an initial delay of α pulses. Then, the received vector can be written as

$$\begin{aligned}\mathbf{y}_n &= \sum_{m=-\alpha+1}^{L-\alpha} \langle \mathcal{S}^{-mT_s}\mathbf{r}, \mathbf{B}\rangle \\ &= \sum_{m=-\alpha+1}^{L-\alpha}\sum_{k=1}^{K} x_{n-m}(k)\langle \mathcal{S}^{-mT_s}\mathbf{u}_k, \mathbf{B}\rangle. \end{aligned} \tag{13}$$

Define

$$\mathbf{H}_m = [\langle \mathcal{S}^{-mT_s}\mathbf{u}_1, \mathbf{B}\rangle, \ldots, \langle \mathcal{S}^{-mT_s}\mathbf{u}_K, \mathbf{B}\rangle] \in \mathbb{R}^{K\times K}, \tag{14}$$

and the received vector can now be written as

$$\mathbf{y}_n = \sum_{m=-\alpha+1}^{L-\alpha} \mathbf{H}_m \mathbf{x}_{n-m} = \mathbf{H}_n * \mathbf{x}_n. \tag{15}$$

Eq. (15) states that the ISI part of the received vector can be large, depending on L. For instance, if we use Daubechies 4 wavelets as basis functions [3], we have that $L > 12$. Thus, the received symbol vector $\mathbf{y}_n$ is influenced by more than 12 symbol vectors.

4.2. Serial transmission

Assume that the overall channel $h(t)$ has a time support length equal to λT_c, $\lambda > 1$, $L = \lceil \lambda \rceil$, and assume that $L < K$ and $M < K - L + 1$. Furthermore, in the case of zero ISI, $x_n(k)$ can be recovered after an initial delay of α chip pulses. Then the received vector is

$$\begin{aligned}\mathbf{y}_n &= \sum_{m=-1}^{1}\sum_{k=1}^{K} x_{n-m}(k)\langle \mathcal{S}^{-mT_s}\mathbf{u}_k, \mathbf{B}\rangle \\ &= \mathbf{H}_{-1}\mathbf{x}_{n+1} + \mathbf{H}_0\mathbf{x}_n + \mathbf{H}_1\mathbf{x}_{n-1}.\end{aligned}$$

Since $L < K$, we have that $\mathbf{H}_0$,$\mathbf{H}_1$, and $\mathbf{H}_{-1}$ can be written as

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & \dots & h_{-\alpha+1} & 0 & \dots & 0 \\ \vdots & \ddots & & \ddots & & \vdots \\ h_{L-\alpha} & & & & \ddots & 0 \\ 0 & & & \ddots & & h_{-\alpha+1} \\ \vdots & & \ddots & & & \vdots \\ 0 & \dots & 0 & h_{L-\alpha} & \dots & h_0 \end{bmatrix}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \dots & h_{L-\alpha} & \dots & h_1 \\ \dots & \ddots & 0 & \ddots & \vdots \\ 0 & \dots & \ddots & \dots & h_{L-\alpha} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix} \qquad (16)$$

$$\mathbf{H}_{-1} = \begin{bmatrix} 0 & 0 & \dots & & & 0 \\ \vdots & \vdots & & & & \\ h_{-\alpha+1} & 0 & & & & \vdots \\ \vdots & h_{-\alpha+1} & \ddots & & & \\ & \vdots & \ddots & \ddots & & \\ h_{-1} & h_{-2} & \dots & h_{-\alpha+1} & \dots & 0 \end{bmatrix},$$

where $h_k = \langle \mathbf{u}_l, \mathbf{b}_{l+k} \rangle$, $l + k \in \{1, \dots, k\}$, $\mathcal{S}^{-T_s}\mathbf{u}_k = \mathbf{u}_{k-K}$, and $\mathcal{S}^{T_s}\mathbf{u}_k = \mathbf{u}_{K+k}$.

