A COMPLEX ADAPTIVE DELAY FILTER

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ABSTRACT

In this paper we propose an adaptive delay filter with complex coefficients for identifying an unknown system with complex sparse impulse response. The delay taps can not be determined by the conventional real adaptive delay filter, which evaluates the mean squared error with one constant gain in the system with complex coefficients. In the proposed method, a modified evaluation function, which consists of the mean squared error value with a complex constant gain and one with the conjugate value of the complex constant gain, is introduced in order to estimate the delay taps correctly. Using simulations we also clarify that the identification error is significantly reduced by means of the proposed complex adaptive delay filter.

1. INTRODUCTION

In some system identification problems, such as in multipath equalization and echo canceling, we encounter many unknown systems whose impulse response is sparse, i.e., many of the coefficients are zero. When the standard adaptive filter identifies such a sparse impulse response system, a large number of filter taps might be required resulting in misadjustment in identification and significant increases in computational requirements. In order to reduce the misadjustment and computational requirements, an adaptive filter with variable delay taps and gains, that is, an adaptive delay filter, has been proposed [1]-[3]. In the adaptive delay filter, it is obvious that accuracy in the estimation of the number of delay taps strongly effects the identification. Cheng and Etter have analyzed the mean squared error surfaces and proposed an algorithm for the processing of real signals that sequentially estimates the delay taps and the corresponding gains [1].

However, in some application such as echo canceling in radar or communication systems, the unknown system has a complex impulse response and the input and output are complex signals. Therefore, Cheng and Etter's algorithm (the conventional algorithm mentioned above) can never be applied to systems with complex coefficients because the algorithm requires the impulse response and signals to be real.

To overcome this problem, we propose a modified evaluation function, which consists of the mean squared error value with a complex constant gain and one with the conjugate value of the complex constant gain, in order to determine the delay taps. We demonstrate the effectiveness of the proposed adaptive delay filter with complex coefficients with a computer simulation.

2. REAL ADAPTIVE DELAY FILTER

Figure 1 shows a block diagram of the conventional adaptive delay filter with K pairs of the delay z^{-d_k} and the corresponding gain g_k $\left(k=1,2,\cdots,K\right)$ [1]. The estimation process at each stage in the conventional adaptive delay filter consists of two steps. The first is the estimation of the delay in the sparse system. The second is the adaptive control of their corresponding gain. During the process of estimating the delay d_k, the corresponding gain value g_k has to be held constant. Here, this constant gain value is denoted as g_c. Let the unknown system be a finite impulse

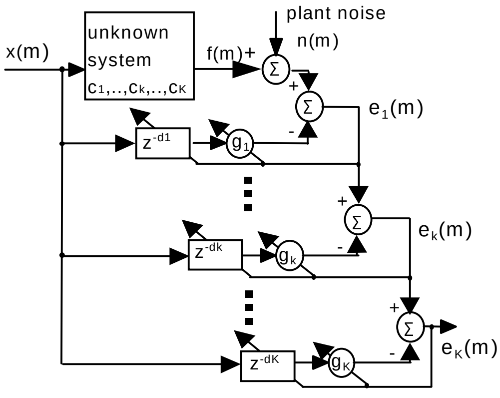

Fig. 1 Adaptive delay filter with serial structure [1].

response (FIR) system having a set of coefficients $\left(c_1,\cdots,c_k\right)$. The output of the unknown system $f\left(m\right)$ can be written as:

$$f(m) = \sum_{i=1}^{K} c_i x(m - d_i) \qquad (1)$$

where $x(m)$ is the input signal, d_i is the delay with the coefficient c_i and m is the sample number.

