A Recurrent RBF Network For Non-Linear Channel

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abstract

On the conventional method to design the recurrent RBF networks for channel equalizer, firstly, the impulse response of channel is estimated with an adaptive FIR filter. Secondly, all noise free received signals are estimated with the estimate of impulse response. However, the performance of the network designed with this method is degraded down if the channel would be nonlinear.

In order to overcome this drawback, we apply Miyake's method to the conventional training method of recurrent RBF networks. In the proposed method, we estimate the received signals of noise free with the relation between the training signals and received signal. Then we design the recurrent RBF networks with noise free received signals.

Key words: Nonlinear channel, recurrent RBF network, noise-free received signal, noise variance

1. INTRODUCTION

Many digital communications channels are subject to noise, intersymbol interference and nonlinear distortion in a modulation and demodulation process. The equalizers are the systems to eliminate these distortions from received signal.

The RBF equalizer is a kind of nonlinear equalizers. This equalizer has the structure which is a linear combination of radial basis function (RBF) that has two parameters, vector called center and parameter called the width of RBF. Thus, the optimal RBF equalizer can be designed with adjusting weights if the centers and the width of RBF are optimal.

Now, it is known that the performance of optimal RBF equalizers can be improved by increasing the size of received signal vector in the case that this equalizer is used on the digital communication systems. Thus, in order to design the equalizer with high performance, we need to large size of recieved signal vector. However, the RBF equalizer has a drawback that the capacity of memory in an RBF equalizer grows up exponentially with the size of received signal vector.

So, Cid-Sueiro et al.[3] proposed the recurrent RBF network and training method of this network for equalizing.

In the recurrent RBF network, this network is designed so that the capacity of the memory to store the input signal to the network is minimized as smally as possible. Then the performance of this network is kept with using the past outputs of RBF recursively.

In the Cid-Sueiro's training method for recurrent RBF network, the recurrent RBF network is designed with the noise-free received signal that is estimated under the assumption that the channel is linear system. When channel is, however, nonlinear channel, the performance of recurrent RBF network designed with Cid-Sueiro's training method may be degraded down.

In this paper, we propose the method that is used to train the recurrent RBF network for nonlinear channel. It is known that the recurrent RBF network can be designed by obtaining all noise-free received signals and the noise variance. Thus, using the method to estimate all noise-free received signals of nonlinear channel and noise variance, we can design recurrent RBF network for the digital communication systems in which the characteristics of the channel are nonlinear channel.

In section 2 we show the channel model and notation for this paper. Section 3 presents some quantities used in this paper, the recurrent RBF network[3], the Cid-Sueiro's method[3] and Miyake's method[4]. Section 4 shows the problem of conventional method and proposed method. Simulation results are given in Section 5. Conclusions are summarized in Section 6

2. NONLINEAR CHANNEL [4]

The model of data communication system studied in this paper is shown in Fig.1 We define the $((N+1)\times 1)$ transmitted signal vector $X(t)$ as following

$$X(t) = [x(t), x(t-1), \cdots, x(t-N)]^T, \tag{1}$$

where a transmitted signal $x(t)$ is binary signal $\{+1,-1\}$ assumed to have an equal probability. The output of nonlinear channel $o(t)$ assumed to be described with the transmitted signal vector $X(t)$ as following N-th order nonlinear system:

$$o(t) = f(X(t)), \tag{2}$$

where the function $f()$ transforms the $((N\times 1)\times 1)$ vector $X(t)$ to the scalar output $o(t)$. The received signal $y(t)$ can be described as following:

$$y(t) = o(t) + e(t) \tag{3}$$

where $e(t)$ is assumed to be a Gaussian white noise having zero-mean and variance σ^2.