Thus, the elements $y_n(1), \dots, y_n(L-\alpha)$ and $y_n(K-\alpha+2), \dots, y_n(K)$ of the received symbol vector $\mathbf{y}_n$ will contain ISI from the $(n-1)$th and $(n+1)$th symbols. An oftenly used method to cope with this effect, is to discard precisely those elements [5]. Define $\overline{\mathbf{B}} = [\mathbf{b}_{L-\alpha+1}, \dots, \mathbf{b}_{K-\alpha+1}]^T$, and $\overline{\mathbf{y}}_n = [y_n(L-\alpha+1), \dots, y_n(K-\alpha+1)]^T$, then,

$$\overline{\mathbf{y}}_n = \sum_{k=1}^{K} \langle \mathbf{u}_k, \overline{\mathbf{B}} \rangle = \langle \mathbf{r}, \overline{\mathbf{B}} \rangle = \overline{\mathbf{H}}_0 \mathbf{x}_n, \qquad (17)$$

where $\overline{\mathbf{H}}_0 = \mathbf{G}\mathbf{H}_0$, and

$$\mathbf{G}^T = \begin{bmatrix} \mathbf{0}_{(L-\alpha)\times(K-L+1)} \\ \mathbf{I}_{(K-L+1)\times(K-L+1)} \\ \mathbf{0}_{(\alpha-1)\times(K-L+1)} \end{bmatrix}. \qquad (18)$$

5. SUBSPACE DETECTORS

Based on the received vector $\mathbf{y}_n$ we must decide whether a bit zero or a bit one was sent. Since the transmitted vectors are subspace signals, *Matched Subspace Detectors* are well suited detectors [6]. The benefit of such detectors is that the decision criterion is independent of the additive noise variance $\sigma_n^2 = \mathcal{N}_0/2$. We define the orthogonal projection operator as [6]

$$\mathbf{P}_{F_i} = \mathbf{F}_i(\mathbf{F}_i^T\mathbf{F}_i)^{-1}\mathbf{F}_i^T, \quad i = 0, 1 \qquad (19)$$

so that $\mathbf{P}_{F_i}\mathbf{r}$ is a projection of $\mathbf{r}$ onto the subspace $\langle \mathbf{F}_i \rangle$.

5.1. No ISI

In this case the received vector is given by Eq. (7), and the decision criterion is that we choose class Ω_0 if

$$\mathbf{y}_n^T \mathbf{P}_{F_0} \mathbf{y}_n > \mathbf{y}_n^T \mathbf{P}_{F_1} \mathbf{y}_n, \qquad (20)$$

and otherwise choose class Ω_1. The detector measures the amount of the received energy that resides in subspace $\langle \mathbf{F}_i \rangle$, and then chooses the class corresponding to the subspace containing the largest amount of energy [3].

An exact expression for the bit-error probability in the case of orthonormal subspaces will be given in section 5.3.

5.2. Serial transmission and ISI

In section 4.2 we suggested to use the receiver matrix $\mathbf{G}$ to discard the elements in $\mathbf{y}_n$ that contain ISI. Using a receiver matrix, the classification criterion is to choose class Ω_0 if

$$\overline{\mathbf{y}}_n^T \mathbf{P}_{GH_0F_0} \overline{\mathbf{y}}_n > \overline{\mathbf{y}}_n^T \mathbf{P}_{GH_0F_1} \overline{\mathbf{y}}_n, \qquad (21)$$

and otherwise choose class Ω_1. The channel and the receiver matrix $\mathbf{G}$ causes the signal subspaces to change from a M-dimensional subspace $\langle \mathbf{F}_i \rangle$ in $\mathbb{R}^K$ to a M_i-dimensional subspace $\langle \mathbf{G}\mathbf{H}_0\mathbf{F}_i \rangle$ in $\mathbb{R}^{K-L+1}$, where $M_i \leq M$.