When the input signal $x(m)$ is real, the evaluation function $\hat{\Phi}_k(d)$, to determine the delay d_k is expressed as follows [1]:

$$\begin{aligned}
\hat{\Phi}_k(d) &= \frac{1}{N}\sum_{m=0}^{N-1}\left[\left|e_k(m)\right|^2\right] \\
&= \frac{1}{N}\sum_{m=0}^{N-1}\left[e_{k-1}(m) - g_c x(m-d)\right]^2 \\
&= \sum_{i=1}^{K} c_i^2 r_{xx}(0) \\
&\quad + 2\sum_{1 \le i < j \le K} c_i c_j r_{xx}(d_i - d_j) \\
&\quad + g_c^2\left[\frac{1}{N}\sum_{m=0}^{N-1} x^2(m-d)\right] \\
&\quad - 2 g_c \sum_{i=1}^{K} c_i r_{xx}(d - d_i)
\end{aligned} \qquad (2)$$

where d is the variable covering a range from 0 to some predetermined maximum delay, $e_k(m)$ is the error signal of the k th stage, N is the number of samples used in calculating the mean squared error and r_{xx} is the autocorrelation function of the input signal $x(m)$.

Since the first and third terms of (2) are constant, the second term is real, and $r_{xx}(d - d_i)$ is the autocorrelation function. The expectation of the mean squared error has a maximum value at $d = d_i$. We can

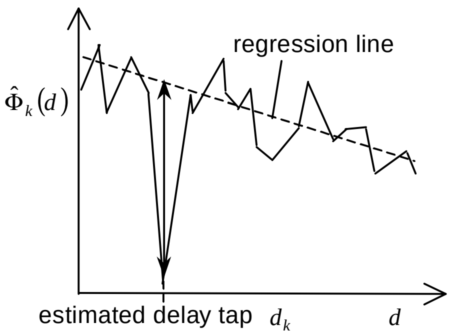

Fig.2. Evaluation function $\hat{\Phi}_k(d)$

determine the delay d_k by searching for the largest deviation between the mean squared error value and its simple regression line. Figure 2 shows an example of the evaluation function $\hat{\Phi}_k(d)$ and its regression line.

If the input signal and the impulse response of the unknown system are complex, $\hat{\Phi}_k(d)$ is expressed as:

$$\begin{aligned}
\hat{\Phi}_k(d) &= \frac{1}{N}\sum_{m=0}^{N-1}\left[\left|e_k(m)\right|^2\right] \\
&= \sum_{i=1}^{K}\sum_{j=1}^{K} c_i^* c_j r_{xx}(d_i - d_j) \\
&\quad + \left|g_c\right|^2 \frac{1}{N}\sum_{m=0}^{N-1}\left|x(m-d)\right|^2 \\
&\quad - 2\,\mathrm{Re}\left[g_c \sum_{i=1}^{K} c_i^* r_{xx}(d - d_i)\right]
\end{aligned} \qquad (3)$$

where the asterisk denotes complex conjugate. The evaluation function $\hat{\Phi}_k(d)$ in the conventional algorithm might not have the largest deviation at $d = d_i$. This is because the third term of (3) becomes zero when the product of the fixed gain and the conjugate coefficient, $g_c c_i^*$, is a purely imaginary number, so that the third term can not have a maximum value at $d = d_i$. Therefore, when the unknown system is complex, the delay d_k might not be determined by the conventional algorithm.

3. COMPLEX ADAPTIVE DELAY FILTER

In this section, we propose a novel algorithm for a complex adaptive delay filter in order to identify a system with complex impulse response. A modified evaluation function is introduced using a constant complex gain value g_c and its conjugate value g_c^* so as to estimate the delay d_k correctly even if the impulse response of the system is complex. We propose the modified evaluation function $\hat{\Phi}'_k(d)$ as follows:

$$\hat{\Phi}'_k(d) = \hat{\Phi}_{g_k}(d) + \hat{\Phi}_{g_k^*}(d) \qquad (4)$$

$$\hat{\Phi}_{g_k}(d) = \frac{1}{N'}\sum_{m=0}^{N'-1}\left[\left|e_{k-1}(m) - g_c x(m-d)\right|^2\right] \qquad (5)$$

$$\hat{\Phi}_{g_k^*}(d) = \frac{1}{(N - N')}\sum_{m=N'}^{N-1}\left[\left|e_{k-1}(m) - g_c^* x(m-d)\right|^2\right] \qquad (6)$$

where N' and $N - N'$ (> 0) are the number of samples used for calculating the mean squared error function. Figure 3 shows a processing diagram of the proposed delay estimation technique of the k th stage for the