The received signal $y(t)$ is the input to the recurrent RBF equalizer. The output of this equalizer is transfered to the slicer to give the estimate $\hat{x}(t-d)$ where the delay d is set to satisfy:

$$0 \le d \le N \tag{4}$$

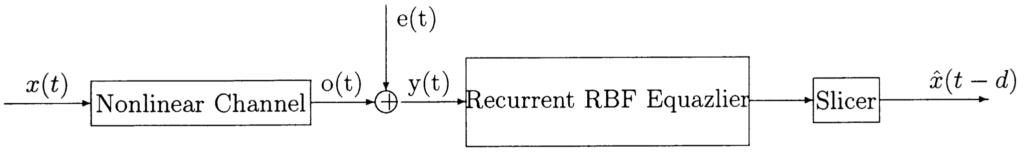

Figure 1: Digital Communication System

The slicer is the device given by the sign function:

$$Sgn(r) = \begin{cases} +1, & r \geq 0 \\ -1, & r < 0 \end{cases} \tag{5}$$

3. CONVENSIONAL METHODS

This section presents some quantities used in this paper, a recurrent RBF equalizer, the Cid-Sueiro's method and Miyake's method.

3.1. Preparation

Here, we define some quantities, which are used in order to explain Miyake's method and the Cid-Sueiro's recurrent RBF network.

We define S_x as the set which consists of all transmitted signal $X(t)$. Also, we define the function $L()$ as follows:

$$L(X) = \sum_{i=0}^{N} 2^{N-i} s(x_i) \tag{6}$$

where X denotes an element of set S_x and x_i denotes the i-th element of vector X and the function $s()$ is following function:

$$s(r) = \begin{cases} +1, & r \geq 0 \\ 0, & r < 0 \end{cases} \tag{7}$$

The function value $L(X(t))$ equals to value to be decimalited the following bit sequence

$$[s(x(t)), s(x(t-1)), \cdots, s(x(t-N))] \tag{8}$$

The function $L()$ is the one-to-one mapping from the set S_x to the set which consists of the integers from zero through $2^{N+1}-1$. Then we define X_l as the vector which is one of element of set S_x and which is mapped to integer l with the function $L()$. Finally, the noise-free received signal for $X(t) = X_l$ is defined as follows:

$$y_l = f(X_l) \tag{9}$$

3.2. The Recurrent RBF Network and training method of Cid-Sueiro[3]

Figure.2 shows the structure of recurrent RBF network proposed by Cid-Sueiro et al. In the recurrent RBF network, firstly, we calculate the value $g_j(t) (j = 0, 1, \cdots, 2^N - 1)$ as follows:

$$\begin{aligned} g_j(t) &= \frac{1}{2} \exp\left\{ -\frac{(y(t) - y_{2j})^2}{2\sigma^2} \right\} g_{\epsilon(j)}(t-1) \\ &+ \frac{1}{2} \exp\left\{ -\frac{(y(t) - y_{2j+1})^2}{2\sigma^2} \right\} g_{\epsilon(j)+1}(t-1) \end{aligned}$$

where $\epsilon(j)$ is defined as the following function:

$$\epsilon(j) = 2(j \ mod \ 2^{N-1}) \tag{10}$$

and y_{2j}, y_{2j+1} describe the noise-free received signal, y_l (where $l = 2 \cdot j$) and y_l(where $l = 2 \cdot j + 1$) respectively. Also, $g_{\epsilon(j)}(t-1)$ and $g_{\epsilon(j)+1}(t-1)$ describe $g_k(t-1)$ where $(k = \epsilon(j))$, and $g_k(t-1)$ where $(k = \epsilon(j)+1)$ respectively. Also, the term $(A \ mod \ B)$ denotes the remainder that A is divisible by B. For example, if $j = 3$ and $N = 2$, y_{2j},y_{2j+1},$g_{\epsilon(j)}(t-1)$ and $g_{\epsilon(j)+1}(t-1)$ are y_6, y_7, $g_2(t-1)$ and $g_3(t-1)$, respectively. In the recurrent RBF networks, the transmitted signal, $x(t-d)$, is estimated as follows:

$$\hat{x}(t-d) = s(f_e(t,d)) \tag{11}$$

where $f_e(t,d)$ is given by the following function:

$$f_e(t,d) = \sum_{j|b(j,d)=1} g_j(k) - \sum_{j|b(j,d)=0} g_j(k) \tag{12}$$

where $\sum_{j|b(j,d)=-1} g_j(k)$ denotes the sum of all $g_j(t)$ with j satisfing the condition,$b(j,d) = -1$ where

$$b(j,d) = ((j \ mod \ 2^{N-d}) \ div \ 2^{N-1-d}) \tag{13}$$

where $(A \ div \ B)$ denotes a quotient that A is divisible by B.