5.3. Exact bit-error probability

We will give an exact expression of the bit-error probability (BEP) under the assumption of a frequency nonselective channel, orthogonal signal subspaces, $\mathbf{s}_i \sim \mathcal{N}(\mathbf{0}, \sigma_s^2\mathbf{I})$, $i = 0, 1$, and no normalization of the noise vector. Assume that bit one is sent. Then, $\mathbf{F}_1^T\mathbf{y}_n$ and $\mathbf{F}_0^T\mathbf{y}_n$ are distributed as

$$\mathbf{F}_1^T\mathbf{y}_n \sim \mathcal{N}(\mathbf{0}, (\sigma_s^2 + \sigma_n^2)\mathbf{I}) \qquad (22)$$

$$\mathbf{F}_0^T\mathbf{y}_n \sim \mathcal{N}(\mathbf{0}, \sigma_n^2\mathbf{I}). \qquad (23)$$

Define $Z_i = \mathbf{y}_n^T\mathbf{F}_i\mathbf{F}_i^T\mathbf{y}_n = \mathbf{y}_n^T\mathbf{P}_{F_i}\mathbf{y}_n$, then the probability density functions of Z_0 and Z_1 are obviously

$$f_{Z_1}(z) = Bz^{M/2-1}\exp(-bz) \qquad (24)$$

$$f_{Z_0}(z) = Az^{M/2-1}\exp(-az), \qquad (25)$$

where

$$a = \frac{1}{2\sigma_n^2} \qquad b = \frac{1}{\sigma_s^2 + \sigma_n^2} \qquad (26)$$

$$A = \frac{a^{M/2}}{(M/2-1)!} \qquad B = \frac{b^{M/2}}{(M/2-1)!} \qquad (27)$$

The BEP can therefore be written as [1]

$$P_e = \int_0^\infty \left\{ f_{Z_1}(z_1) \int_{z_1}^\infty f_{Z_0}(z)\, dz_0 \right\} dz_1$$
$$= \frac{1}{(1+a/b)^{M/2}} \left[\sum_{j=0}^{M/2-1} \frac{(M/2+j-1)!}{(M/2-1)!j!} \left(\frac{a/b}{1+a/b} \right)^j \right]. \qquad (28)$$

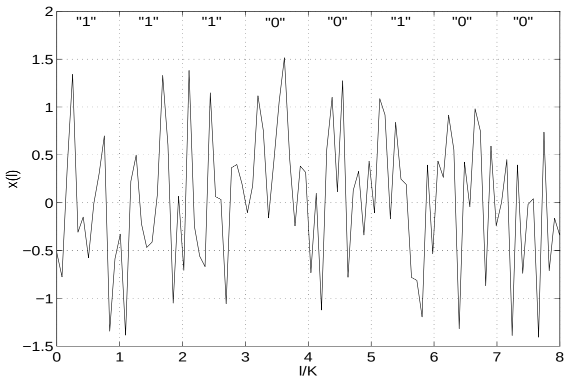

Fig. 2. The chip sequence $x(l)$ of the message '11100100'.

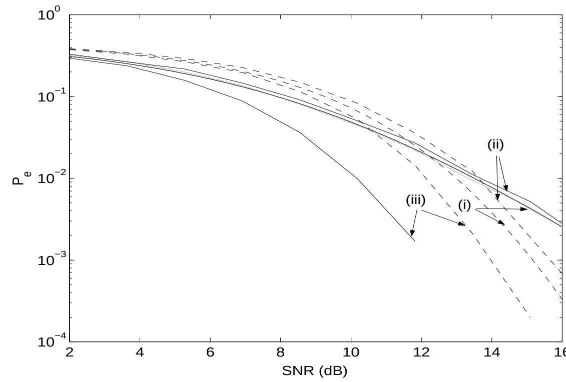

Fig. 3. Bit-error probability as a function of SNR. Full lines: $M = 6$ and $K = 16$. Dashed lines: $M = 24$ and $K = 52$. Labels: (i) Exact BEP with zero ISI and no gain normalization, (ii) Monte Carlo simulations of the BEP, including ISI and normalization of noise vector, and (iii) Monte Carlo simulations of the BEP with zero ISI and noise vector normalization.