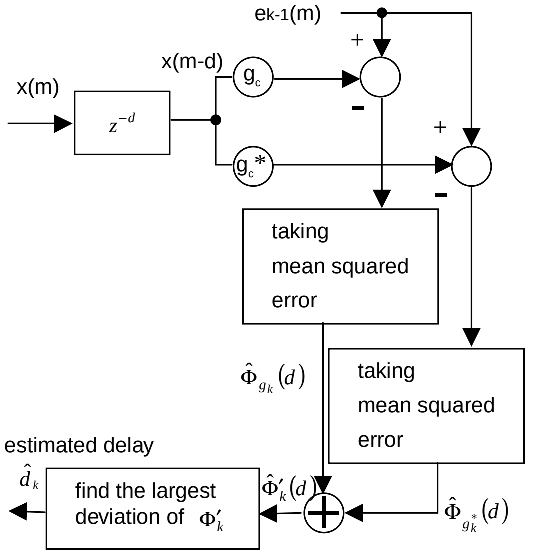

Fig.3. Processing diagram for delay estimation in the proposed technique

system with complex coefficients. Equation (4) can be rewritten as follows:

$$\begin{aligned}\hat{\Phi}'_k(d) &= \sum_{i=1}^{K}\sum_{j=1}^{K} c_i^* c_j r_{xx}(d_i - d_j) \\ &+ |g_c|^2 \frac{1}{N'}\sum_{m=0}^{N'-1}|x(m-d)|^2 \\ &+ \sum_{i=1}^{K}\sum_{j=1}^{K} c_i^* c_j r'_{xx}(d_i - d_j) \\ &+ |g_c^*|^2 \frac{1}{(N-N')}\sum_{m=N}^{N-1}|x(m-d)|^2 \\ &- \mathrm{Re}\left[g_c \sum_{i=1}^{K} c_i^* r_{xx}(d-d_i)\right] \\ &- \mathrm{Re}\left[g_c^* \sum_{i=1}^{K} c_i^* r'_{xx}(d-d_i)\right]\end{aligned} \qquad (7)$$

where r'_{xx} denotes the approximated autocorrelation function using $(N-N')$ samples of the input signal $x(m)$. In (7), even if the product $g_c c_i^*$ is a purely imaginary number or the fifth term becomes zero, the sixth term has a maximum value at $d = d_i$. Therefore, when the input signal and the impulse response of the unknown system are complex, we can correctly estimate the delay d_k by searching the maximum deviation in the modified evaluation function $\hat{\Phi}'_k(d)$.

After determining the delay d_k in the k th stage, the corresponding gain value g_k can be estimated by means of a normalized least mean squared (NLMS) algorithm [4] as follows:

$$\begin{aligned} g_k(m+1) &= g_k(m) \\ &+ \frac{\alpha}{\beta + |x(m-d_k)|^2} x^*(m-d_k) e_{k-1}(m) \end{aligned} \qquad (8)$$

$$e_k(m) = e_{k-1}(m) - x(m-d_k) g_k(m) \qquad (9)$$

where α and β denote step size parameters. By means of the adaptation described above for all the stages sequentially, we can identify the unknown system with a complex sparse impulse response. The adaptive algorithm adopting the proposed delay estimation technique is summarized below.

I. Set $k = 1$.
II. Set some complex constant gain g_c in place of g_k. Then form g_c^*.
III. Calculate the modified evaluation function $\hat{\Phi}'_k(d)$ using eq. (4).
IV. Determine the delay d_k by searching for the maximum deviation between the evaluation function $\hat{\Phi}'_k(d)$ and its regression line.
V. Compute gain g_k by means of NLMS algorithm shown in eq. (8).
VI. Calculate the error signal $e_k(m)$ given by eq. (9).
VII. If $k < K$ then $k = k+1$, and go to II. Otherwise, end.