As shown in eq.(10), we need all noise-free received signals and noise-variance to design the recurrent RBF network. In the Cid-Sueiro's training method for this network, the channel is assumed to be linear and the channel impulse response is estimated by adaptive FIR filter. All noise-free received signal and noise variance are estimated with this estimate of channel response.

3.3. Miyake's method

The Miyake's method is used to train the RBF equalizer. This method consists of two-step. The first step is estimating noise-free received signal vector and noise variance with quasi-random noise whose period is longer than the number of all noise-free received signals. The second step is that the RBF equalizer is designed with this result. In this paper, we show the first procedure of Miyake's method, which is used to estimate all noise-free received signal and noise variance.

In Miyake's method, the estimate $\hat{y}_l(t)$ of y_l at time t is calculated by the following equation.

$$\hat{y}_l(t) = \begin{cases} \frac{r_l(t)}{r_l(t)+1}\hat{y}_l(t-1) + \frac{1}{r_l(t)+1}y(t) & (l = L(X(t))) \\ \hat{y}_l(t-1) & (l \neq L(X(t))) \end{cases} \tag{14}$$

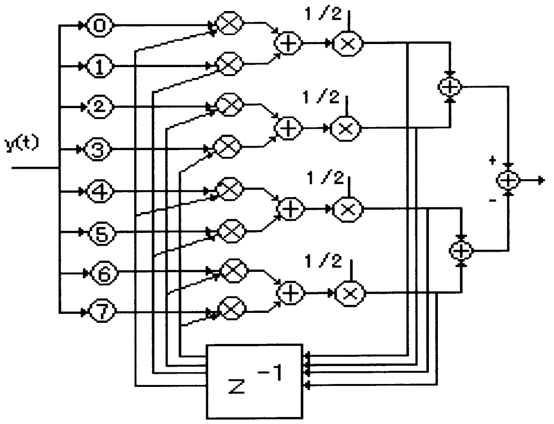

Figure 2: The Recurrent RBF Network with N=2,d=0. In this figure, the circle ① denotes nonlinear element $\exp\left\{-\frac{(y(t)-y_l)^2}{2\sigma^2}\right\}$

where $r_l(t)$ denotes the training number of $\hat{y}_l(t)$, and the estimate $\hat{\sigma}(t)$ of noise variance is calculate with the following equation

$$\hat{\sigma}^2(t) = \sum_{l=0}^{2^{N+1}-1} e_l(t) \tag{15}$$

where $e_i(t)$ is calculated as follows

$$e_l(t) = \begin{cases} \frac{r_l(t)}{r_l(t)+1} e_l(t) + \frac{(y(t)-\hat{y}_l(t))^2}{r_l+1} & (l = L(X(t))) \\ e_l(t-1) & (l \neq L(X(t))) \end{cases} \tag{16}$$

Finally, $r_l(t)$ is updated by the following equation.

$$r_l(t) = \begin{cases} r_l(t-1)+1 & (l = L(X(t))) \\ r_l(t-1) & (l \neq L(X(t))) \end{cases} \tag{17}$$

4. PROPOSED METHOD

4.1. The drawback of conventional method

The recurrent RBF network has an advantage that it can be designed with smaller memory than that of RBF network and has higher performance than that of RBF network. However, Cid-Sueiro's method has the drawback that is is difficult to design optimally recurrent RBF network when the channel is nonlinear system. While Miyake's method has the feature that it can get the received signal vectors of noise-free without estimating the characteristics of the channel and that can design the optimal RBF network, is has the drawbacks that the larger the size of the received signal vector become, the more the capacity of the memory grow up exponentially, in order to improve the performance of the RBF network.

In this paper, we discuss the method, in which the merits of above two algorithms are introduce, in order to design recurrent RBF network optimaly.

1.Initialization:
$g_j(0) = \begin{cases} 1 & ,j=0 \\ 0 & ,1 \leq j \leq 2^N-1 \end{cases}$
$y_l(0) = 0, (l = 0, 1, \cdots, 2^{N+1}-1)$
$e_l(0) = 0, (l = 0, 1, \cdots, 2^{N+1}-1)$
$r_l(0) = 0, (l = 0, 1, \cdots, 2^{N+1}-1)$
2.In the training period, $\hat{y}_l(t), e_l(t), \hat{\sigma^2}(t), r_l(t)$ are calculated by eq.(14) $\sim$ eq.(17).
3.Estimating the transmitted signal $x(t-d)$ by using eq.(10) $\sim$ eq.(13).