6. SIMULATIONS

To demonstrate the proposed digital modulation scheme, we now present some numerical simulations.

In our simulations the subspace matrices $\mathbf{F}_0$ and $\mathbf{F}_1$ are chosen to be orthogonal [6], and constructed from the orthonormal eigenvectors of the $K \times K$ covariance matrix of an AR(2) process with $a_1 = 0.81$, $a_2 = 0.35$, and $\sigma^2 = 1$. This is a simple way of constructing the subspace matrices, but obviously not the only possibility. Furthermore, $p_{\boldsymbol{\theta}_0}(\mathbf{s}) = p_{\boldsymbol{\theta}_1}(\mathbf{s})$ are both multivariate Gaussian $\mathcal{N}(\mathbf{0}, \mathbf{I})$.

Fig. 2 shows an example of a transmitted chip sequence representing the message '11100100'. One single bit is represented by $K = 12$ chip values. Observe that two subsequent equal source bits have different waveforms, due to the randomness of the symbol vector $\mathbf{x}_n$. Note also that the pulse length is hidden, and that there are no periodicities in the information carrying signal.

To demonstrate the detector given in Eq. (21) we now consider a FIR channel $h(z) = 0.2z + 1 + 0.4z^{-1} - 0.2z^{-2}$, and $\alpha = 2$ and $L = 4$.

As in conventional communications we define the signal-to-noise ratio (SNR) as $\mathrm{SNR} = E_b/\mathcal{N}_0$, which in our case can be written as $\mathrm{SNR} = M||\mathbf{h}||^2/\mathcal{N}_0$. We have chosen $K = 2M + L$.

Fig. 3 shows the exact and Monte Carlo simulations of the BEP of the subspace detectors with and without ISI. The full lines represent $M = 6$ and $K = 16$, whereas the dashed lines represent $M = 24$ and $K = 52$. The curves labeled (i) are exact BEP of the detector given in Eq. (20), as calculated by Eq. (28). The curves labeled (ii) are Monte Carlo simulations of the BEP of the detector given in Eq. (21), with channel $h(z)$ and noise vector normalization, and the curves labeled (iii) are Monte Carlo simulations of the BEP of the detector given in Eq. (20) with normalized noise vector. From Fig. 3 we see, as expected, that the BEP decreases as a function of increasing SNR. Furthermore, notice that the performance increases significantly when $\mathbf{s}_n^{norm}$ is used instead of $\mathbf{s}_n$. We also see that the channel $h(z)$ yields a decrease in performance due to that the new subspaces $\langle \mathbf{GH}_0\mathbf{F}_0 \rangle$ and $\langle \mathbf{GH}_0\mathbf{F}_1 \rangle$ are not (in general) orthogonal.

For low-probability-of-intercept communications, the dimension of $\mathbf{F}_0$ and $\mathbf{F}_1$ must be chosen such that the signal power per unit bandwidth is below the noise spectral density. In the simulations above, $M = 24$ implies the use of waveforms with higher frequency components than for $M = 6$. Thus, larger values of M spreads the transmitted signal over a wider frequency band. Since the energy of the transmitted baseband waveforms in the case of $M = 6$ and $M = 24$ are equal, the signal power per unit bandwidth is lower for the case of $M = 24$.

7. CONCLUSIONS

We have presented a new digital modulation technique that introduces some degree covertness in a simple fashion. The transmitted waveform is noiselike, and would therefore not attract the attention of unfriendly receivers. Using a subspace formalism, a simple and efficient zero-forcing subspace detector is constructed to cope with the channel effects.

8. REFERENCES

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