4. SIMULATION RESULTS

We demonstrate the effectiveness of the proposed complex adaptive delay filter by computer simulation. We assume that the unknown system has the following transfer function $H(z)$.

$$\begin{aligned} H(z) &= (1-j)z^{-9} + 2.5z^{-10} \\ &+ (1.05 - j1.5)z^{-51} - j2z^{-71} \end{aligned} \qquad (10)$$

This means that $c_1 = 1 - j, c_2 = 2.5, c_3 = 1.05 - j1.5,$ $c_4 = -j2$ and $d_1 = 9, d_2 = 10, d_3 = 51, d_4 = 71$. Table 1 shows the true delays and impulse responses of the complex sparse unknown system. In this simulation model, the input signal is complex Gaussian white noise with zero mean and variance one for both the real and imaginary part of the signal. A plant white noise at 0.5 power is added to the output of the system. One hundred samples are used to determine the delays d_k $(N = 100)$. In the proposed method, the number of samples used for calculating $\Phi_{g_k}(d)$ and $\Phi^*_{g_k}(d)$ are 60 and 40, respectively $(N' = 60)$. Three hundred samples are used for estimating the gain value g_k . In this simulation the constant gain value g_c is set to $2 - j1.4$, the range for searching the delay from 0 to 80 and $\alpha = 0.025, \beta = 0.05$. It should be noted that the product $g_c c_3^*$ is equal to a purely imaginary value $((2 - j1.4) \times (1.05 - j1.5) = -j4.47)$.

Table 1 The delays and coefficients
of the unknown system

i	delay d	coefficient c
1	9	1-j
2	10	2.5
3	51	1.05-j1.5
4	71	-j2

Table 2 shows the estimated delays and gains. From Table 2, we find that the misestimation occurs in the conventional technique at not only $d = 51$, in which the product $g_c c_3^*$ is a purely imaginary value, but also at other delay points. The estimation of the gain is also incorrect due to the misestimation of delays. On the other hand, the proposed complex adaptive delay filter can estimate the delays correctly; i. e. $\hat{d} = d$.

Table 2 The estimated delays and gains

	proposed		conventional	
k	delay $\hat{d}$	gain $\hat{g}$	delay $\hat{d}$	gain $\hat{g}$
1	10	2.58+j0.33	50	0.61-j0.68
2	51	0.59-j1.48	59	0.75-j0.26
3	9	0.09-j0.09	19	-0.10-j0.15
4	9	1.00-j0.61	11	-0.59+j0.75
5	71	-0.01-j1.10	33	0.15+j0.27

Figure 4 illustrates the mean squared errors between the output signal of the unknown system and that of the adaptive delay filter for one hundred independent trials. From Fig. 4, we observed that the mean squared error of the proposed complex adaptive filter converges to one, which is equal to the noise power. On the other hand, the mean squared error of the conventional technique does not converge. This means that the proposed method is much more accurate than the conventional one for identification of a system with complex coefficients.

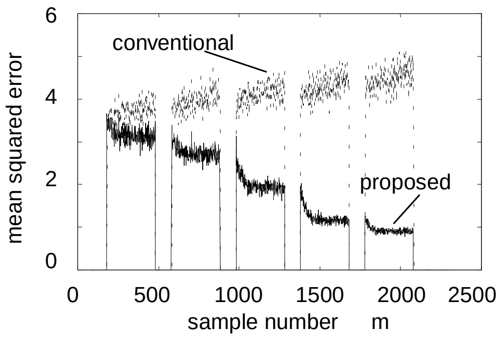

Fig. 4. The mean squared errors of the conventional and proposed complex adaptive filters

5. CONCLUSIONS

In this paper we have presented the complex adaptive delay filter in order to identify an unknown system with a complex sparse impulse response. With this technique, the delays are determined by means of the modified evaluation function, which consists of the mean squared error value with a complex constant gain and one with the conjugate of the constant gain. From the simulation results, this technique is able to identify an unknown system with complex sparse impulse response that the conventional method can not identify.

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