Table 1: The proposed method to train the recurrent RBF Network

4.2. Proposed method

Here, we propose the training method of recurrent RBF network for nonlinear channel.

The Cid-Sueiro's method to design the recurrent RBF network is not available on nonlinear channel because that method can be used on the assumption that channel is linear. However, it is known that the recurrent RBF network can be designed by obtaining all noise-free received signals and the noise variance if the characteristic of channel is not known. On the other hands, the method to estimate the noise free received signals is proposed, it is Miyake's method. Thus, using the estimate of received signals that is estimated with Miyake's method, we can design the recurrent RBF network for nonlinear channel.

On proposed method, firstly, the received signals of noise free are estimated with Miyake's method in Section 3.3. Then, the recurrent RBF equalizer is designed with the result above.

The flows of our method is shown in table.1.

5. SIMULATION RESULTS

In this section, we compare the proposed method with Miyake's RBF network with respect to both the convergence speed and the convergence accuracy.

Fig.3 shows the simulation result under the following condition.

1. The size of received signal vector, which is inputted to Miyake's RBF equalizer, are equal to 2 or 6. The reasons why we select this condtion are as follows:
   (a) The Miyake's equalizer that the size of received signal vector equal to 2 has the highest convergence speed of all Miyake's equalizer.
   (b) The Miyake's equalizer that the size of received signal vector equal to 6 has the highest convergence accuracy of all practical Miyake's equalizer.
2. The relation $f()$ between a transmitted signal vector $X(t)$ and noise-free received signal$o(t)$.

$$f(X(t)) = H(X(t)) + 0.2(H(X(t)))^2 \tag{18}$$

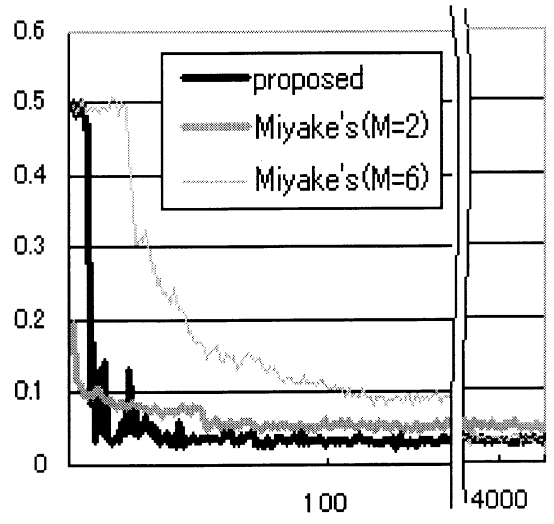

Figure 3: The result of Simulation

where

$$H(X(t)) = 0.3482x(t) + 0.8704x(t-1) + 0.3482x(t-2) \quad (19)$$

3. $e(t)$: white Gaussian noise that SNR equals to 10dB
4. Delay:$d = 1$
5. The performance of the proposed is evaluated by bit error rate(BER).

Fig.3 shows that the network trained with our method has the convergence speed as well as Miyake's equalizer with $M = 2$, and BER of the network trained our method is smaller than that of Miyake's RBF equalizer. These results show that the proposed method is useful.

Additionally, Fig.3 shows that the recurrent RBF network trained with our method has the performance as well as the Miyake's RBF equalizer with $M = 6$. However, the network trained our method has the higher convergence speed than the Miyake's equalizer with $M = 6$. Thus, this result shows that the proposed equalizer is useful.

In these results, the usefulness of proposed method is shown.

6. CONCLUSIONS

In this paper, we proposed the method that is used to train recurrent RBF network for nonlinear channel. We verify the usefulness of the proposed method through the computer simulation.

In the future, we would like to research the recurrent RBF network for nonlinear channel of which characterization is time-variant.

[Acknowledgement]
We acknowledge with thank our indebtedness to Prof Nozomu Hamada, Keio University, who helped produce this paper.

7. REFERENCES

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